sequential relativity

real scator algebra

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exploring physics with reality
INAOE - Puebla 2008

colaboradores y expertise

- Felipe Saldívar - algebra and number theory
- Gilberto Fuentes - geometrical algebra and general relativity
- Carlos Zagoya - graduate student
- Ángeles Eraña - philosophy

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sequence

freedom of choice

- There is liberty regarding the choice of mathematical structure selected in order to describe and predict physical phenomena.
- Within such a structure it is necessary to state relationships between physical variables in accordance with observational facts

Poincaré established this idea, according to Carnap, in the following terms "*No matter what observational facts are found, the physicist is free to ascribe to physical space any one of the mathematically possible geometrical structures, provided he makes suitable adjustments in the laws of mechanics and optics and consequently in the rules for measuring length*" [1]

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velocities of material bodies in inertial frames

The addition of velocities in special relativity ought to be consistent with the two fundamental postulates:

1. the constancy of the speed of light
2. the equivalence of all observers in free inertial motion

additional ingredient to establish addition of velocities rule:

- the separation between space - time events is chosen as the square root of a quadratic form with appropriate signature

Length - Lorentz metric

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real scator algebra

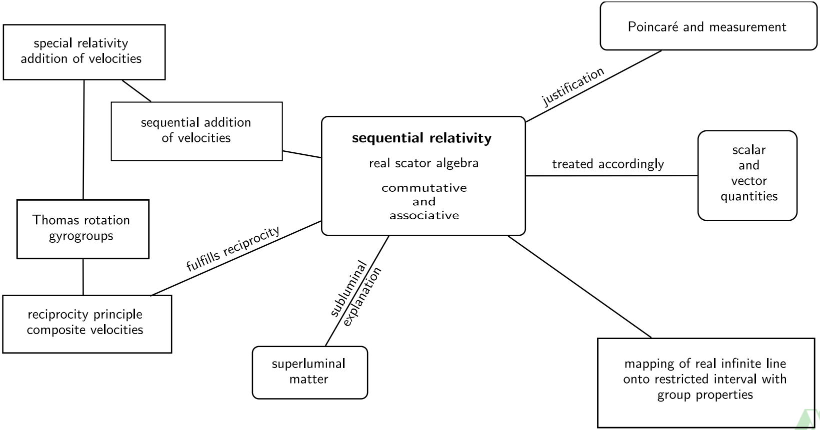

Figure: sequential relativity

sequential relativity approach

- philosophical - epistemological
- mathematical
- prevailing theoretical model and concepts
- experimental

modified Lorentz metric

- doubly[2] or triply[3] special relativity
- modification of the metric in the Planck scale
- other metrics in more general frames like FRW

[2] Amelino-Camelia, G., "Doubly-special relativity: First results and open key problems", International Journal of Modern Physics D, vol. 11, no. 10, pp. 1643 - 1669, 2002.

[3] Kowalski-Glikman, J. and Smolin, L., "Triply special relativity", Physical Review D, vol. 70, pp. 065020, 2004.

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mathematical formalism

- four vectors used in relativity contain four vector components
  - as a consequence time is a vector
- tensors - 4-vectors with 4 x 4 matrix products
- in contrast, quaterions are elements in $\mathbb{R}^4$ with one scalar part and three vector components (Hamilton)

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graded algebras
geometric algebra

Clifford algebras

- Algebras with elements in $\mathbb{R}^n$ dimensions
- spinors are a particular case (Pauli matrices a subgroup)

The geometric product

$$\mathbf{ab} = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b}$$

makes sense although the addends have different degree[4]. Dot and cross product may be defined from the geometric product. (wedge product is $\mathbf{a} \wedge \mathbf{b} = i\mathbf{a} \times \mathbf{b}$)

[4] D. Hestenes, *New foundations for classical mechanics*, CIP Kluwer (1990)

algebras
evolution

properties not fulfilled

- Algebras with group properties - real and complex ($\mathbb{R}^1$ and $\mathbb{R}^2$)
- Algebras without commutativity ($\mathbb{R}^4$ and $\mathbb{R}^n$)
- Algebras without associativity

The distributivity of the product over addition

- non-distributive algebras (?)

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addition of velocities

Let an event have velocity $\mathbf{u} = (u_1, u_2, u_3)$ in a given frame K' which, in turn has a relative velocity $\mathbf{v} = (v_1, v_2, v_3)$ with respect to another frame K. The velocity of the event in the frame K is given by

Definition

addition of velocities

$$\mathbf{v} \oplus \mathbf{u} \equiv \left(\frac{v_1 + u_1}{1 + \frac{v_1 u_1}{c^2}}, \frac{v_2 + u_2}{1 + \frac{v_2 u_2}{c^2}}, \frac{v_3 + u_3}{1 + \frac{v_3 u_3}{c^2}} \right)$$

If the relative velocity lies in the $\hat{\mathbf{e}}_1$ direction

$$v_1 \hat{\mathbf{e}}_1 \oplus \mathbf{u} = \left(\frac{v_1 + u_1}{1 + \frac{v_1 u_1}{c^2}}, v_2, v_3 \right)$$

magnitude and admissibility

Definition

The magnitude of an arbitrary velocity is

$$\|\mathbf{u}\| \equiv c\left[1-\prod_{j=1}^{3}\left(1-\frac{u_j^2}{c^2}\right)\right]^{\frac{1}{2}}$$

$$\|\mathbf{u}\| = \left[\left(u_1^2+u_2^2+u_3^2\right)-\left(u_1^2u_2^2+u_2^2u_3^2+u_3^2u_1^2\right)+u_1^2u_2^2u_3^2\right]^{\frac{1}{2}}$$

Definition

An admissible velocity is defined by a velocity element whose individual components are less or equal to the velocity of light in vacuum $u_i^2 < c^2$.

addition of colinear velocities

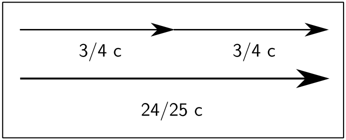

Figure: colinear velocities

$$\frac{v_1 + u_1}{1 + \frac{v_1 u_1}{c^2}} = \frac{\frac{3}{4}c + \frac{3}{4}c}{1 + \frac{\frac{3}{4}c \times \frac{3}{4}c}{c^2}} = \frac{\frac{3}{2}c}{1 + \frac{9}{16}} = \frac{3 \times 16}{25 \times 2}c = \frac{24}{25}c$$

special relativity and sequential relativity yield the same result

Galilean addition

$\frac{3}{4}c + \frac{3}{4}c = 1.5c$

addition of perpendicular velocities

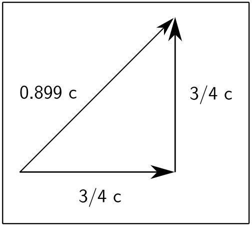

Figure: orthogonal velocities

$$\|\mathbf{u}\| = c\left[1-\left(1-\frac{u_1^2}{c^2}\right)\left(1-\frac{u_2^2}{c^2}\right)\right]^{\frac{1}{2}} = \sqrt{1-\left(1-\frac{9}{16}\right)^2}\,c$$

$$\|\mathbf{u}\| = \sqrt{1-\left(\frac{7}{16}\right)^2}\,c = \sqrt{1-\frac{49}{256}}\,c = \sqrt{\frac{207}{256}}\,c \approx 0.9c$$

addition of perpendicular velocities

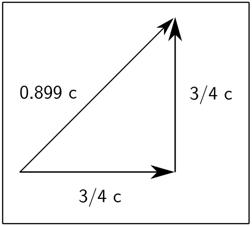

Figure: orthogonal velocities

sequential relativity - magnitude of velocity is always less than c if each component is smaller than c.

Special relativity and Galilean magnitude

$$\sqrt{u_1^2 + u_2^2} = \sqrt{\frac{9}{16} + \frac{9}{16}}c = \sqrt{\frac{18}{16}}c \approx 1.06c$$

magnitude and admissibility (SR)

special relativity

- magnitude of an arbitrary velocity in SR $\|\mathbf{u}\| \equiv \left[\sum u_j^2\right]^{\frac{1}{2}}$.
- Admissible velocity in SR $\sum u_j^2 < c^2$.

Addition of velocities in SR

$$\mathbf{u}'_{\parallel} = \frac{\mathbf{u}_{\parallel} + \mathbf{v}}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}}{c^2}\right)}, \quad \mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\perp}}{\gamma_v \left(1 + \frac{\mathbf{v}\cdot\mathbf{u}}{c^2}\right)}$$

previous case: velocities are orthogonal $\mathbf{u}_{\parallel} = 0$, perpendicular velocity is $\mathbf{u}_{\perp} = u_1$, relative velocity $v = u_2$. Einstein's velocity addition of $(u_1, 0)$ and $(0, u_2)$ is $\left(\mathbf{u}'_{\perp}, \mathbf{u}'_{\parallel}\right) = (u'_1, u'_2) = \left(\frac{u_1}{\gamma_v}, u_2\right)$ magnitude is $\left|\left(u_1\sqrt{1 - \frac{u_2^2}{c^2}}, u_2\right)\right| \le c$

sum of velocities

recapitulate

- sequential relativity - addition of velocities[5] and definition of magnitude[6] are simultaneously redefined[7].
- in one dimension, sequential relativity and special relativity addition schemes are identical
- a velocity element with components of say, $\mathbf{u} = \frac{3}{4}c\hat{e}_1 + \frac{3}{4}c\hat{e}_2$, has a magnitude in this new scheme equal to $\|\mathbf{u}\| \approx 0.9c$ rather than the usual (Euclidian) SR velocity magnitude $\|\mathbf{u}\| = \sqrt{18/16}c \approx 1.06c$!

[5] algebraic or *geometrical structure* in Poincaré's statement
[6] *length* in Poincaré's statement
[7] However, not regardless of *observational facts*

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astronomical observations

Observations, their interpretation and comparison with theories are the ultimate test for the validity of a physical model.

Recent observations of jets in some astronomical objects such as radio galaxies, quasars and microquasars exhibit very high transverse velocities leading to (apparent) superluminal speeds:

- superluminal motion in galaxy M87 jet[8]
- 3C 120 radio jet[9]
- micro cosmic x-ray source GRS1915+105 in our galaxy [10]

[8] J. A. Biretta et al., Hubble space telescope observations of superluminal motion in the M87 jet, Astrophysical J., **520**, no. 2, pp. 621 - 626, 1999

[9] R. C. Walker et al., The structure and motions of the 3C 120 radio jet . . . , Astrophysical J., **556**, no. 2, pp. 756 - 772, 2001

[10] Mirabel, I. F., Rodriguez, L. F., A Superluminal Source in the Galaxy, Nature **371**, NO. 6492/SEP1, P. 46, 1994

observations II

- It should be most interesting to analyze the astronomical observed data within the present formulation[11]
- systematical study of astronomical sources with apparent superluminal motion (?)
- detailed explanation within SR framework (?)
- other experiments or observations with relativistic orthogonal velocities (?)

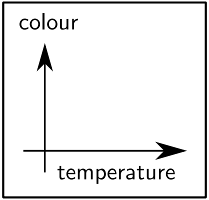
[11] (Subluminal speeds are to be expected since the proposed scheme always leads to subluminal velocities even if the parallel and transverse velocities are arbitrarily close to c.)

the conceptual problem

categories

- quantities that do not possess the quality of direction - magnitudes, temperature, charge[12], probability, ¿time? ...
- quantities that do possess the quality of direction - position, velocity, force, energy flow ...

independent variables $\longrightarrow$ orthogonal representation



[12] do not confuse direction and sense ($\pm$), or whether the quantity is positive definite

time

an elusive concept

- What type of variable should we ascribe to time?
- It does have a sense (±) although we often call it the direction or arrow of time
- Does it make an angle with respect to something else?
- time is very often the independent variable ⟶ thus we represent it on an orthogonal axis

time

time should be treated as a scalar[a] quantity

[a] scalar in the meaning as without direction, not synonym of invariant

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algebra

substrate to describe physical models . . .

fundamental constituents of an algebraic system

- elements
- definition of two operations between elements
- operations properties
- order parameter
- geometrical interpretation

elements

elements of algebra in 1+3 dimensions

$$\overset{o}{\alpha} = (a_0; a_1, a_2, a_3) \in \mathbb{R}^4$$

$$\overset{o}{\alpha} = a_0 + a_1\hat{\mathbf{e}}_1 + a_2\hat{\mathbf{e}}_2 + a_3\hat{\mathbf{e}}_3$$

- a letter with an ellipse above represents a scator element
- $\hat{\mathbf{e}}_j$ are unit vectors
- the ordered tetrad may represent time in the first variable and space in the remaining three variables

sca-tor comes from the contraction of scalar and director (vector).

esca-tor proviene de la contracción de escalar y director (vector).

addition

Scators $\overset{o}{\alpha} = (a_0; a_1, a_2, a_3)$ and $\overset{o}{\beta} = (b_0; b_1, b_2, b_3)$ have an

addition operation

$$\overset{o}{\sigma} = \overset{o}{\alpha} + \overset{o}{\beta} = (a_0 + b_0, a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

the sum fulfills group properties. Neutral is $(0; 0, 0, 0)$.

group properties

- conmutativity $\quad a \circledast b = b \circledast a$
- asociativity $\quad (a \circledast b) \circledast c = a \circledast (b \circledast c)$
- existence of neutral element $\quad a \circledast \emptyset = a$
- existence of inverse $\quad a \circledast a^{(inv)} = \emptyset$
- closure $\quad a, b \in \mathbb{A} \Rightarrow a \circledast b \in \mathbb{A}$

product

product operation

$$\overset{o}{\alpha}\overset{o}{\beta} = a_0 b_0 \prod_{k=1} \left(1 + \frac{a_k b_k}{a_0 b_0}\right) + a_0 b_0 \sum_{j=1}^{n} \prod_{k=1}^{n} \left(1 + \frac{a_k b_k}{a_0 b_0}\right) \frac{\left(\frac{a_j}{a_0} + \frac{b_j}{b_0}\right)}{\left(1 + \frac{a_j b_j}{a_0 b_0}\right)} \hat{\mathbf{e}}_j$$

The product is commutative, associative, neutral exists $(1; 0, 0, 0)$, closed.

inverse exists if divisors of zero are excluded:

$a_0, b_0 \neq 0$, that is, *scalar part should be different from zero.*

scalar scator

Consider $\overset{o}{\alpha} = (a_0; a_1 = 0, a_2 = 0, a_3 = 0)$, that is $\overset{o}{\alpha} = (a_0; 0, 0, 0)$ and an arbitrary scator $\overset{o}{\beta} = (b_0; b_1, b_2, b_3)$

$$\overset{o}{\alpha} = (a_0; 0, 0, 0) \otimes (b_0; b_1, b_2, b_3) = (a_0 b_0; a_0 b_1, a_0 b_2, a_0 b_3)$$

since the scalar part is $g_0 = a_0 b_0 \prod_{k=1} \left(1 + \frac{a_k b_k}{a_0 b_0}\right) = a_0 b_0$, director components have the form $g_j = a_0 b_j$

- the scalar distributes over each component of the scator
- this case is analogous to the product of a real number λ times a vector that scales the whole vector space.

Corollary

a scalar is a scator with one component
vectors are not a subset of scators

product - scalar factored

may be written as

$$\overset{o}{\alpha}\overset{o}{\beta} = a_0 b_0 \prod_{k=1} \left(1 + \frac{a_k b_k}{a_0 b_0}\right) \left[1 + \sum_{j=1}^{n} \frac{\left(\frac{a_j}{a_0} + \frac{b_j}{b_0}\right)}{\left(1 + \frac{a_j b_j}{a_0 b_0}\right)} \hat{\mathbf{e}}_j\right]$$

or

$$\overset{o}{\alpha}\overset{o}{\beta} = a_0 b_0 \prod_{k=1} \left(1 + \frac{a_k b_k}{a_0 b_0}\right) \left(1, \frac{\left(\frac{a_j}{a_0} + \frac{b_j}{b_0}\right)}{\left(1 + \frac{a_j b_j}{a_0 b_0}\right)}, \frac{\left(\frac{a_j}{a_0} + \frac{b_j}{b_0}\right)}{\left(1 + \frac{a_j b_j}{a_0 b_0}\right)}, \frac{\left(\frac{a_j}{a_0} + \frac{b_j}{b_0}\right)}{\left(1 + \frac{a_j b_j}{a_0 b_0}\right)}\right)$$

mmm ... reminiscent of the addition of velocities proposal.
The product in real scator algebra with constant (unit) scalar corresponds to composition of velocities in sequential relativity.

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$$\overset{o}{\alpha}\overset{o}{\beta} = a_0 b_0 \prod_{k=1} \left(1 + \frac{a_k b_k}{a_0 b_0}\right) \left[1 + \sum_{j=1}^{n} \frac{\left(\frac{a_j}{a_0} + \frac{b_j}{b_0}\right)}{\left(1 + \frac{a_j b_j}{a_0 b_0}\right)} \hat{\mathbf{e}}_j\right]$$

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conjugation and metric

conjugate operation

The director components change sign, the scalar remains the same

$$\overset{o}{\alpha}{}^{*} = (a_0; -a_1, -a_2, -a_3)$$

magnitude

is the positive square root of the scator times its conjugate

$$\left\| \overset{o}{\alpha} \right\| = \sqrt{\overset{o}{\alpha}\overset{o}{\alpha}{}^{*}} = \left[\left(a_0^2 \left(1 - \frac{a_1^2}{a_0^2}\right)\left(1 - \frac{a_2^2}{a_0^2}\right)\left(1 - \frac{a_3^2}{a_0^2}\right); 0, 0, 0 \right) \right]^{\frac{1}{2}}$$

The magnitude is a scalar that establishes an order parameter.

Length!

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Length!

restricted space

The *restricted space* requires that the scalar is greater[13] than any director component.

$$a_0^2 > a_j^2 \quad \forall \quad j \neq 0$$

with the appropriate scaling $a_0 \to ct$ this is the restriction imposed by the admissible velocities.

[13] or equal only if all director components are zero

time priority

time precedes space

why does a unit scator represent velocity?

coordinate and velocity

The coordinate or time - space scator is

$$\overset{o}{\mathbf{x}} = (x_0; x_1, x_2, x_3)$$

interval

$$d\overset{o}{\mathbf{x}} = (cdt; dx_1, dx_2, dx_3)$$

velocity

$$\frac{1}{c}\frac{d\overset{o}{\mathbf{x}}}{dt} = (1; \beta_{u1}, \beta_{u2}, \beta_{u3})$$

where $\beta_{uj} \equiv \frac{u_j}{c} = \frac{dx_j}{dt}$.

Scator with unit scalar $\overset{o}{\mathbf{x}} \to \frac{\overset{o}{\mathbf{x}}}{x_0}$.

composite velocity

The product of two scators with unit scalar $\overset{o}{\alpha} = (1; a_1, a_2, a_3)$ and $\overset{o}{\beta} = (1; b_1, b_2, b_3)$ is

$$\overset{o}{\alpha}\overset{o}{\beta} = \prod_{k=1} (1 + a_k b_k) + \sum_{j=1}^{n} \prod_{k=1}^{n} (1 + a_k b_k) \frac{(a_j + b_j)}{(1 + a_j b_j)} \hat{e}_j$$

the scalar component of the product is no longer one. Introduce a factor to insure that the resulting scator also has unit scalar

composition of velocities in scator algebra

$$\overset{o}{\alpha}' = \frac{1}{\prod_{k=1} (1 + a_k b_k)} \overset{o}{\alpha}\overset{o}{\beta} = 1 + \sum_{j=1}^{n} \frac{(a_j + b_j)}{(1 + a_j b_j)} \hat{e}_j$$

metric invariance

The scator with unit magnitude

$$\overset{o}{\alpha} \rightarrow \frac{\overset{o}{\alpha}}{\left\| \overset{o}{\alpha} \right\|} = \frac{\overset{o}{\alpha}}{\sqrt{\overset{o}{\alpha}\overset{o}{\alpha}^{*}}} = \frac{\overset{o}{\alpha}}{a_0 \prod_{k=1} \sqrt{1 - \frac{a_k^2}{a_0^2}}}$$

magnitude of product = product of magnitudes $\left\| \overset{o}{\alpha}\overset{o}{\beta} \right\| = \left\| \overset{o}{\alpha} \right\| \left\| \overset{o}{\beta} \right\|$

Therefore, product with a unit scator preserves metric.

The scator $\gamma_u \left(1; \beta_{u1}, \beta_{u2}, \beta_{u3}\right)$ has unit magnitude for

$$\gamma_u = \frac{1}{\sqrt{\prod_j \left(1 - \beta_{uj}^2\right)}}.$$

Scator velocity transformations preserving metric are the counterpart to energy - momentum 4-vector transformations in SR.

scator proper time

γ_u written explicitly for three components

$$\gamma_u = \frac{1}{\sqrt{1 - (\beta_1^2 + \beta_2^2 + \beta_3^2) + (\beta_1^2\beta_2^2 + \beta_2^2\beta_3^2 + \beta_3^2\beta_1^2) - \beta_1^2\beta_2^2\beta_3^2}},$$

the γ_u involved in the scator proper time $d\tau = dt\gamma_u^{-1}$ reduces to the special relativity case $\gamma_u^{(SR)} = \left(1 - \frac{\mathbf{u}\cdot\mathbf{u}}{c^2}\right)^{-\frac{1}{2}}$ for either small β's or one spatial dimension (paraxial).

spatial magnitude

$$\sqrt{\underbrace{a_0^2 - \left(a_1^2 + a_2^2 + a_3^2\right)}_{\text{Lorentz}} + \left(\frac{a_1^2 a_2^2}{a_0^2} + \frac{a_2^2 a_3^2}{a_0^2} + \frac{a_3^2 a_1^2}{a_0^2}\right) - \frac{a_1^2 a_2^2 a_3^2}{a_0^4}}$$

scator and Euclidian magnitude: spatial part

$$\sqrt{a_0^2 - \underbrace{\left(a_1^2 + a_2^2 + a_3^2\right) + \left(\frac{a_1^2 a_2^2}{a_0^2} + \frac{a_2^2 a_3^2}{a_0^2} + \frac{a_3^2 a_1^2}{a_0^2}\right) - \frac{a_1^2 a_2^2 a_3^2}{a_0^4}}_{\text{spatial magnitude}}}$$

- temporal variable becomes admixed with spatial magnitude

comparison - velocity

	sequential relativity	special relativity
elements	real scators	4-vectors
tetrad	$\overset{o}{\chi} = (x_0; x_1, x_2, x_3)$	$\mathbf{x} = (x_0, x_1, x_2, x_3)$
metric	$\left\lVert \overset{o}{\chi} \right\rVert = \left(x_0^2 \prod_{j=1} \left(1 - \frac{x_j^2}{x_0^2} \right) \right)^{\frac{1}{2}}$	$\lVert \mathbf{x} \rVert = \left(x_0^2 - \sum_{j=1} x_j^2 \right)^{\frac{1}{2}}$
$\frac{dt}{d\tau} = \gamma_u$	$\prod_{j=1} \left(1 - \frac{u_j^2}{c^2} \right)^{-\frac{1}{2}}$	$\left(1 - \sum_{j=1} \frac{u_j^2}{c^2} \right)^{-\frac{1}{2}}$
u_0	u_0'	u_0'
$\mathbf{u}_\parallel$	$\frac{\mathbf{u}_\parallel' + \mathbf{v}}{\left(1 + \frac{\mathbf{v} \cdot \mathbf{u}'}{c^2}\right)}$	$\frac{\mathbf{u}_\parallel' + \mathbf{v}}{\left(1 + \frac{\mathbf{v} \cdot \mathbf{u}'}{c^2}\right)}$
$\mathbf{u}_\perp$	$\mathbf{u}_\perp'$	$\frac{\mathbf{u}_\perp'}{\gamma_v \left(1 + \frac{\mathbf{v} \cdot \mathbf{u}'}{c^2}\right)}$

circuado

sequential versus special

sister theories

- similarities
  - identical in one dimension (time and space are transformed), similar in paraxial limit
  - equal transformations for parallel velocities
- differences
  - metric is different [*fundamental departure*]
  - magnitude allows for two close to c components in scators
  - different transformations for perpendicular velocities
  - composition of velocities is commutative in scators

why sequential?

SR sequence

- an event is static in an umprimed frame
- event is seen in a primed frame with relative velocity in x
- primed frame seen in biprimed frame with relative velocity in y
- biprimed frame seen in triprimed frame with relative velocity in z

SR transformations

$$\mathbf{u}'_{\parallel} = \frac{\mathbf{u}_{\parallel} + \mathbf{v}}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}}{c^2}\right)} \qquad \mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\perp}}{\gamma_v \left(1 + \frac{\mathbf{v}\cdot\mathbf{u}}{c^2}\right)}$$

The parallel and perpendicular sub indices are the decomposition of the velocities $\mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}$ with respect to the relative velocity $\mathbf{v}$ between frames such that $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}_{\parallel} \cdot \mathbf{v}$. Since the velocities are always perpendicular in the sequence

$$\mathbf{u}'_{\parallel} = \mathbf{v} \qquad \mathbf{u}'_{\perp} = \gamma_v^{-1} \mathbf{u}_{\perp} = \sqrt{1 - \frac{v^2}{c^2}}\,\mathbf{u}_{\perp}$$

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velocity composition in SR

Event at rest in a reference frame $\Re$ $\mathbf{u}_{\|}=0$, $\mathbf{u}_{\perp}=0$; $\mathbf{u}=(0,0,0)$.
Primed reference frame $\Re'$ has relative velocity $\mathbf{v}_{01}\left(\Re,\Re'\right)=v_x\hat{\mathbf{e}}_x$
The velocity of the event in the primed system

$$\mathbf{u}'=\left(v_x,0,0\right).$$

Double primed frame $\Re''$ with relative velocity $\mathbf{v}_{12}\left(\Re',\Re''\right)=v_y\hat{\mathbf{e}}_y$
The velocity of the event in the double primed system

$$\mathbf{u}''=\left(v_x\left(1-\frac{v_y^2}{c^2}\right)^{\frac{1}{2}},v_y,0\right)$$

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z velocity in SR

Triple primed frame $\mathfrak{R}'''$ with relative velocity $\mathbf{v}_{23}\left(\mathfrak{R}'', \mathfrak{R}'''\right) = v_z \hat{\mathbf{e}}_z$

The velocity of the event in the triple primed system

$$\mathbf{u}''' = \left(v_x \left(1 - \frac{v_y^2}{c^2}\right)^{\frac{1}{2}} \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_y \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_z \right)$$

The velocity squared in the triple primed system is

$$\left|\mathbf{u}'''\right|^2 = v_x^2 \left(1 - \frac{v_y^2}{c^2}\right)\left(1 - \frac{v_z^2}{c^2}\right) + v_y^2 \left(1 - \frac{v_z^2}{c^2}\right) + v_z^2.$$

If we expand the products, we obtain

$$\left|\mathbf{u}'''\right|^2 = v_x^2 + v_y^2 + v_z^2 - \frac{v_x^2 v_y^2}{c^2} - \frac{v_y^2 v_z^2}{c^2} - \frac{v_z^2 v_x^2}{c^2} + \frac{v_x^2 v_y^2 v_z^2}{c^4},$$

z vel in SR cont ...

that may be regrouped together to yield

$$\left|\mathbf{u}'''\right|^2 = c^2\left[1-\left(1-\frac{v_x^2}{c^2}\right)\left(1-\frac{v_y^2}{c^2}\right)\left(1-\frac{v_z^2}{c^2}\right)\right].$$

But this is the metric form in sequential relativity!
scator with velocity $(c; v_x, v_y, v_z)$ *measured from triprimed frame.*
in SR, velocity vector

$$\mathbf{u}''' = \left(c, v_x\left(1-\frac{v_y^2}{c^2}\right)^{\frac{1}{2}}\left(1-\frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_y\left(1-\frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_z\right)$$

However, vector (v_x, v_y, v_z) has magnitude $\left|\mathbf{u}'''\right|^2 = v_x^2 + v_y^2 + v_z^2$

z vel in SR cont ...

that may be regrouped together to yield

$$\left|\mathbf{u}'''\right|^2 = c^2\left[1-\left(1-\frac{v_x^2}{c^2}\right)\left(1-\frac{v_y^2}{c^2}\right)\left(1-\frac{v_z^2}{c^2}\right)\right].$$

But this is the metric form in sequential relativity!
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$$\mathbf{u}''' = \left(c, v_x\left(1-\frac{v_y^2}{c^2}\right)^{\frac{1}{2}}\left(1-\frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_y\left(1-\frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_z\right)$$

However, vector (v_x, v_y, v_z) has magnitude $\left|\mathbf{u}'''\right|^2 = v_x^2 + v_y^2 + v_z^2$

reciprocity principle

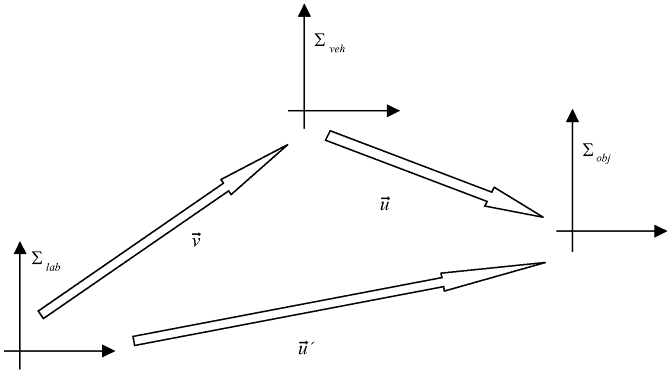

Figure: composition of velocities

Mocanu paradox

addition of velocities $\mathbf{u}'_{lab} = \mathbf{v} \boxplus \mathbf{u}$, seen from $\sum_{lab}$
addition as seen from $\sum_{obj}$ is $\mathbf{u}'_{obj} = (-\mathbf{u}) \boxplus (-\mathbf{v}) = -(\mathbf{u} \boxplus \mathbf{v})$

reciprocity principle

$\mathbf{u}'_{lab} = -\mathbf{u}'_{obj}$, therefore the sum $\mathbf{v} \boxplus \mathbf{u} = \mathbf{u} \boxplus \mathbf{v}$ commutes.

coupling between relative velocities and frame orientations "has disentangled" this Mocanu paradox [14].
Thomas precession plays a determinant role. A rotation operator in the velocity addition laws, allows for gyrocommutative and gyroassociative properties to produce a gyrogroup [15].

[14] C. I. Mocanu, On the relativistic velocity composition paradox and the Thomas rotation, Foundations of Physics Letters, vol. 5, no. 5, 1992

[15] A. A. Ungar, The relativistic composite-velocity reciprocity principle, Foundations of Physics, vol. 30, no. 2, pp. 331 - 342, 2000

reciprocity

- Sequential relativity fulfills the reciprocity principle since it is commutative and associative.
- It is not manifest whether it is implicitly including a rotation operator in a comparable fashion as the Thomas rotation is explicitly introduced of in gyrogroups.
- Alternatively, it is possible that this scheme does not involve a rotation of the inertial frames at all.
- A definitive answer to these questions requires a detailed analysis of the time and space transformations consistent with the present velocity addition rules.

finale

general considerations

- an alternative formulation has been presented
  - fulfills the two fundamental axioms of special relativity
  - a new algebraic - geometrical structure is introduced
  - magnitude, or rules to measure length have been modified
    - time has become admixed in spatial measurements *i.e.* $\sqrt{x^2 + y^2 - \frac{x^2 y^2}{c^2 t^2}}$
- from an epistemological - metaphysical view, time is treated as a scalar

finale - relativity

scator relativity

- parallel velocities identical to special relativity
- perpendicular velocities are unaltered in moving frame
- magnitude is such that $(1, c, c)$ has magnitude equal to c
- insensitive to the velocity sequence
    - sequential invariant relativity

finale - algebra

algebraic structure

- vectors [no scalar] $\longrightarrow$ graded algebras [may include scalar] $\longrightarrow$ scators [must include scalar]
- group under addition
- group under product if divisors of zero are excluded
- real magnitude in restricted space
- product does not distribute over addition

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finale - experiments

observations

- perpendicular velocities in astrophysics
- reciprocity of composite velocities
- other experiments . . .

References

- http://investigacion.izt.uam.mx/mfg
- http://luz.izt.uam.mx/mediawiki
- mfg@xanum.uam.mx

comparison - momenta

	sequential relativity	special relativity
elements	real scators	4-vectors
tetrad	$\overset{o}{\chi} = (x_0; x_1, x_2, x_3)$	$\mathbf{x} = (x_0, x_1, x_2, x_3)$
metric	$\left\Vert \overset{o}{\chi} \right\Vert = \left(x_0^2 \prod_{j=1} \left(1 - \frac{x_j^2}{x_0^2} \right) \right)^{\frac{1}{2}}$	$\Vert \mathbf{x} \Vert = \left(x_0^2 - \sum_{j=1} x_j^2 \right)^{\frac{1}{2}}$
proper time $\frac{dt}{d\tau} = \gamma_u$	$\prod_{j=1} \left(1 - \frac{u_j^2}{c^2} \right)^{-\frac{1}{2}}$	$\left(1 - \sum_{j=1} \frac{u_j^2}{c^2} \right)^{-\frac{1}{2}}$
U_0	$\gamma_v U_0' \prod_{k=1} \left(1 + \frac{U_k' v_k}{U_0' c} \right)$	$\gamma_v \left(U_0' + \frac{\mathbf{v}}{c} \cdot \boldsymbol{U}' \right)$
$\mathbf{U}_\parallel$	$\gamma_v \left(\mathbf{U}_\parallel' + U_0' \frac{\mathbf{v}}{c} \right)$	$\gamma_v \left(\mathbf{U}_\parallel' + \frac{\mathbf{v}}{c} U_0' \right)$
$\boldsymbol{U}_\perp$	$\gamma_v \left(1 + \frac{U_\parallel'}{U_0'} \frac{v}{c} \right) \boldsymbol{U}_\perp'$	$\boldsymbol{U}_\perp'$

product by components
1+2 dimensions

$$\overset{o}{\gamma} = \overset{o}{\alpha}\overset{o}{\beta} = (g_0; g_1, g_2, b_3)$$

the scalar part is

$$g_0 = a_0 b_0 \left(1 + \frac{a_1 b_1}{a_0 b_0}\right)\left(1 + \frac{a_2 b_2}{a_0 b_0}\right),$$

the director or component 1 is

$$g_1 = \left(1 + \frac{a_2 b_2}{a_0 b_0}\right)(a_1 b_0 + a_0 b_1),$$

the director or component 2 is

$$g_2 = \left(1 + \frac{a_1 b_1}{a_0 b_0}\right)(a_2 b_0 + a_0 b_2).$$

mag calculation

$$\|\mathbf{u}\| = c\left[1-\left(1-\frac{u_1^2}{c^2}\right)\left(1-\frac{u_2^2}{c^2}\right)\right]^{\frac{1}{2}} = \sqrt{\frac{u_1^2}{c^2}+\frac{u_2^2}{c^2}-\frac{u_1^2u_2^2}{c^4}}c$$

$$\|\mathbf{u}\| = \sqrt{\frac{9}{16}+\frac{9}{16}-\frac{9}{16}\frac{9}{16}}c = \sqrt{\frac{18\times 16}{256}-\frac{81}{256}}c$$

$$\|\mathbf{u}\| = \sqrt{\frac{288-81}{256}}c = \sqrt{\frac{207}{256}}c \approx 0.9c$$

x velocity SR

Event at rest in a reference frame $\Re$ $\mathbf{u}_{||} = 0$, $\mathbf{u}_{\perp} = 0$; $\mathbf{u} = (0, 0, 0)$.
Primed reference frame $\Re'$ has relative velocity $\mathbf{v}_{01}(\Re, \Re') = v_x \hat{\mathbf{e}}_x$
The velocity components in the primed system are

$$\mathbf{u}'_{||} = \mathbf{v}_{01}(\Re, \Re') = v_x \hat{\mathbf{e}}_x, \quad \mathbf{u}'_{\perp}(\Re, \Re') = 0;$$

The velocity of the event in the primed system is

$$\mathbf{u}' = (v_x, 0, 0).$$

y velocity SR

Double primed frame $\Re''$ with relative velocity $\mathbf{v}_{12}\left(\Re', \Re''\right) = v_y \hat{\mathbf{e}}_y$
The velocity $\mathbf{u}'_{\perp}$ that was perpendicular in the $\left(\Re, \Re'\right)$ transformation is now parallel in $\left(\Re', \Re''\right)$

$$\mathbf{u}'_{\|}\left(\Re', \Re''\right) = \mathbf{u}'_{\perp}\left(\Re, \Re'\right) = 0$$

The velocity components in the primed system are

$$\mathbf{u}''_{\|} = \mathbf{v}_{12}\left(\Re', \Re''\right) = v_y \hat{\mathbf{e}}_y \qquad \mathbf{u}''_{\perp} = \frac{\mathbf{u}'_{\perp}}{\gamma_v} = v_x \left(1 - \frac{v_y^2}{c^2}\right)^{\frac{1}{2}} \hat{\mathbf{e}}_x .$$

The velocity of the event in the double primed system is then

$$\mathbf{u}'' = \left(v_x \left(1 - \frac{v_y^2}{c^2}\right)^{\frac{1}{2}}, v_y, 0 \right)$$

z velocity perpendicular SR

Triple primed frame $\Re'''$ with relative velocity $\mathbf{v}_{23}\left(\Re'', \Re'''\right) = v_z \hat{\mathbf{e}}_z$
The parallel transformation is

$$\mathbf{u}_{\parallel}''' = \mathbf{v}_{23}\left(\Re'', \Re'''\right) = v_z \hat{\mathbf{e}}_z$$

The previous parallel and perpendicular velocities in the $(\Re', \Re'')$ transformation are now both perpendicular in $(\Re'', \Re''')$,

$$\mathbf{u}_{\perp}''\left(\Re'', \Re'''\right) = \mathbf{u}_{\parallel}''\left(\Re', \Re''\right) + \mathbf{u}_{\perp}''\left(\Re', \Re''\right) = v_x \left(1 - \frac{v_y^2}{c^2}\right)^{\frac{1}{2}} \hat{\mathbf{e}}_x + v_y \hat{\mathbf{e}}_y .$$

The perpendicular components are then

$$\mathbf{u}_{\perp}''' = \frac{\mathbf{u}_{\perp}''\left(\Re'', \Re'''\right)}{\gamma_{v_{23}}} = v_x \left(1 - \frac{v_y^2}{c^2}\right)^{\frac{1}{2}} \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}} \hat{\mathbf{e}}_x + v_y \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}} \hat{\mathbf{e}}_y$$

z velocity SR

The velocity of the event in the triple primed system is then

$$\mathbf{u}''' = \left(v_x \left(1 - \frac{v_y^2}{c^2}\right)^{\frac{1}{2}} \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_y \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}}, v_z \right)$$

The velocity squared in the triple primed system is

$$\left|\mathbf{u}'''\right|^2 = v_x^2 \left(1 - \frac{v_y^2}{c^2}\right)\left(1 - \frac{v_z^2}{c^2}\right) + v_y^2 \left(1 - \frac{v_z^2}{c^2}\right) + v_z^2 .$$

If we expand the products, we obtain

$$\left|\mathbf{u}'''\right|^2 = v_x^2 + v_y^2 + v_z^2 - \frac{v_x^2 v_y^2}{c^2} - \frac{v_y^2 v_z^2}{c^2} - \frac{v_z^2 v_x^2}{c^2} + \frac{v_x^2 v_y^2 v_z^2}{c^4},$$

that may be regrouped together to yield

$$\left|\mathbf{u}'''\right|^2 = c^2 \left[1 - \left(1 - \frac{v_x^2}{c^2}\right)\left(1 - \frac{v_y^2}{c^2}\right)\left(1 - \frac{v_z^2}{c^2}\right)\right].$$

coexistance of theories
naturalized philosophy

- theories or formalisms say the same thing or are incompatible
- prevailing view: uniqueness of models - analytical philosophy
- it is rare to encounter complementary theories