Linear and Nonlinear Interference

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Outline

- Introduction
- Wave superposition
- Radiation-matter interaction
  - Interference
  - Nonlinear optics
- Conclusions

Introduction

Electromagnetic waves exist in two independent polarizations and carry three quantities which can be transferred to matter:

- o Energy
- o Momentum
- o Angular momentum.

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}+\omega t+\phi)}$$

The algebra of operating with EM fields is well defined (Jackson, Born Wolf) but has some hidden tricks

Introduction

Over the years I have work both in Interferometry and Nonlinear Optics.

These are traditionally independent and different areas based on its scope.

But at the end I have found them to be basically equivalent.

Wave Superposition

$$E = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$E_1 = E_{01} e^{i(\vec{k}_1\cdot\vec{r} - \omega_1 t)} \qquad E_2 = E_{02} e^{i(\vec{k}_2\cdot\vec{r} - \omega_2 t)}$$

$$E_1 + E_2 = E_{01} e^{i(\vec{k}_1\cdot\vec{r} - \omega_1 t)} + E_{02} e^{i(\vec{k}_2\cdot\vec{r} - \omega_2 t)}$$

Wave Superposition: Interference

Every textbook follows the same steps:

$$E_1 + E_2 = E_{01} e^{i(\vec{k}_1 \cdot \vec{r} - \omega_1 t)} + E_{02} e^{i(\vec{k}_2 \cdot \vec{r} - \omega_2 t)}$$

$$I = \frac{1}{2} c \varepsilon_0 \left| E_1 + E_2 \right|^2$$

$$= \frac{1}{2} c \varepsilon_0 \left| E_{01} e^{i\vec{k}_1 \cdot \vec{r}} + E_{02} e^{i\vec{k}_2 \cdot \vec{r}} \right|^2$$

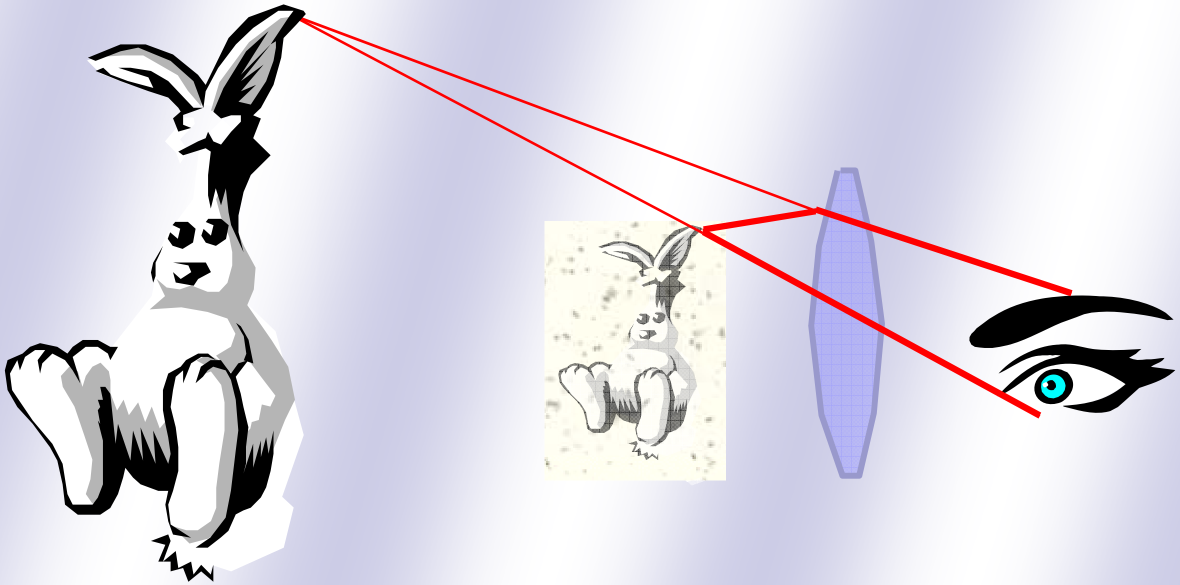
$$= \frac{1}{2} c \varepsilon_0 \left\{ \left| E_{01} \right|^2 + \left| E_{02} \right|^2 + \left| E_{01} \right| \left| E_{02} \right| \cos\left(\left(\vec{k}_1 - \vec{k}_2 \right) \cdot \right) \vec{r} \right\}$$

and so on …

Interference to the limit

Even going one step further, image formation can be considered an interference phenomena.

The best “in focus image” is achieved when the rays coming from the source interfere constructively (somehow incorporated in physical optics math)

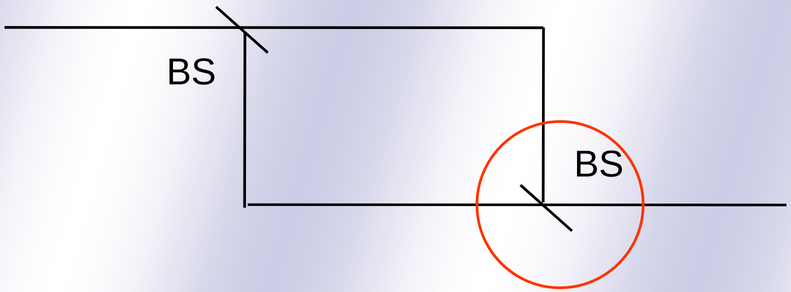


Interference

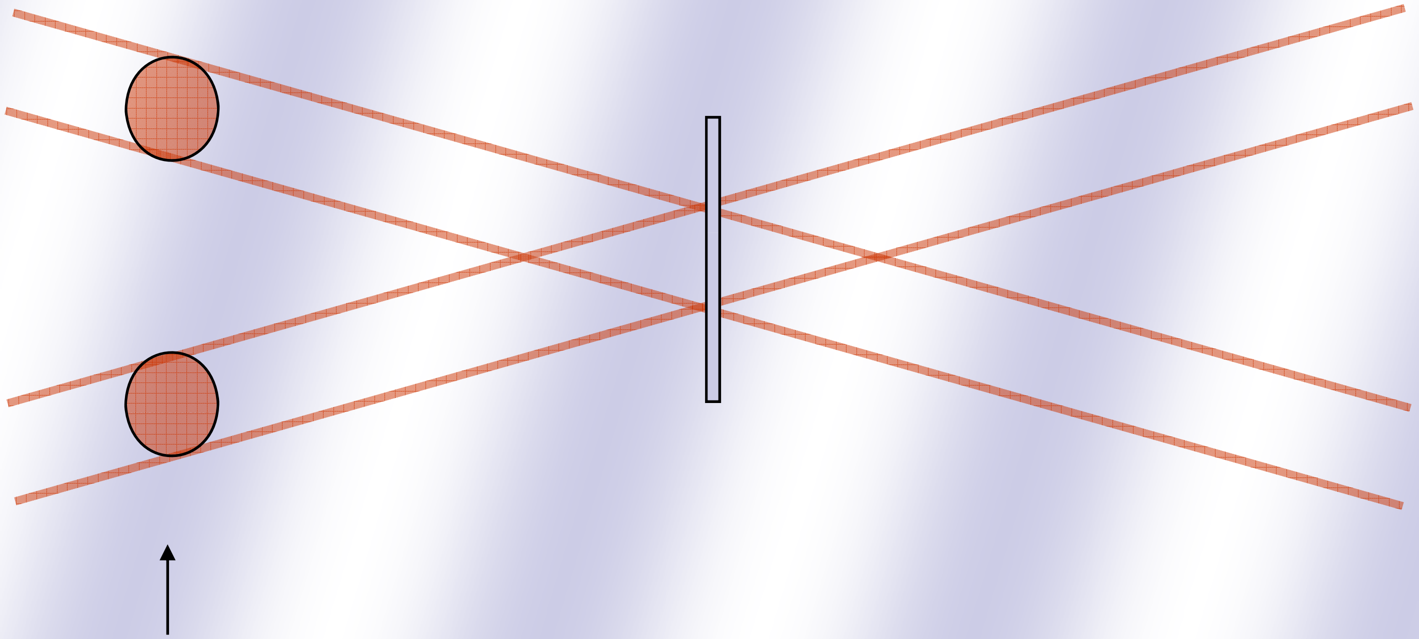
The fundamental step is the observation of the interference pattern:

$$I = \frac{1}{2} c \varepsilon_0 \left| E_1 + E_2 \right|^2$$

Lets see it in more detail on an interferometer:



Interference



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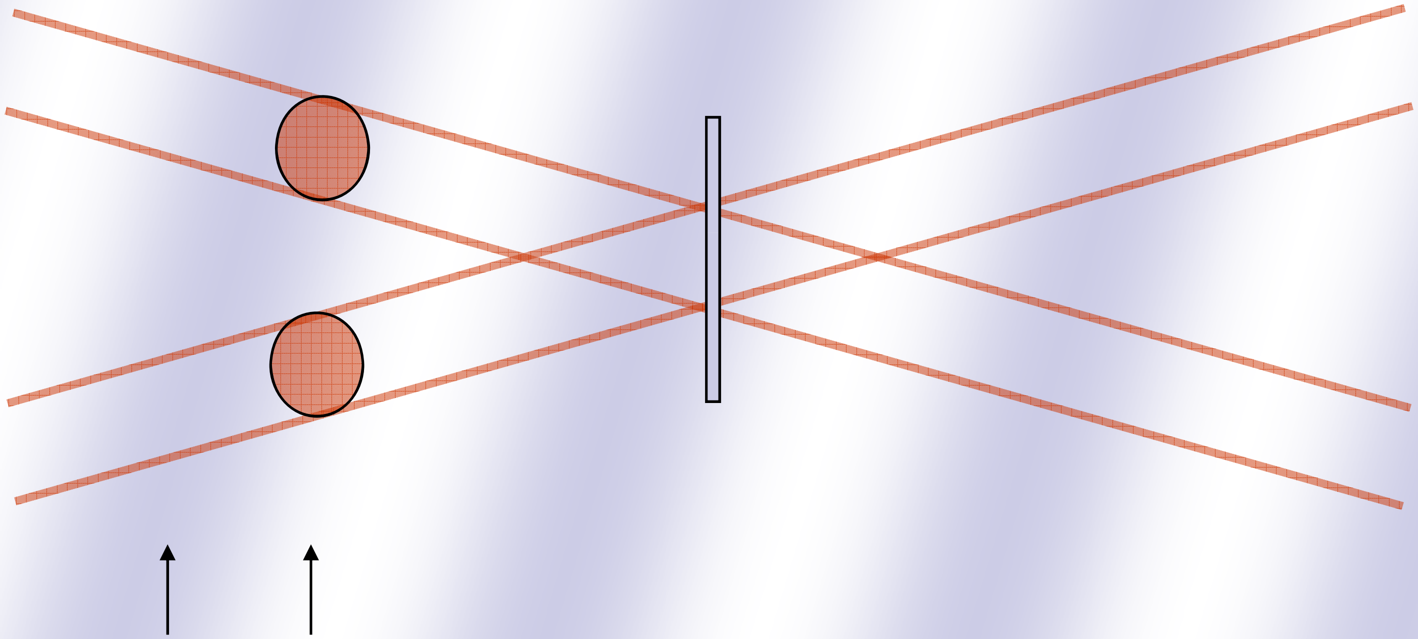

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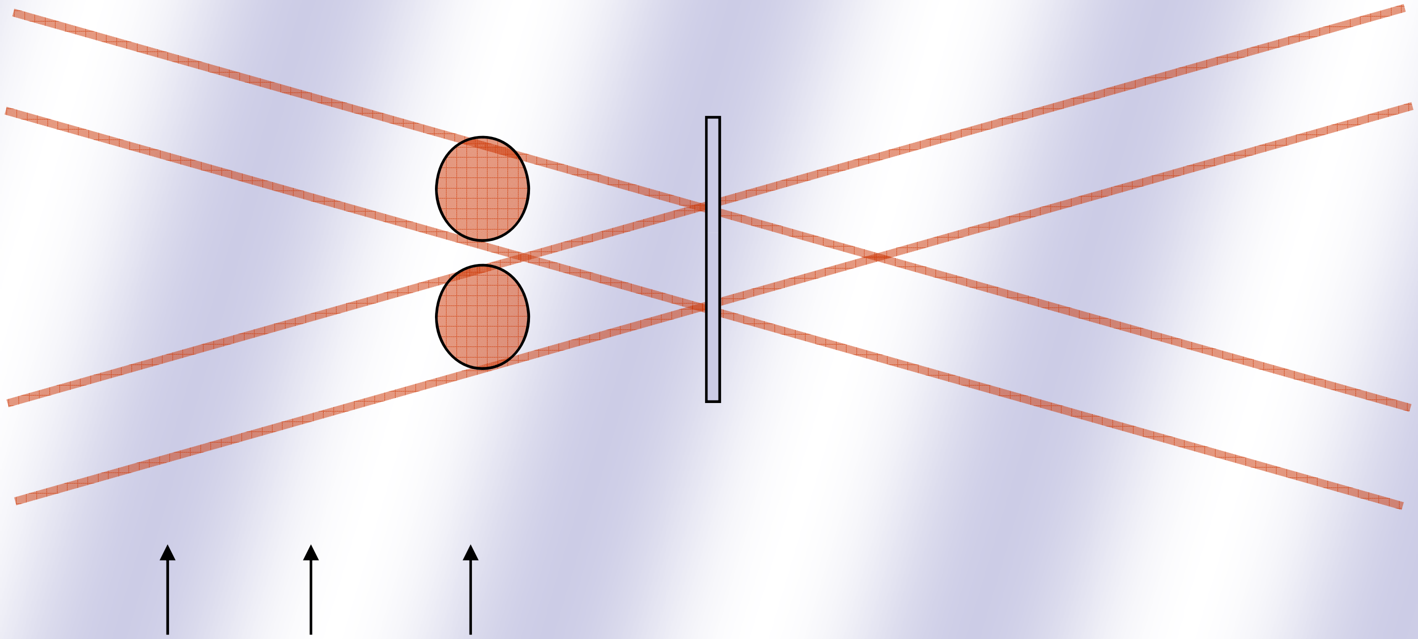

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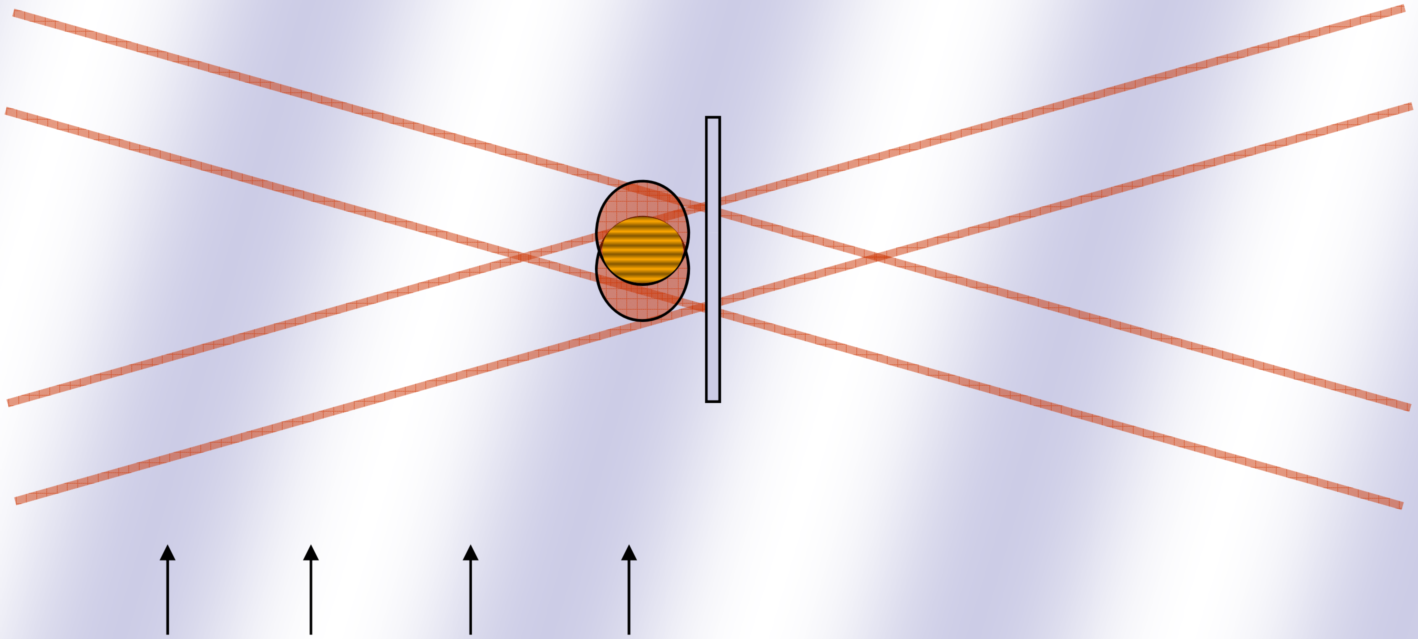

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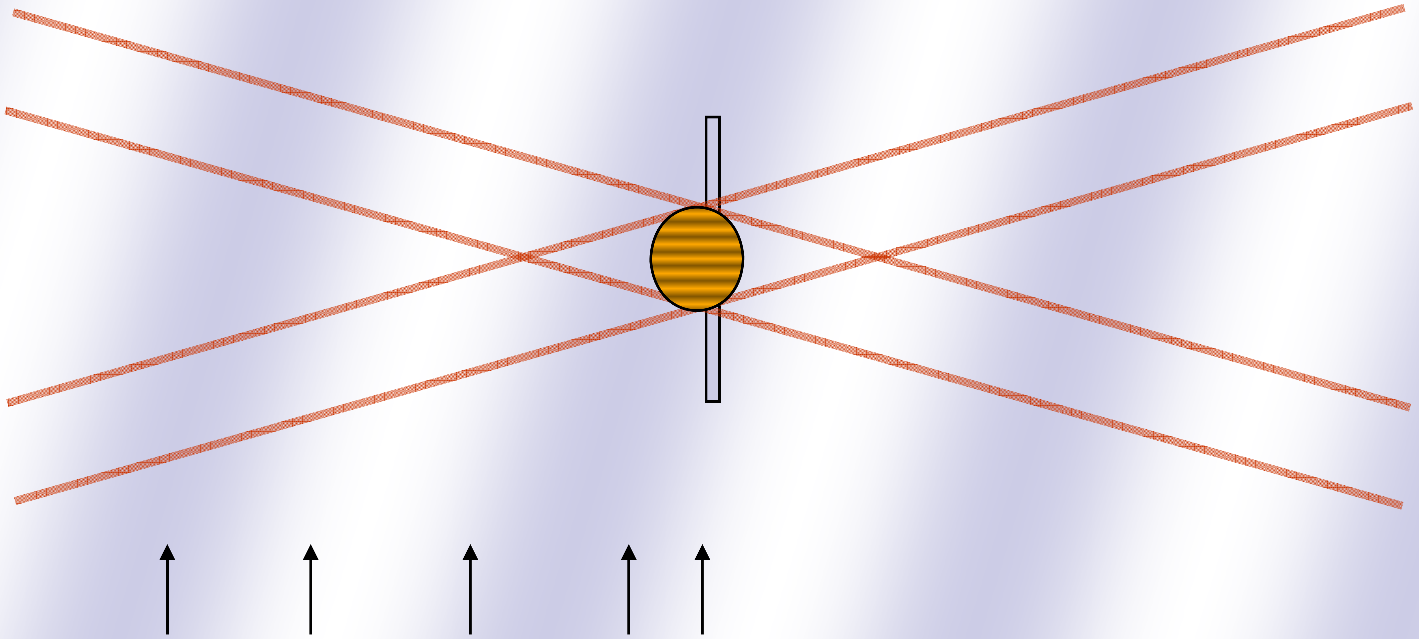

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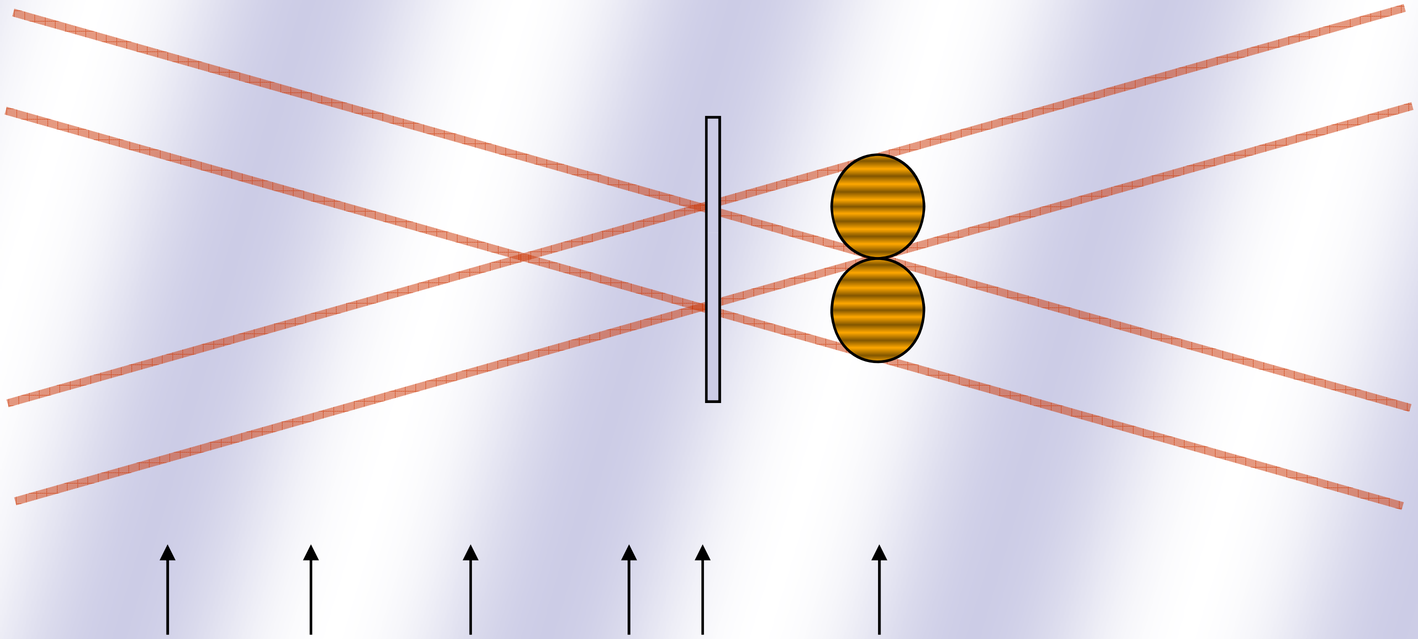

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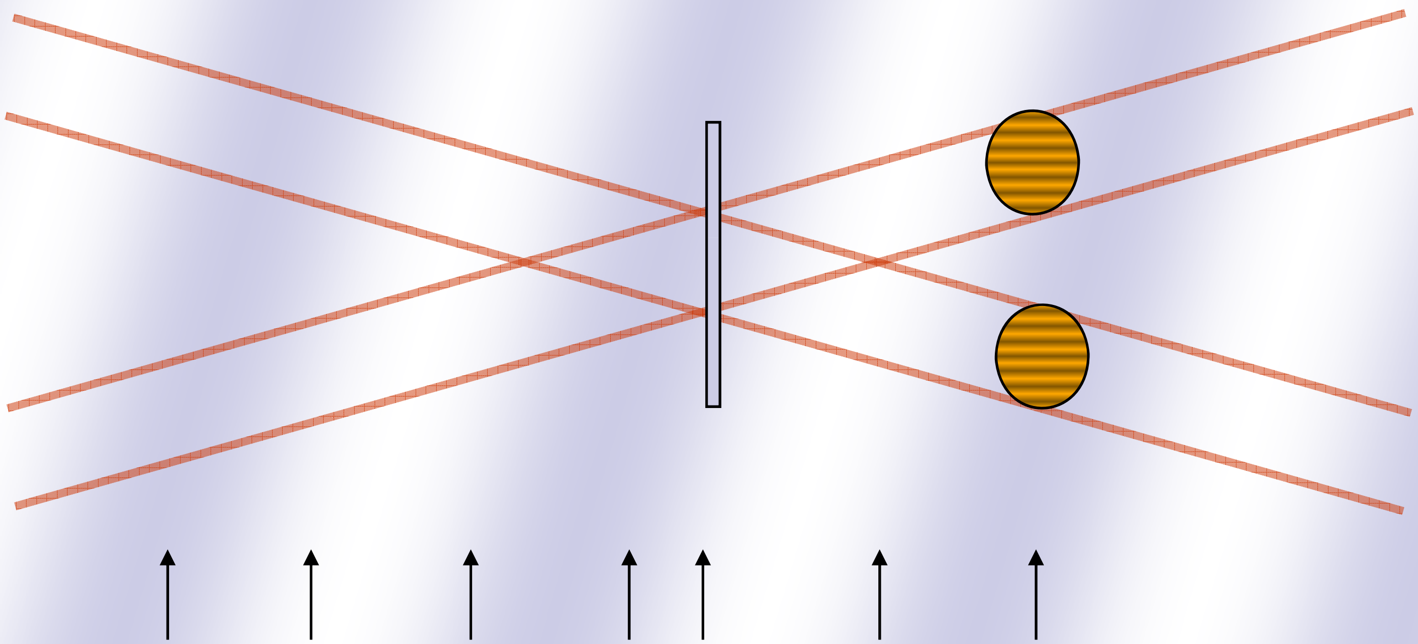

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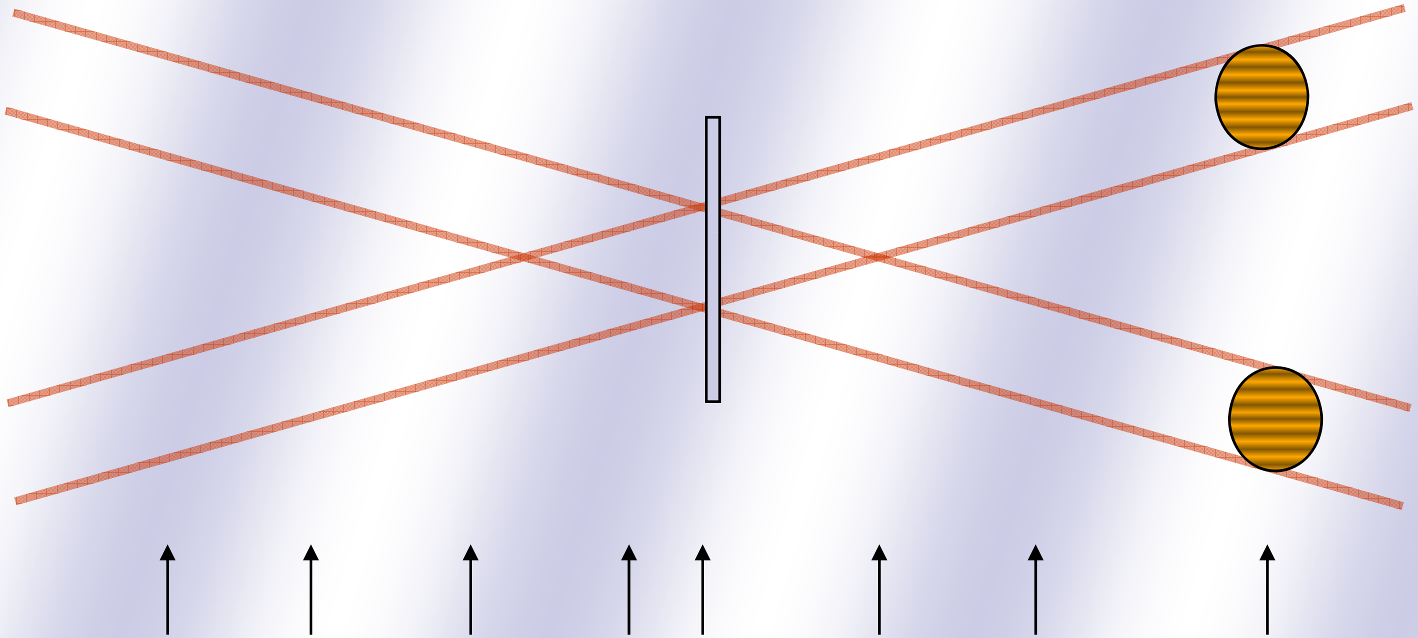


Interference

There is an effective redistribution of energy due to the matter inserted in the optical path

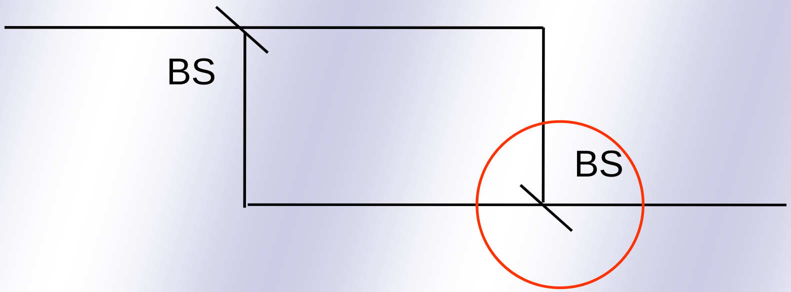
In this case the redistribution is nonlocal

Interference

The fundamental step is the observation of the interference pattern:

$$I = \frac{1}{2} c \varepsilon_0 \left| E_1 + E_2 \right|^2$$

Lets see it in more detail on an interferometer:



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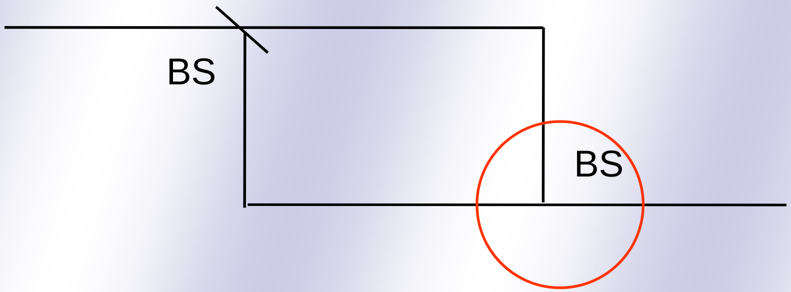

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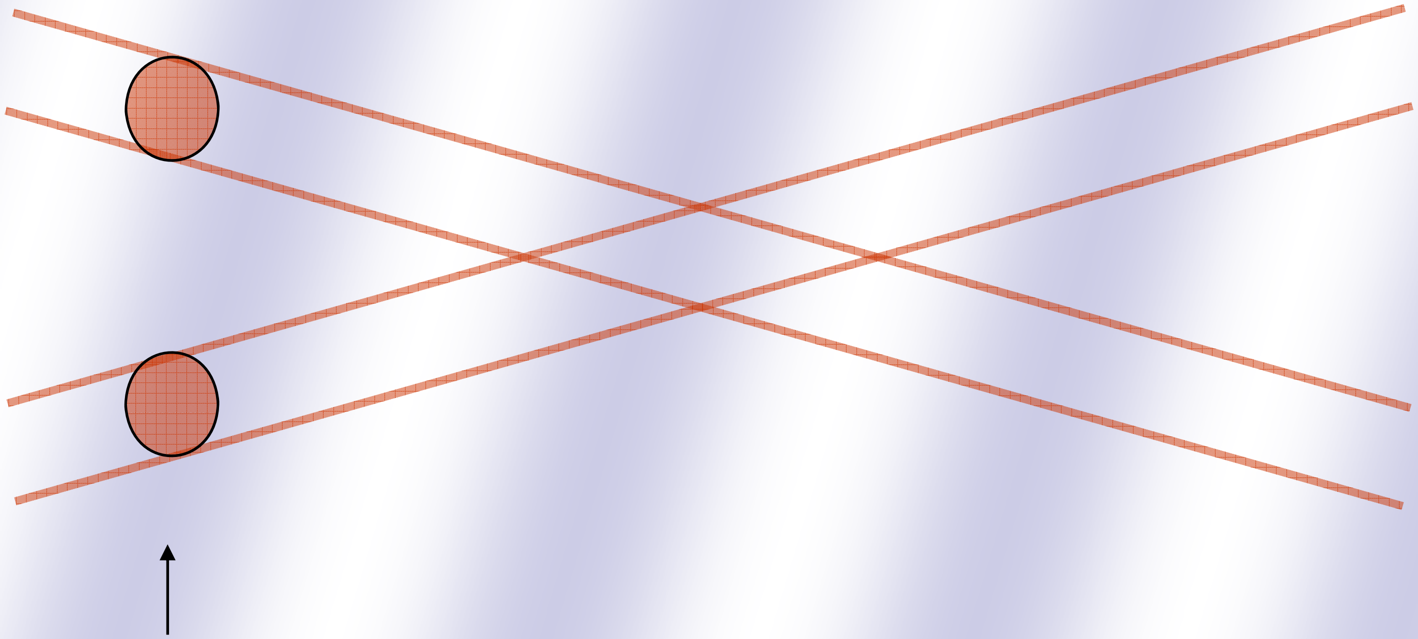

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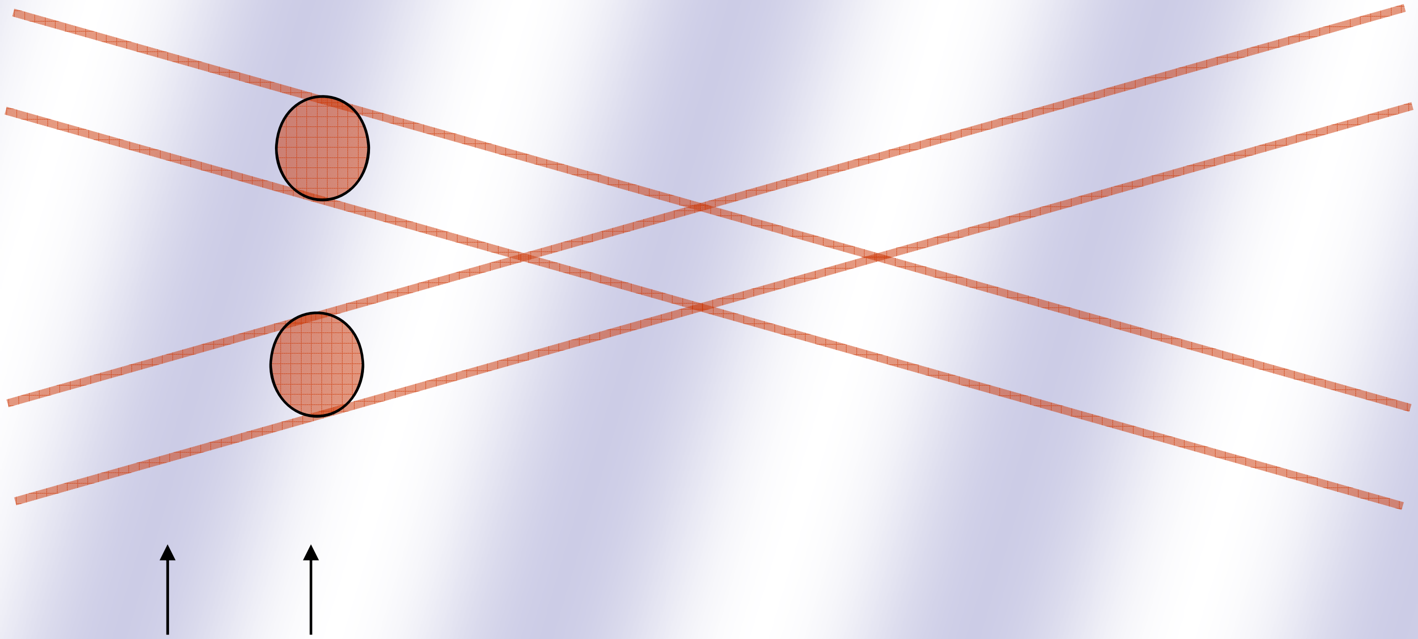

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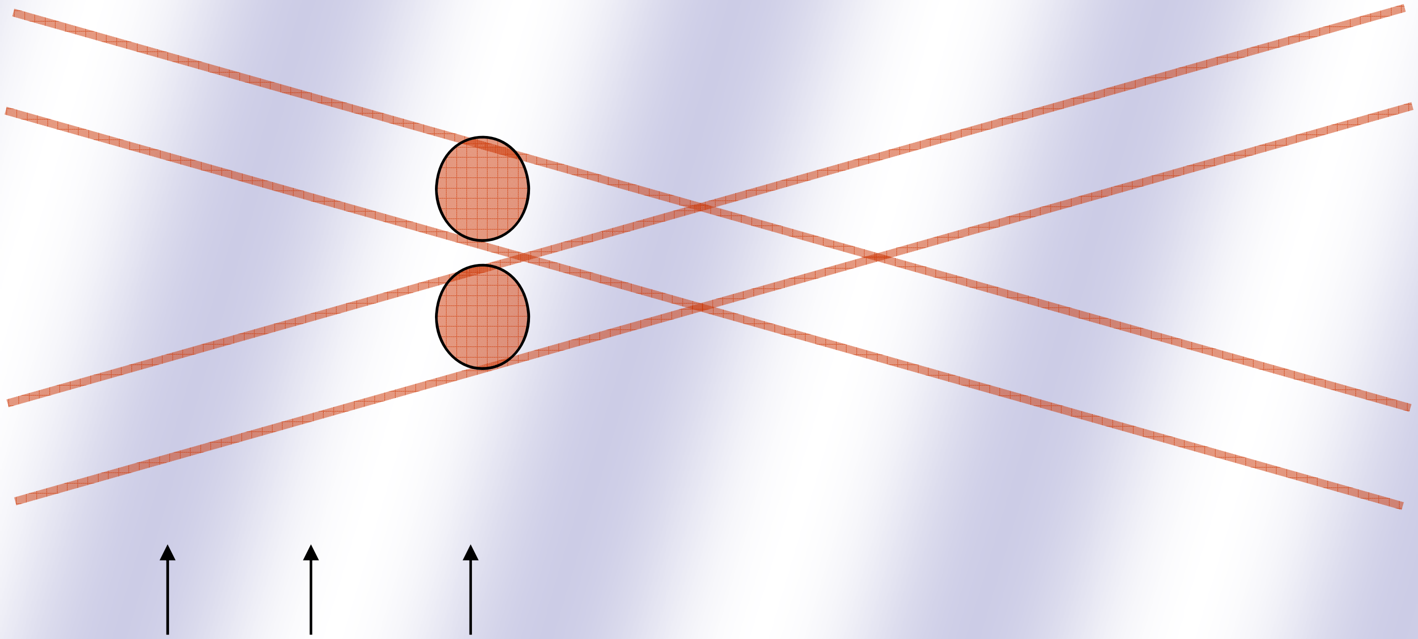

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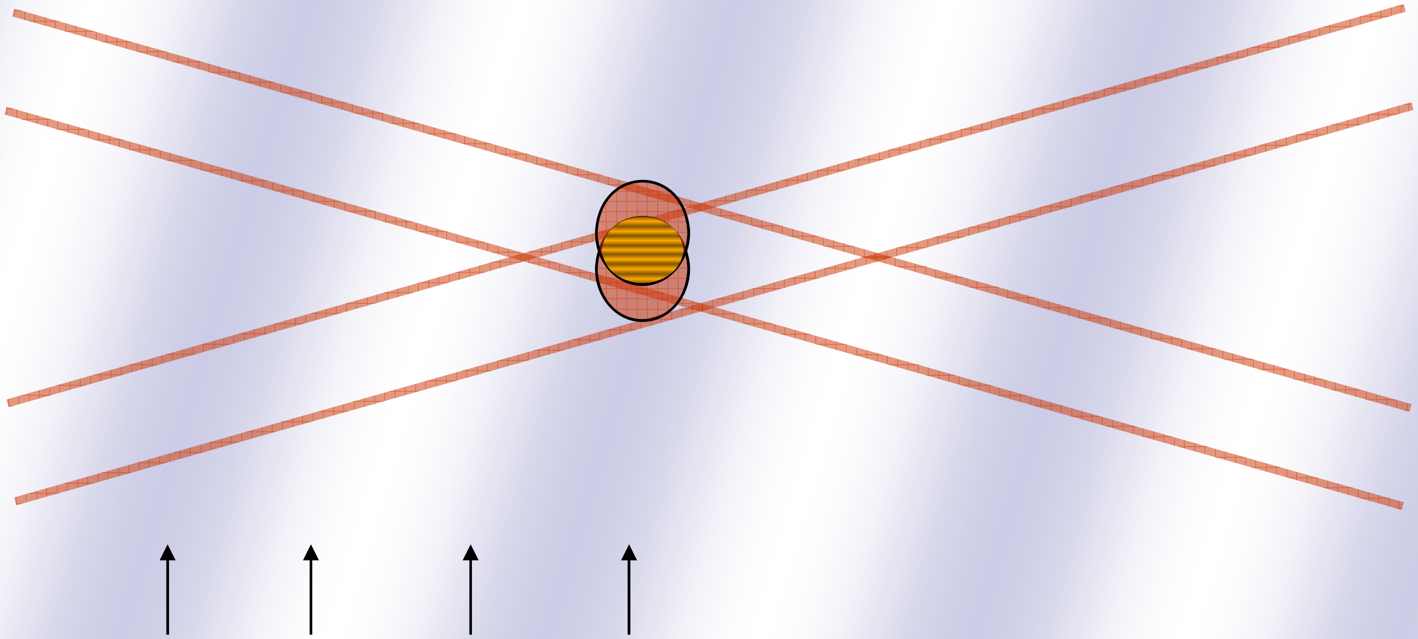

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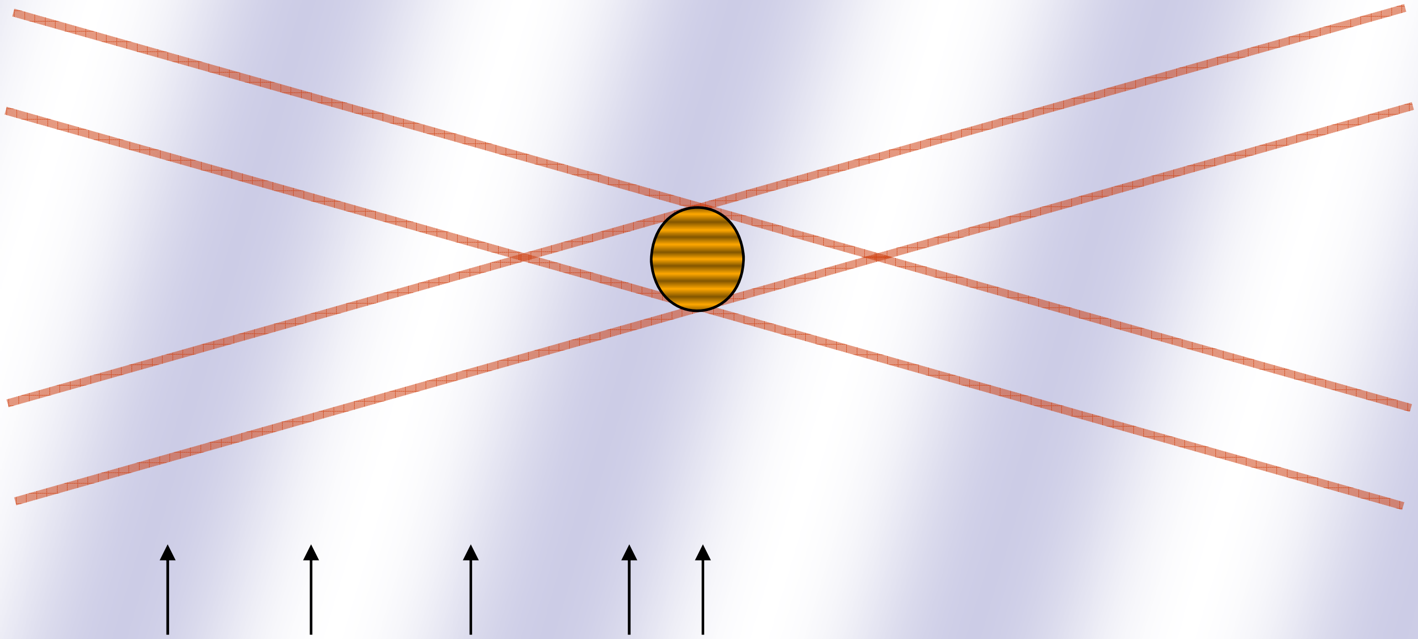

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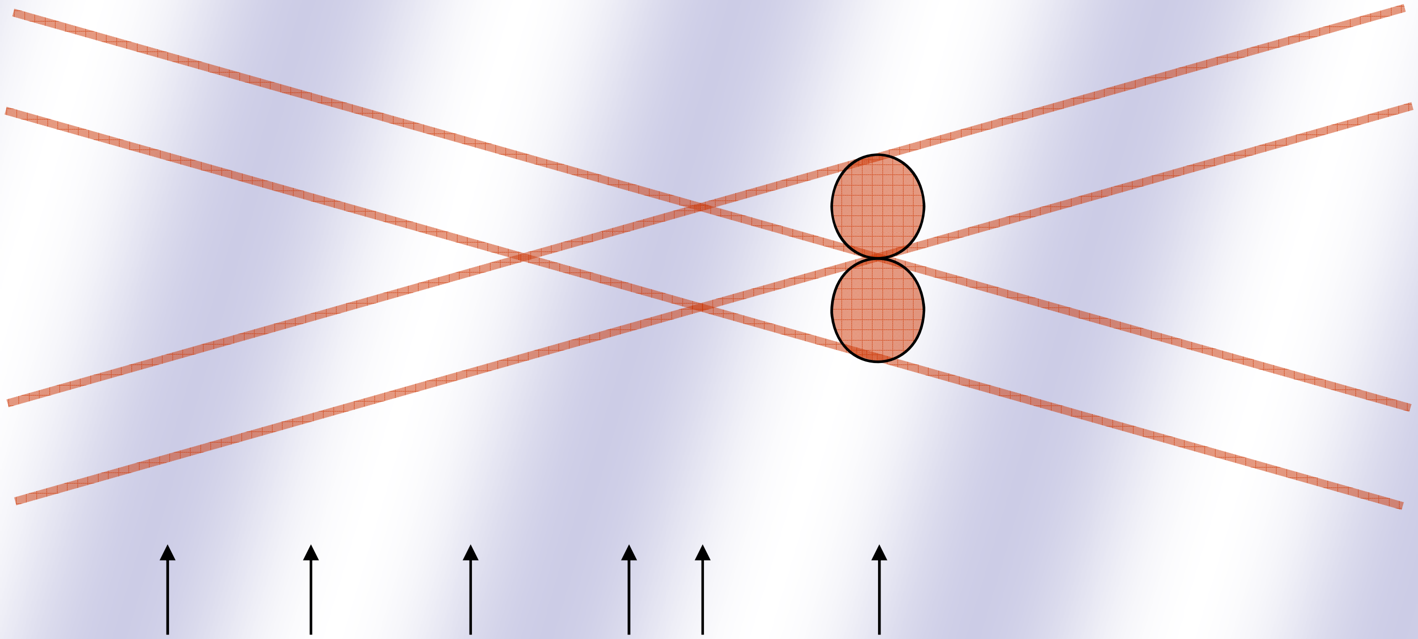

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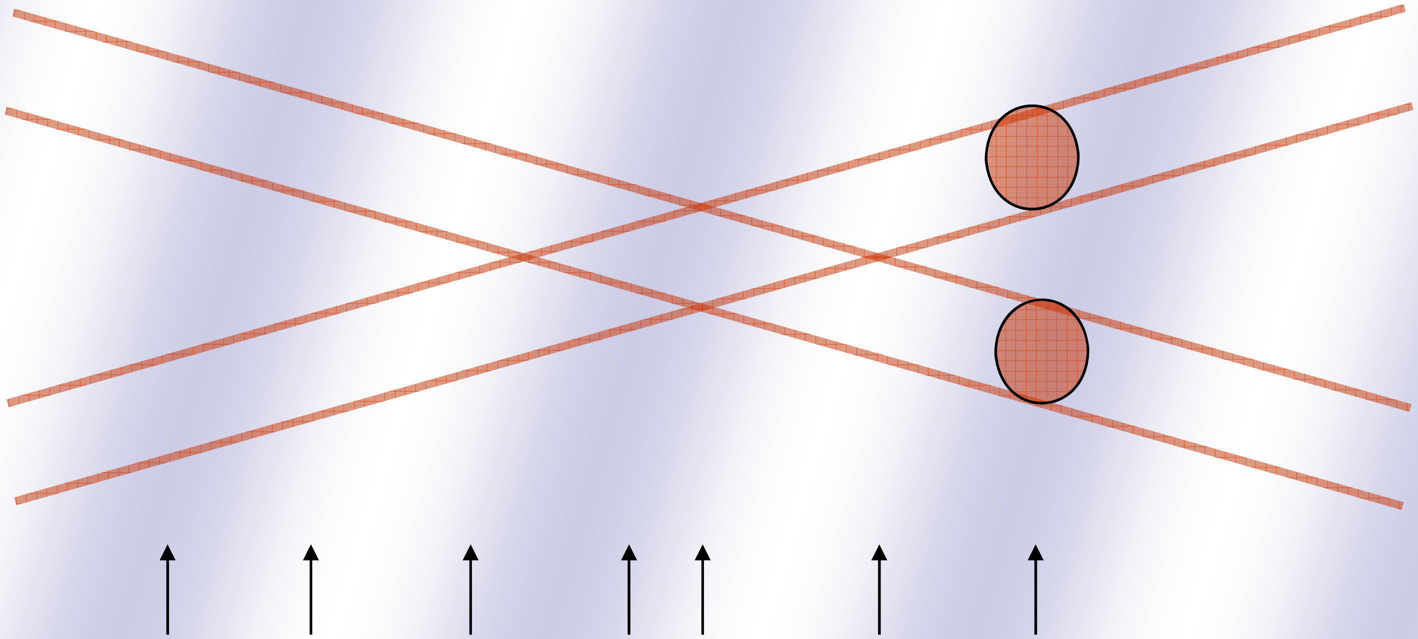

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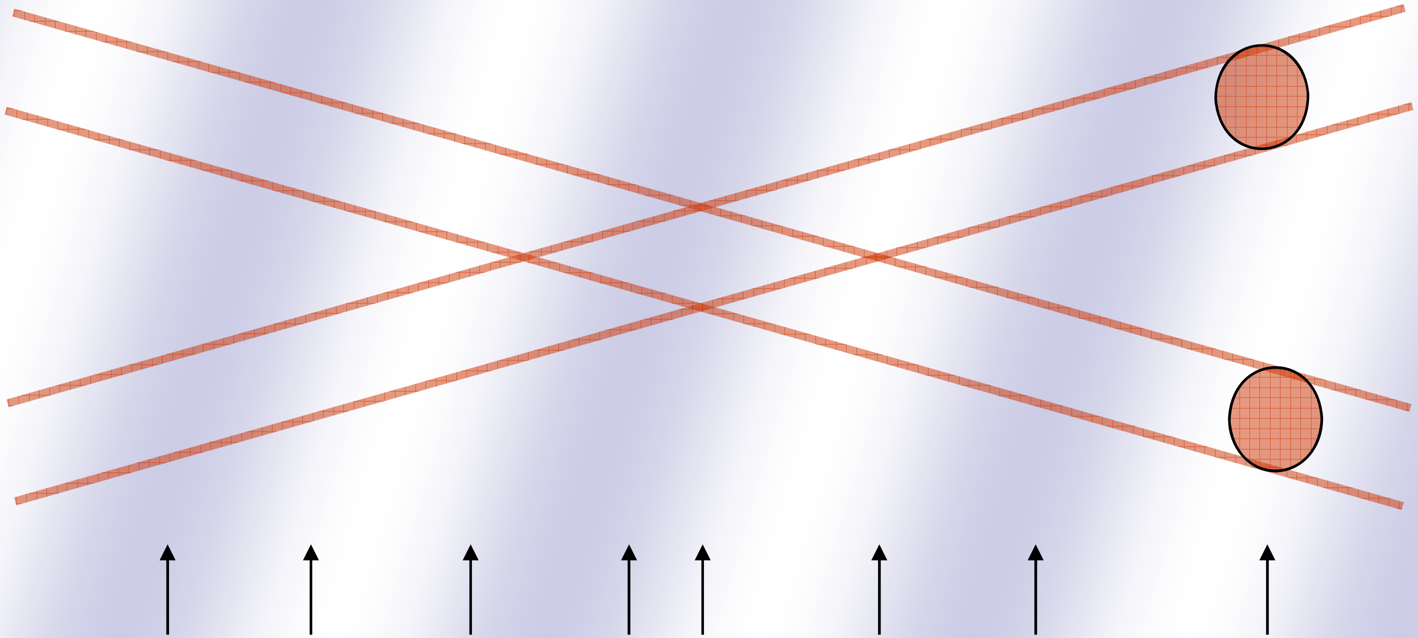


Interference

There is an effective redistribution of energy <u>only</u> on the screen inserted in the optical path

In this case the redistribution is local

Interference

Therefore there is a difference if we insert or not a combining beam-splitter

- **Electric field superposition is common to both scenarios**
- **Interference is not**

Before going any further lets discuss classical NLO

Nonlinear Optics

$$P = \varepsilon_o \overleftrightarrow{\chi} \cdot \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$
$$= \varepsilon_0 (1 + \overleftrightarrow{\chi}) \vec{E}$$

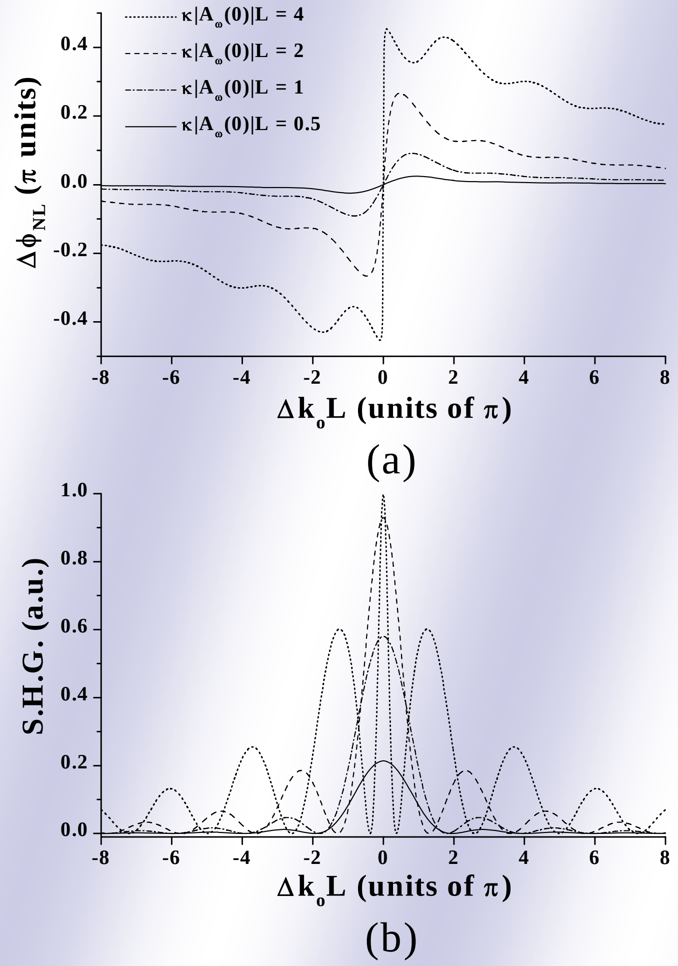
Electronic nonlinearities:

$$P = \varepsilon_o \overleftrightarrow{\chi}^{(1)} \cdot \vec{E} + \varepsilon_0 \overleftrightarrow{\chi}^{(2)} : \vec{E} + \varepsilon_0 \overleftrightarrow{\chi}^{(3)} \vdots \vec{E} + \ldots$$

Second order NLO $\chi^{(2)}$: SHG, DFG, SFG, OR, PG
Third order NLO $\chi^{(3)}$: Kerr, Raman, SPM, XPM, ….

Second Harmonic Generation

Second Harmonic Generation describes the interaction between an intense beam and a noncentrosymetric material which produces a beam with higher energy (double) than the incident



Interference Wave-Matter

- For spatially varying distribution of parameters (type I SHG)

$$\frac{dA_\omega(z)}{dz} + i\,[k_{0,\omega} + \delta k_\omega(z)]\,A_\omega(z) = -i\,\kappa(z)\,A_{2\omega}(z)A_\omega^*(z)$$

$$\frac{dA_{2\omega}(z)}{dz} + i\,[k_{0,2\omega} + \delta k_{2\omega}(z)]\,A_{2\omega}(z) = -i\,\kappa(z)\,A_\omega(z)A_\omega(z)$$

➔ define: - $\Delta k_0 = 2k_{0,\omega} - k_{0,2\omega}$; - $\Delta K(z) = 2\delta k_\omega(z) - \delta k_{2\omega}(z)$

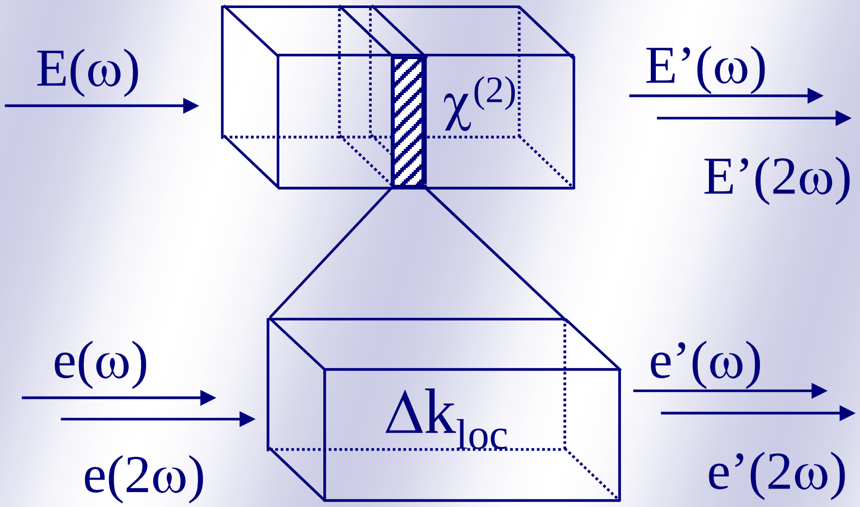
$$^{-}G(z) = L \cdot U(z/L)\kappa(z)e^{i\int_0^L \Delta K(z')dz'}$$

(U(z): *Rectangle* function)

$$\Rightarrow \quad \frac{d\mathcal{A}_\omega(z)}{dz} = -i\,G(z)\mathcal{A}_{2\omega}(z)\mathcal{A}_\omega^*(z)e^{i\,\Delta k_0 z}$$

$$\frac{d\mathcal{A}_{2\omega}(z)}{dz} = -i\,G(z)\mathcal{A}_\omega(z)\mathcal{A}_\omega(z)e^{-i\,\Delta k_0 z}$$

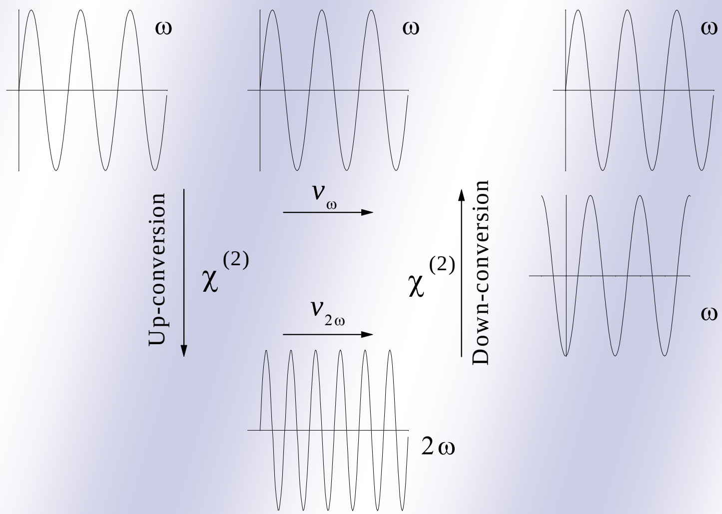
Second Harmonic Generation



- ➔ SHG is a succession of local seeding processes (a non-local process).
- ➔ Local SHG evolution along propagation depends on:
  - ➔ Amplitude and Phase of Transmitted Fundamental (ω).
  - ➔ Amplitude and Phase of Generated SH (2ω).
  - ➔ Local wave-vector mismatch (Δk_{loc}) and nonlinear coupling coefficient.

Second Harmonic Generation

The SHG process is governed by phase-matching conditions to achieve a larger or smaller conversion efficiencies

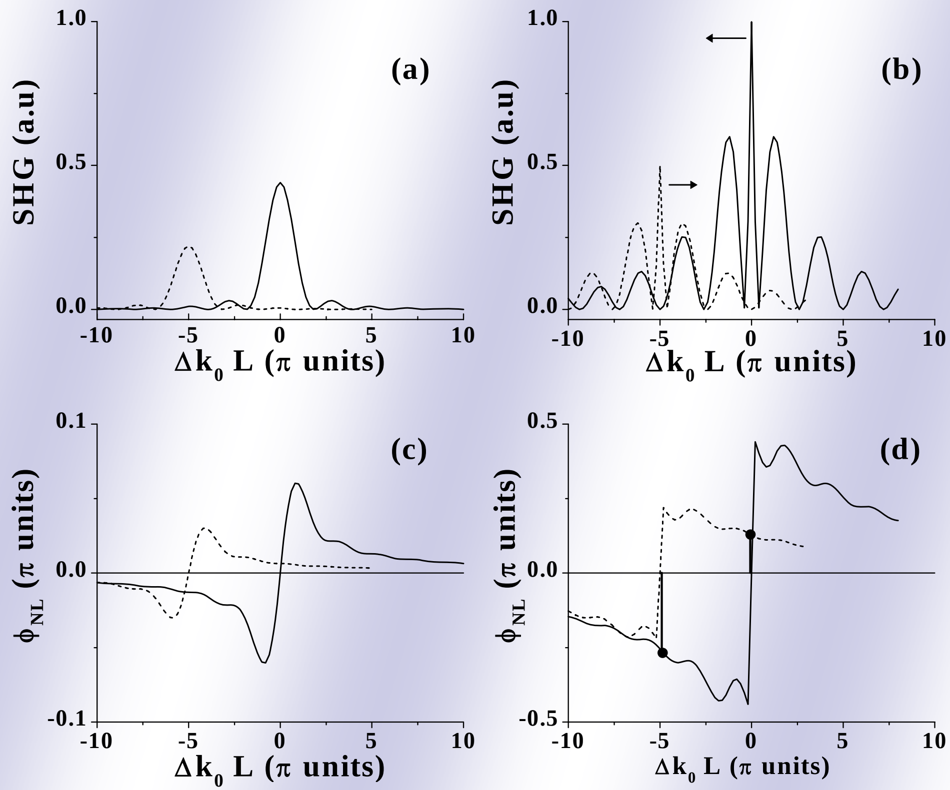


Cascading - interference

Second Harmonic Generation

Cascading - multimode

This is more evident if there are two or more simultaneous "resonances" in which SHG can be achieved



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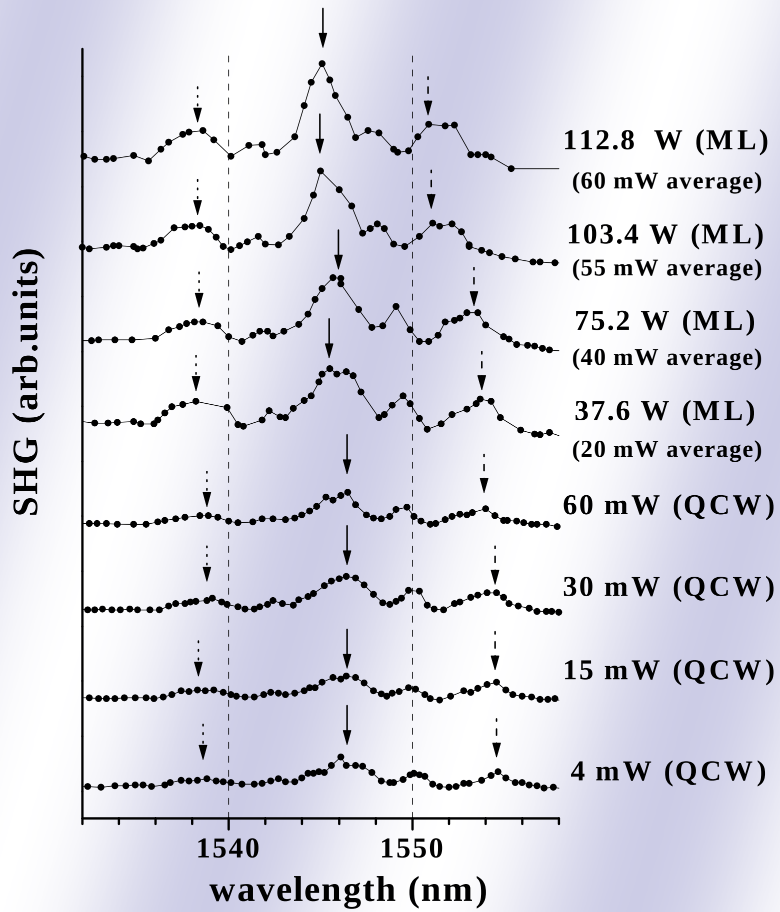

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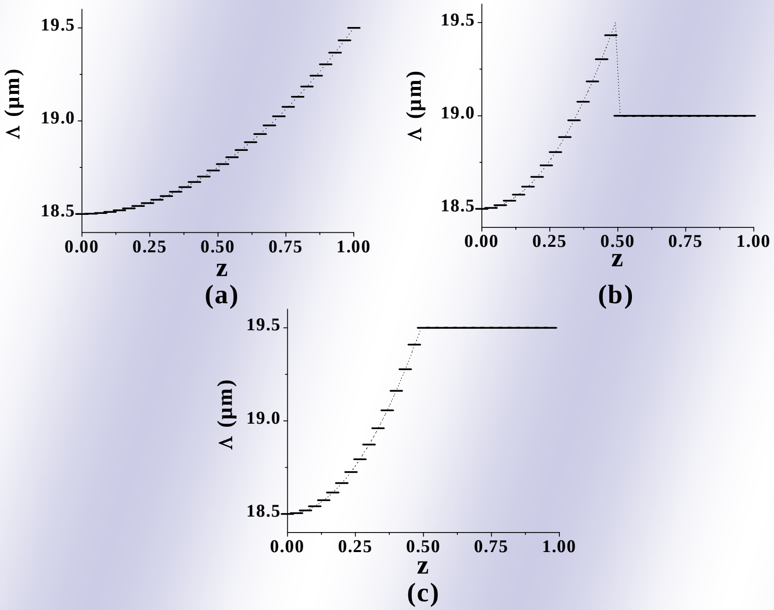
$$\frac{d\mathcal{A}_{2\omega}(z)}{dz} = -i\,G(z)\mathcal{A}_\omega(z)\mathcal{A}_\omega(z)e^{-i\,\Delta k_0 z}$$

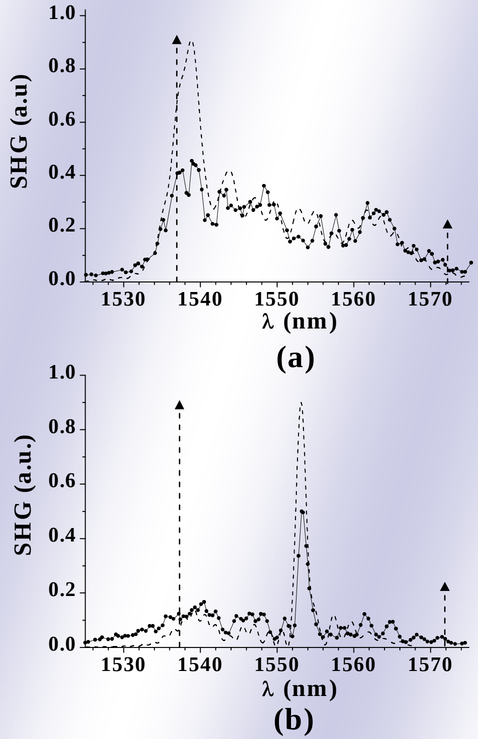
Interference Wave-Matter

The response can be rather complex, but predictable

Nonuniform profiles

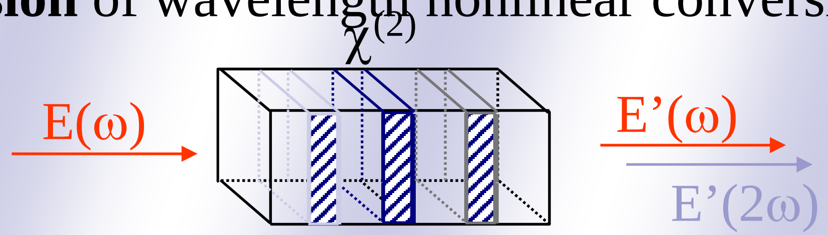


SHG detuning curves

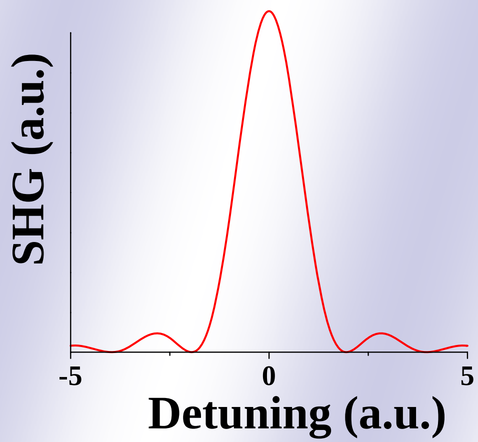


Interference Wave-Matter

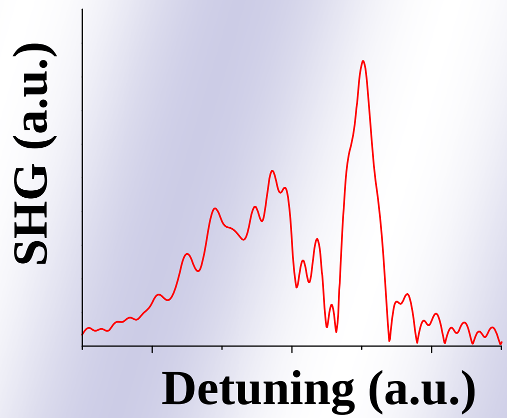
- SHG is a **succession** of wavelength nonlinear conversion process (*nonlocal*)



- The collective behaviour is observed in the SHG detuning curv|e



Uniform → sinc-like



Nonuniform → complex

This is nothing more than standard interferometry in which the medium modulates the fringe "visibility"

Comparing Interferometry and NLO

Interferometry: the <u>light redistribution</u> is used to describe what happened to the <u>light</u> as it is modified by a <u>dense medium</u>, after it has been observed of course – in coordinate space

NLO: the <u>light redistribution</u> is used to describe what happened to the <u>dense medium</u> as it is modified by <u>light</u>, after it has been observed of course – in wavevector space

Comparing Interferometry and NLO

This effect is not particular to interferometry- second order NLO

Both phenomena discussed are related by swapping coordinate and wave-vectors. Similar equivalences can be found between frequency and time (say OCT and local NLO)

This is to be expected because the way these spaces are included in the EM phenomenology

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}+\omega t+\phi)}$$

~~Conclusions~~

Question

1. **Is EM interference a basic phenomena or does it require a dense medium to exist?**
   A: According to me, interference is NOT a basic phenomena
   EM field superposition sampled by radiation-matter interaction IS.