Tunneling Time, the Hartman Effect, and Superluminality: Resolving an Old Paradox

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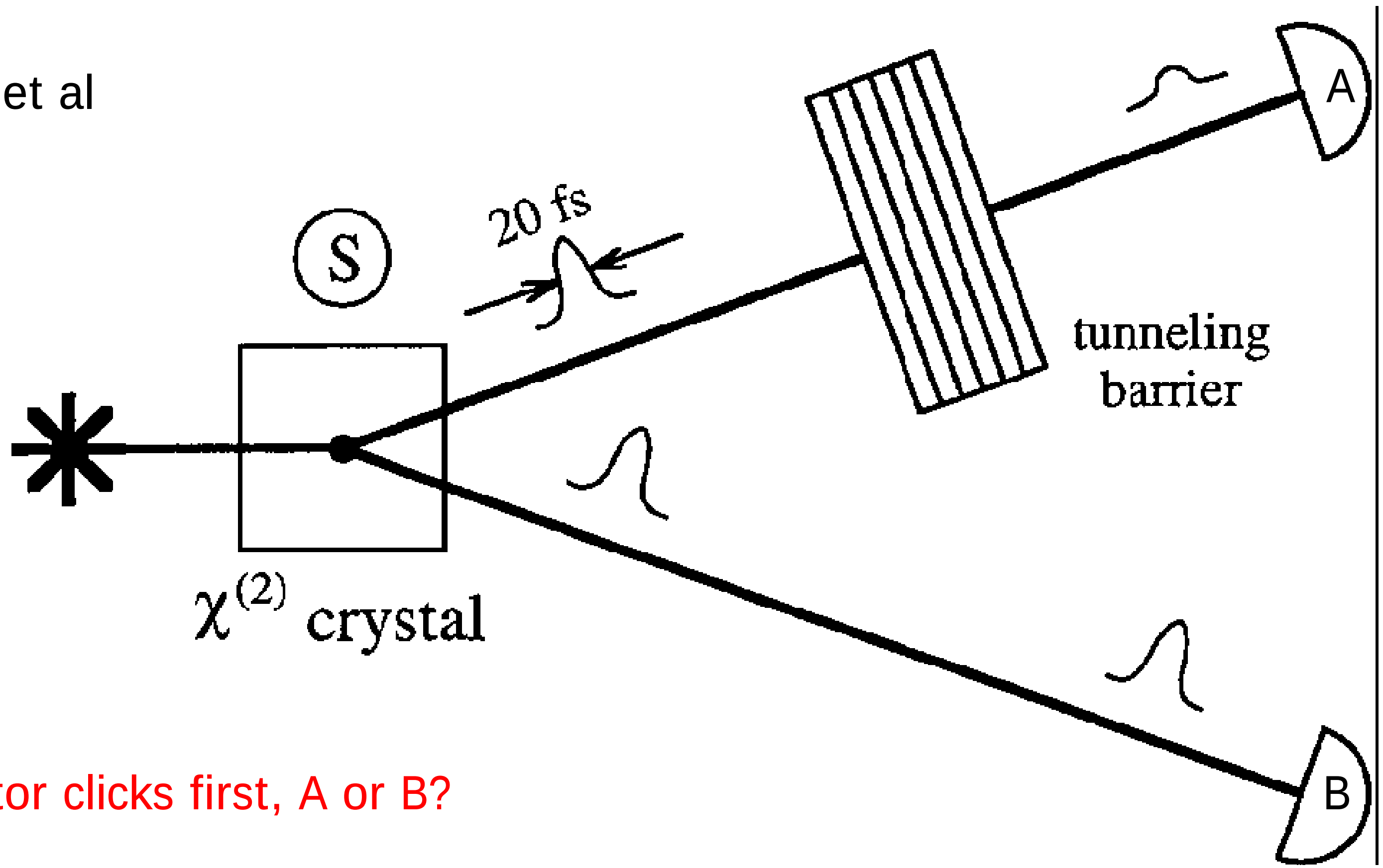
Gedanken Experiment to Measure Tunneling Time

R.Y. Chiao, et al



Which detector clicks first, A or B?

The Orthodox View

- “On average the tunneling photons arrived before those that traveled through air implying an average tunneling velocity of about 1.7 times that of light.”

  [A.M.Steinberg, P.G.Kwiat, and R.Y.Chiao, *PRL*,1993]
- “Experiments show...that the photon passing through that barrier arrives first.” [*Introductory Quantum Optics,* G.C.Gerry and P.L.Knight,2005]
- “Twin-photon experiments DO in fact reveal that light that has passed through a barrier can arrive at a detector much sooner ("superluminally") than light that has propagated freely.” [Anonymous PRL referee,2006]

The Current Belief

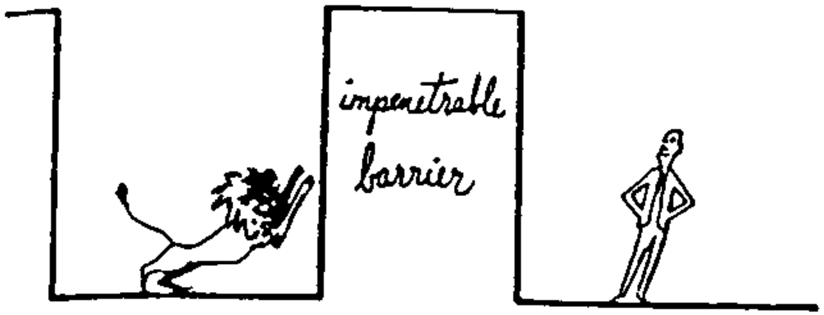
- It is widely believed that tunneling photons travel with a group velocity that exceeds the vacuum speed of light.
- The implication is that electron tunneling can also be superluminal*.

[*Superluminal=faster than light]

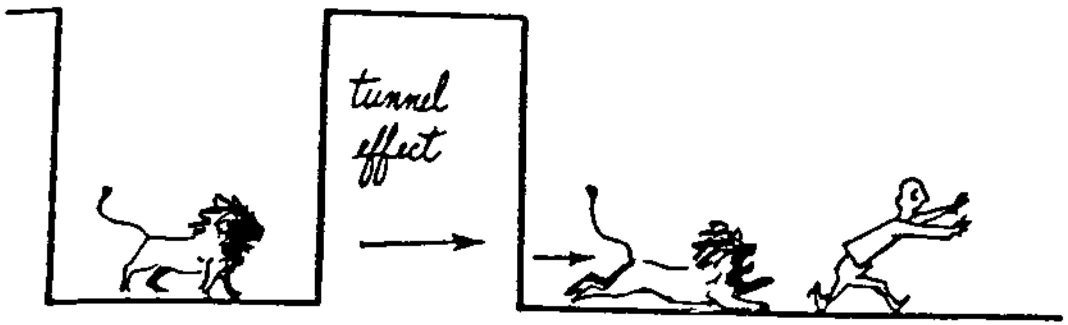
Outline

- A brief history of tunneling time
- Group delay and dwell time
- The Hartman Effect
- “Superluminal” tunneling experiments
- Meaning of tunneling group delay
- Origin of the Hartman Effect
- Reinterpretation of tunneling experimets
- Conclusions

Tunneling



Classical Mechanics

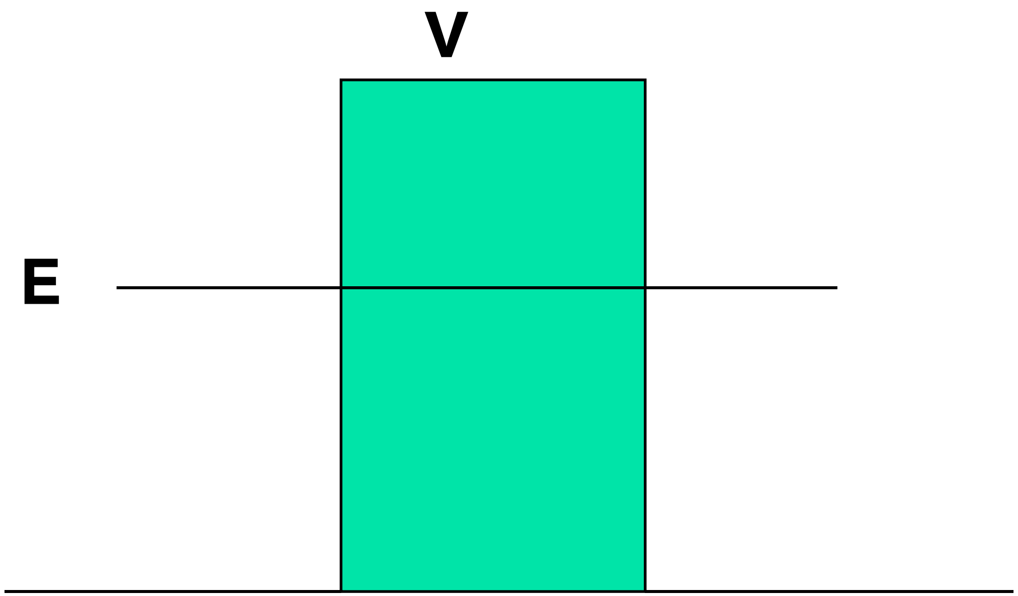


Quantum Mechanics

Importance of tunneling

- Radioactive alpha decay
- Biological processes such as photosynthesis
- Josephson junctions
- Esaki tunnel diodes
- Scanning tunneling microscopy
- ...etc

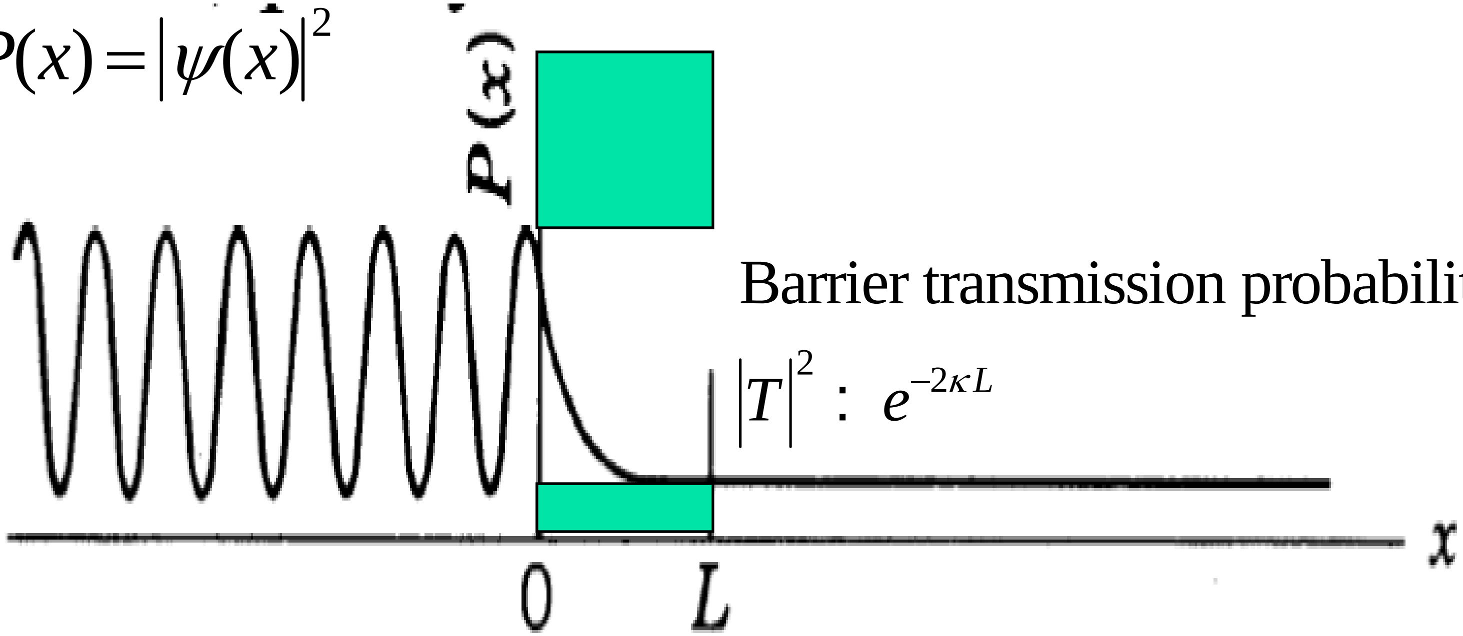
Stationary-state tunneling (1)



$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

Stationary-state tunneling (2)

$$P(x) = |\psi(x)|^2$$

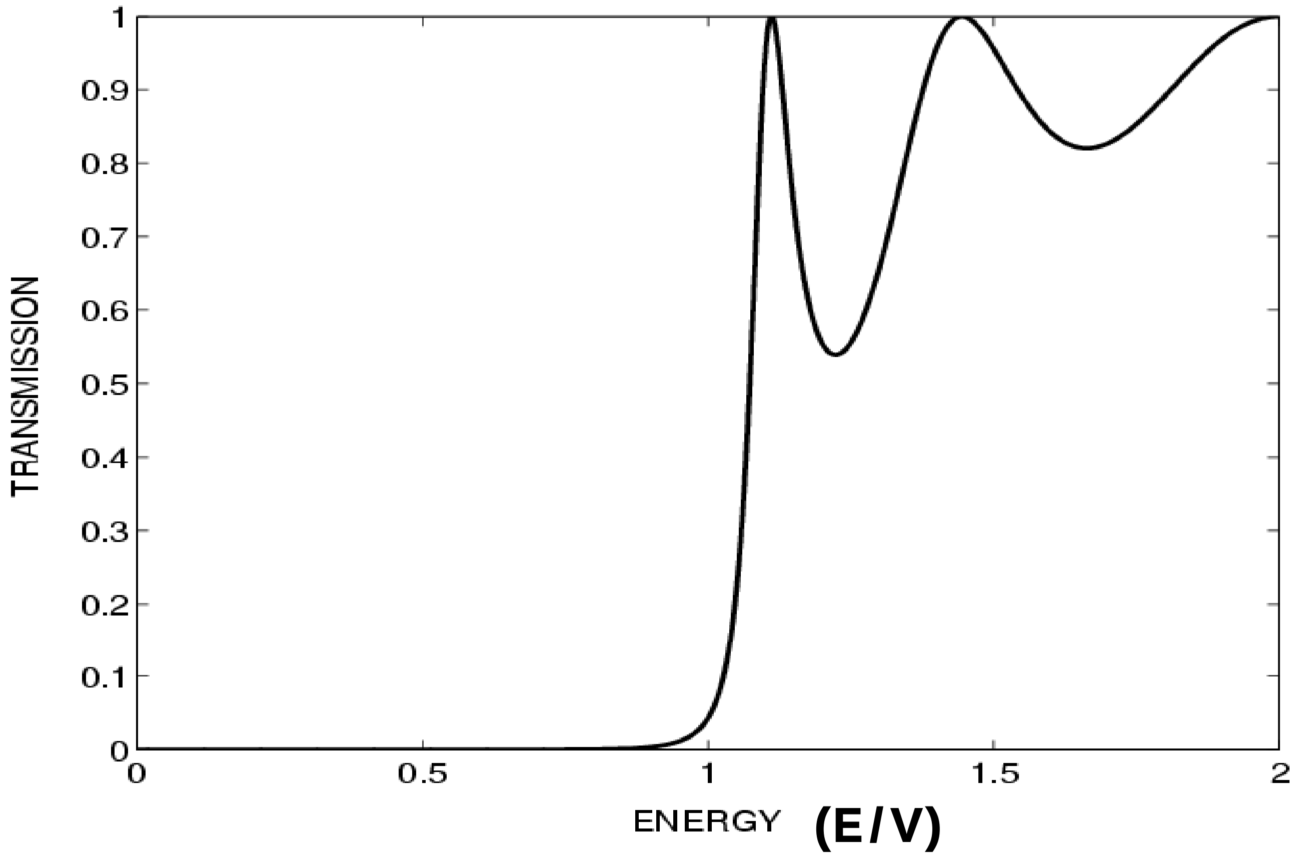


Barrier transmission probability

$$|T|^2 \sim e^{-2\kappa L}$$

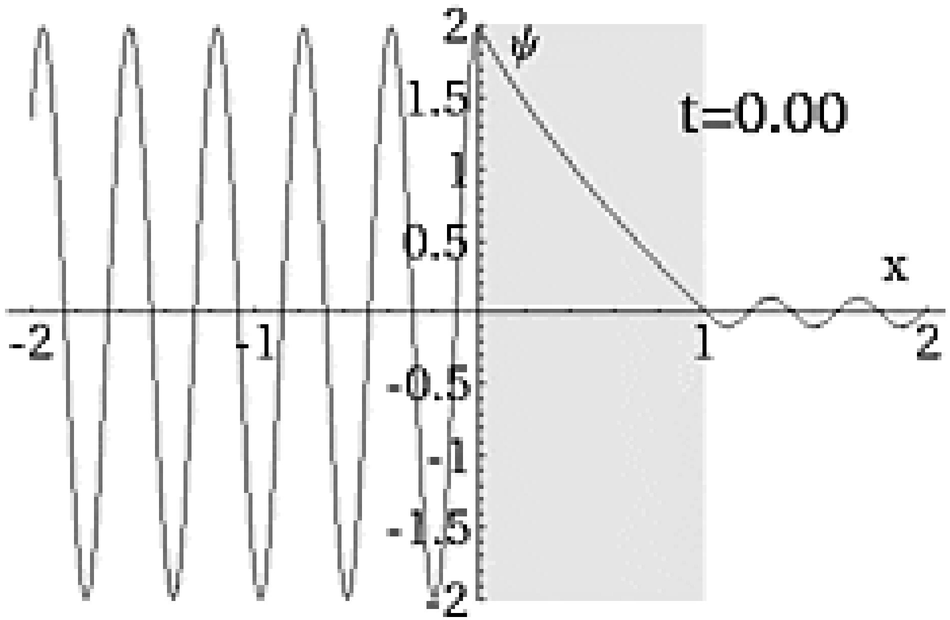
$$\kappa = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

Barrier transmission versus energy



Tunneling: the movie

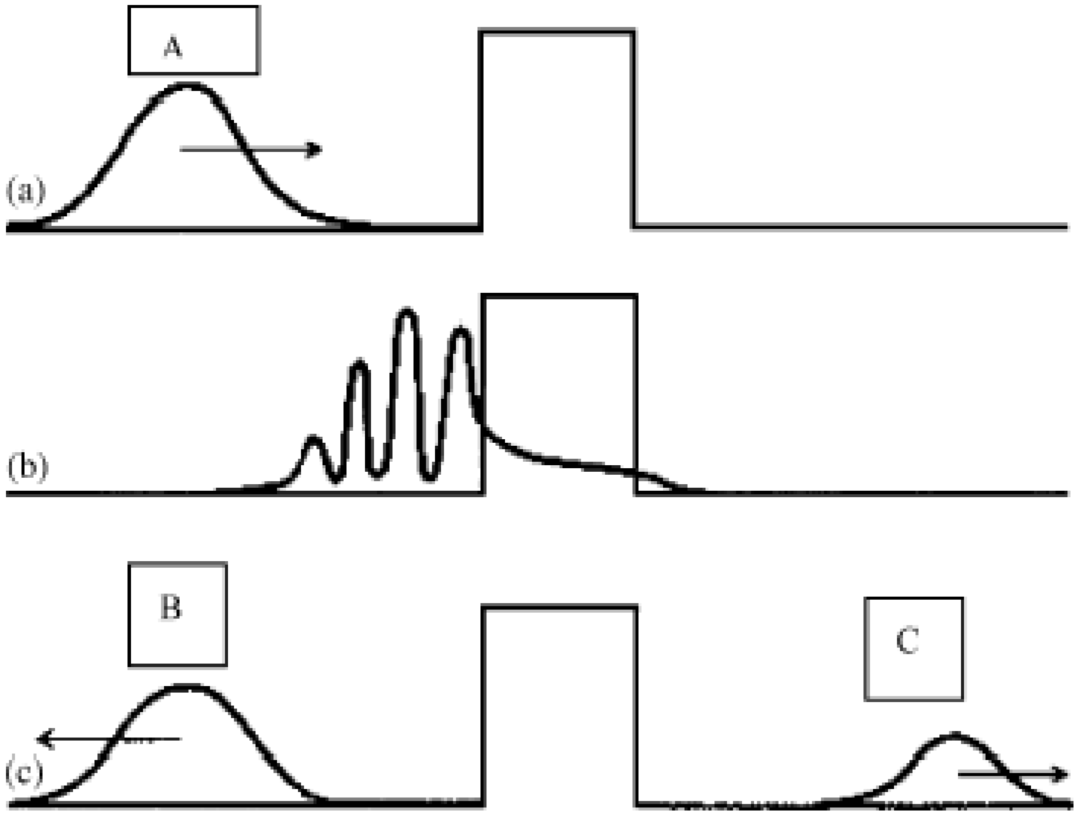
$$\mathrm{Re}\left\{\psi(x)e^{-iEt/\hbar}\right\}$$



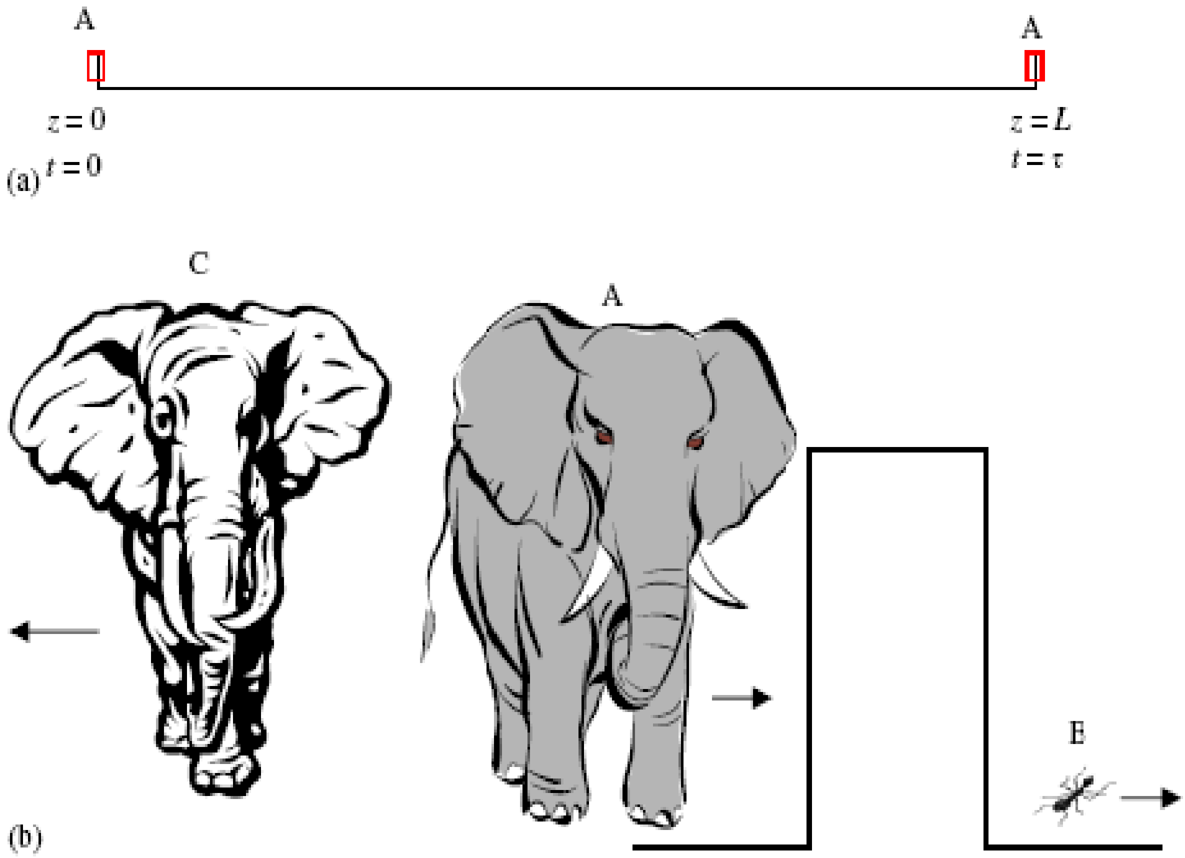
A brief history of tunneling time

- Condon (1931): “How long does it take to tunnel through a barrier?”
- MacColl (1932): “It takes no appreciable time” [*Phys. Rev.* ***40***, *621*].
- Bohm (1948), Wigner, Eisenbud (1952): energy derivative of transmission phase shift should yield time delay.
- Hartman (1962): time delay in tunneling is shorter than the “equal” time; can be faster than light! [*J. Appl. Phys.* ***33***, *3427*]

Tunneling Wavepackets



Relative Sizes of Wave packets



Method of stationary phase and group delay

Incident wave packet $\psi(x,t) = \int_E f(E - E_0)\psi_E(x)e^{-iEt/\hbar}dE$

Transmitted

$$\psi(x,t) = \int_E f(E - E_0)\left|T(E)\right|\exp\left[i\phi_t(E) + ikx - iEt/\hbar\right]dE$$

Group delay $\tau_g = \hbar\frac{\partial}{\partial E}(\phi + kL)$

The Schrödinger equation

$$\left[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V(x)\right]\psi(x,t) = -i\hbar\frac{\partial}{\partial t}\psi(x,t)\,. \qquad (1)$$

For stationary states the wave function separates into a time-independent part and a complex exponential time factor:

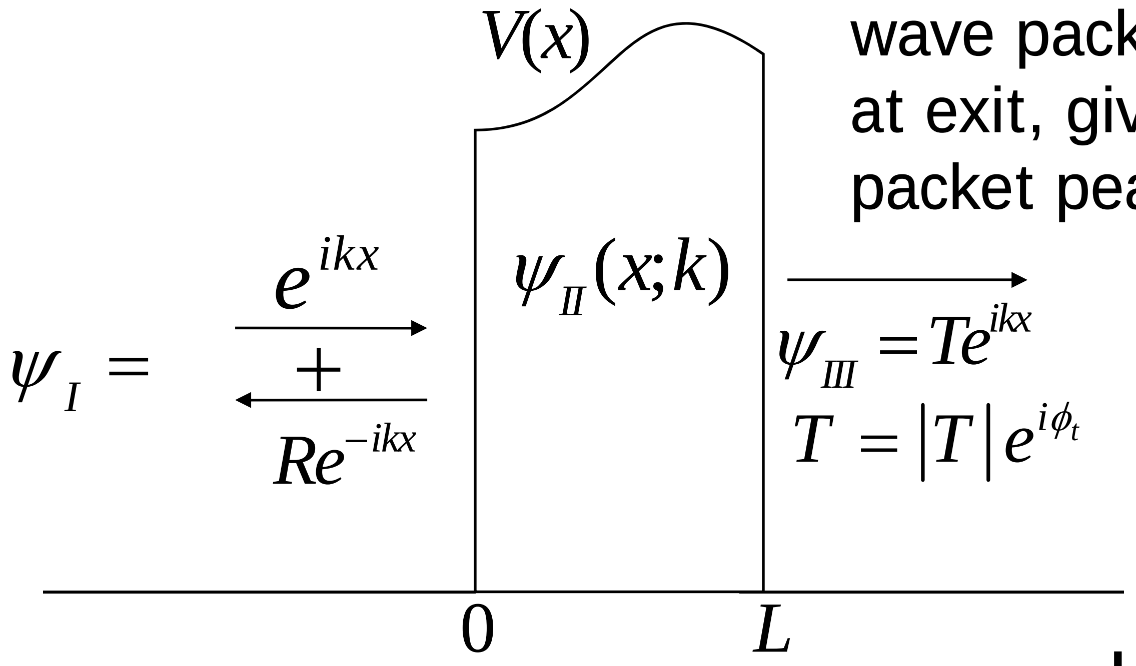
$$\psi(x,t) = \psi_E(x)\exp(-iEt/\hbar)\,,$$

with $\psi_E(x)$ a solution of the time independent Schr dinger equation

$$-\frac{h^2}{2m}\frac{d^2\psi_E}{dx^2} + (V - E)\psi_E = 0$$

Group delay τ_g

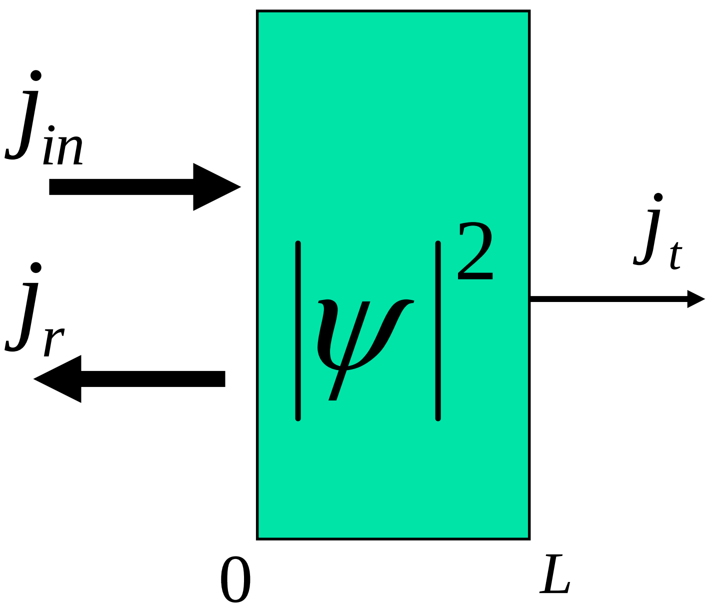
Time at which transmitted wave packet reaches a peak at exit, given that incident packet peaks at t=0 at x=0.



$$\tau_g = \hbar \frac{\partial}{\partial E}\left(\phi_t + kL\right)$$

Dwell Time τ_d

- Time spent in barrier, averaged over all incoming particles.

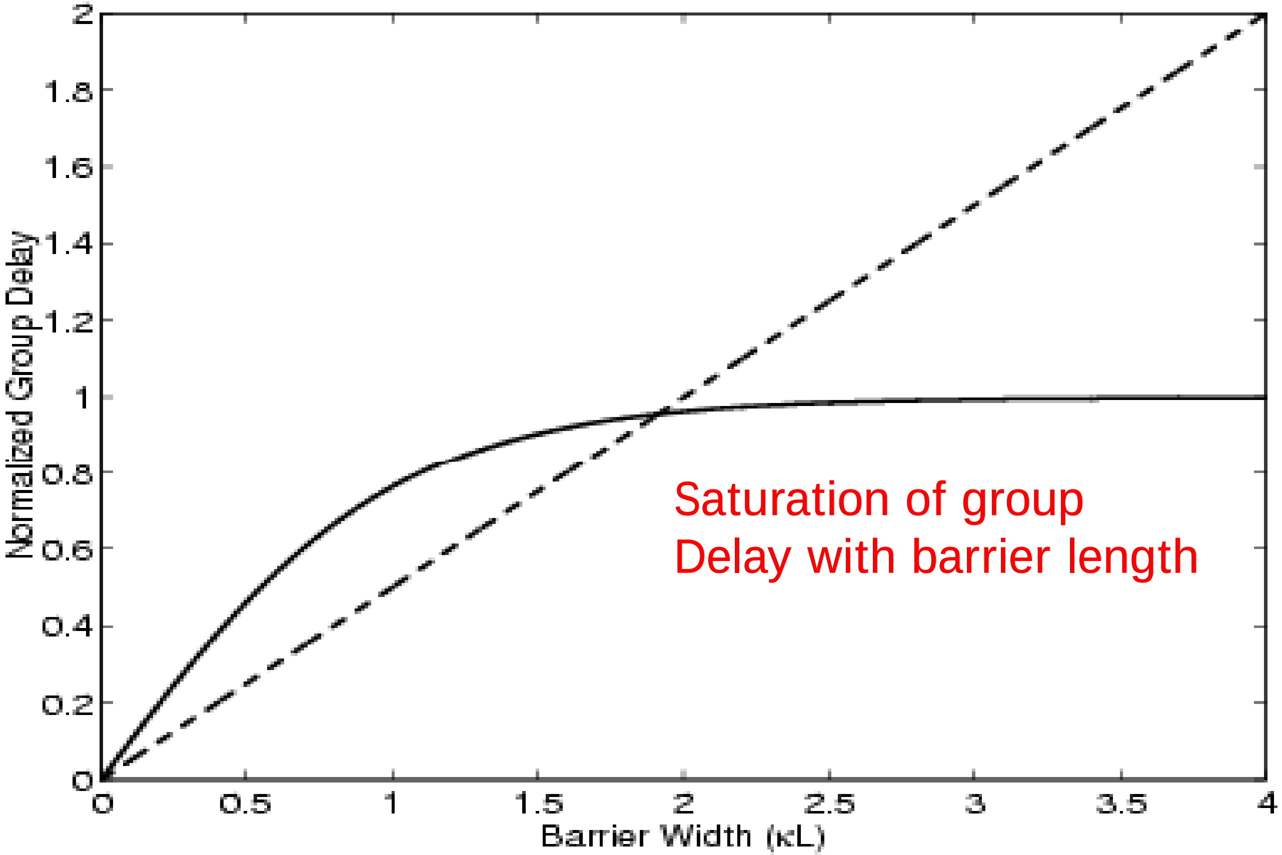


$$\tau_d = \frac{\int_0^L |\psi|^2 \, dx}{j_{in}}$$

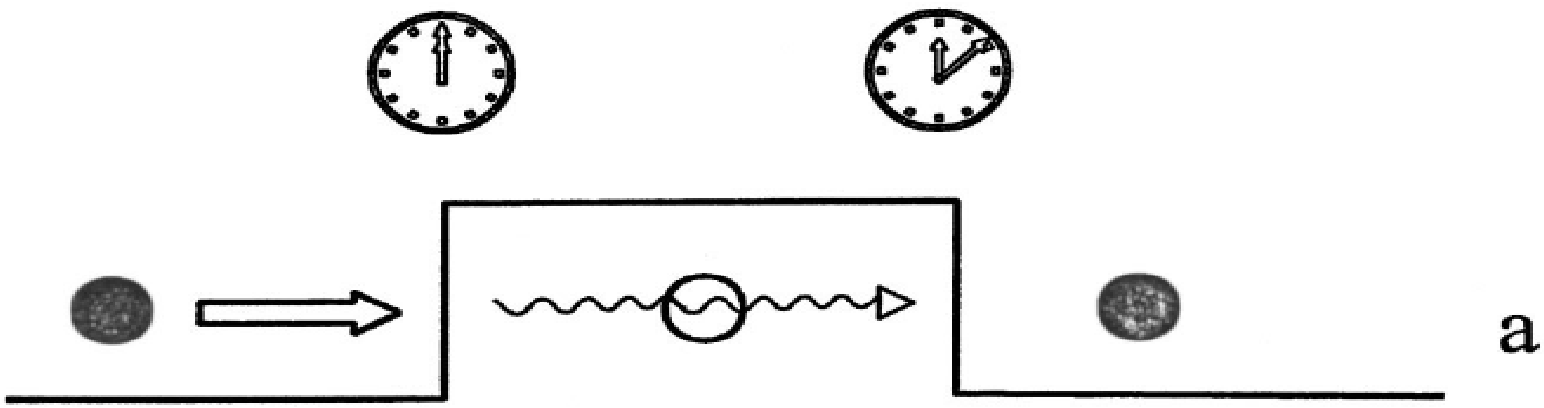
Hartman Effect (1962)

- Tunneling time becomes independent of length for thick barriers. [T.E.Hartman,*J.Appl. Phys. 33,3427 (1962)*]
- If tunneling time becomes independent of length, tunneling velocity must become infinite!
- *Origin of this effect has been a mystery for decades*

Hartman Effect



How to measure tunneling time?



Not an easy task with electrons.

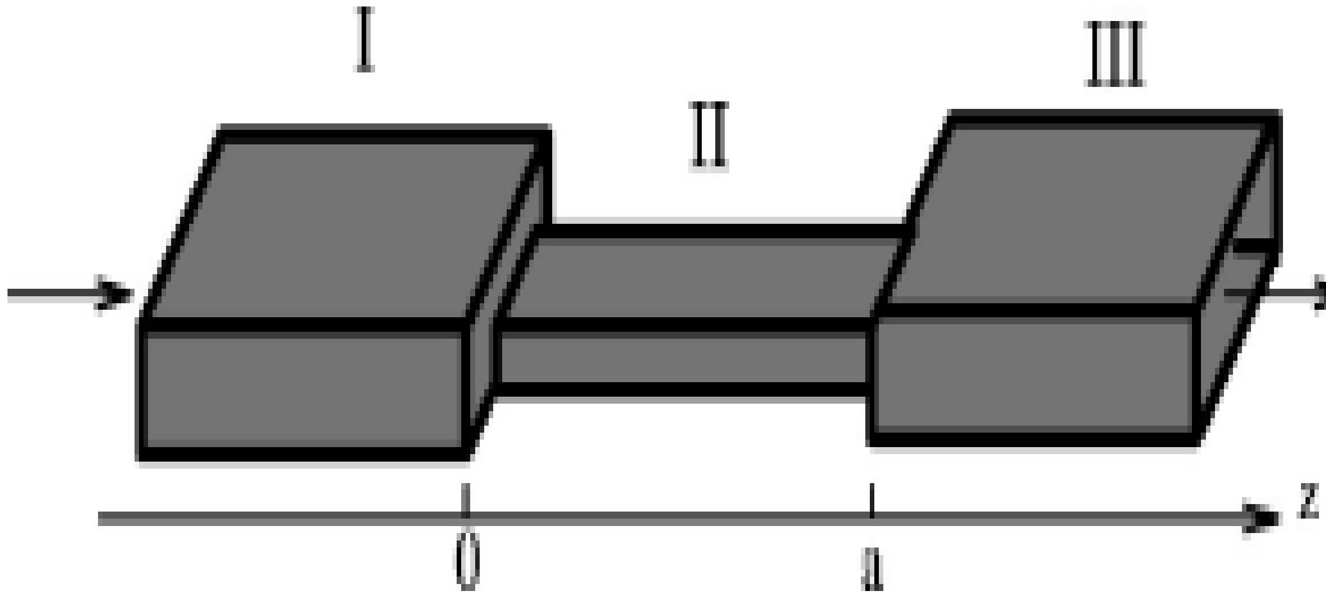
Electromagnetic analogy

- Time-independent Schrödinger equation and Helmholtz equation are identical

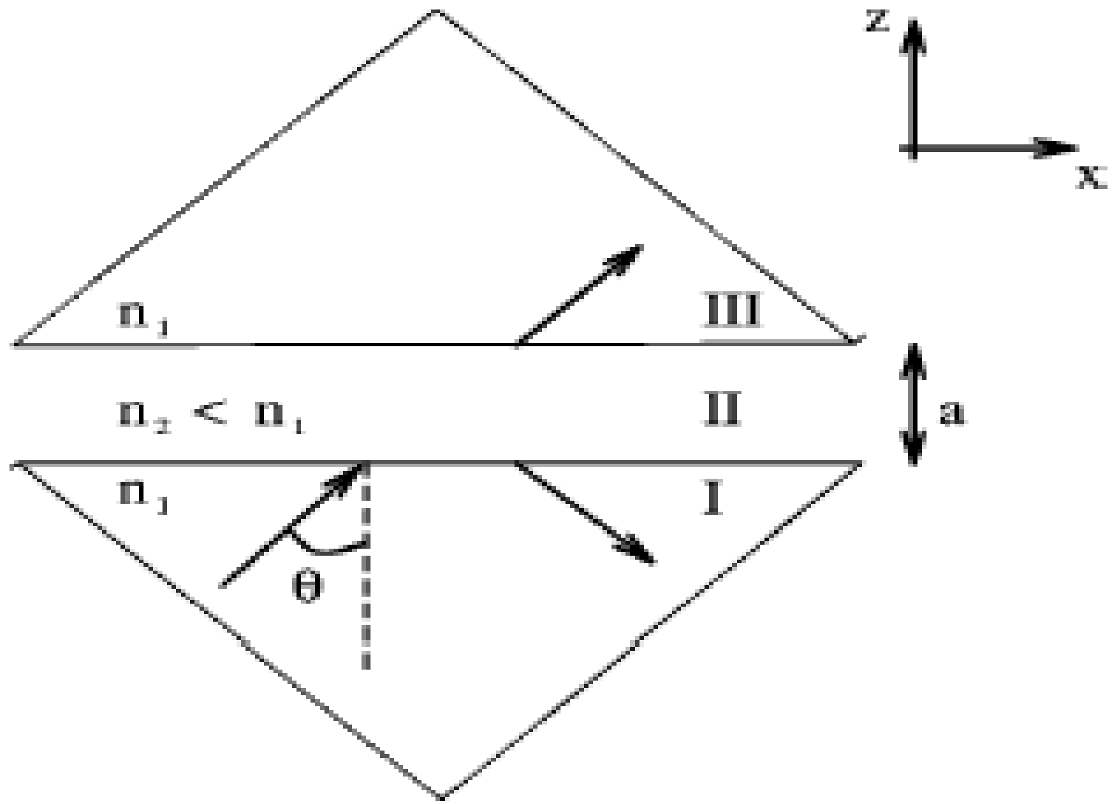
$$\frac{d^2\psi}{dz^2} + \beta^2\psi = 0$$

$$\beta^2 = \begin{cases} \dfrac{2m}{\hbar^2}(E - V) & \text{Quantum} \\ \left(k^2 - k_c^2\right) & \text{Electromagnetic} \end{cases}$$

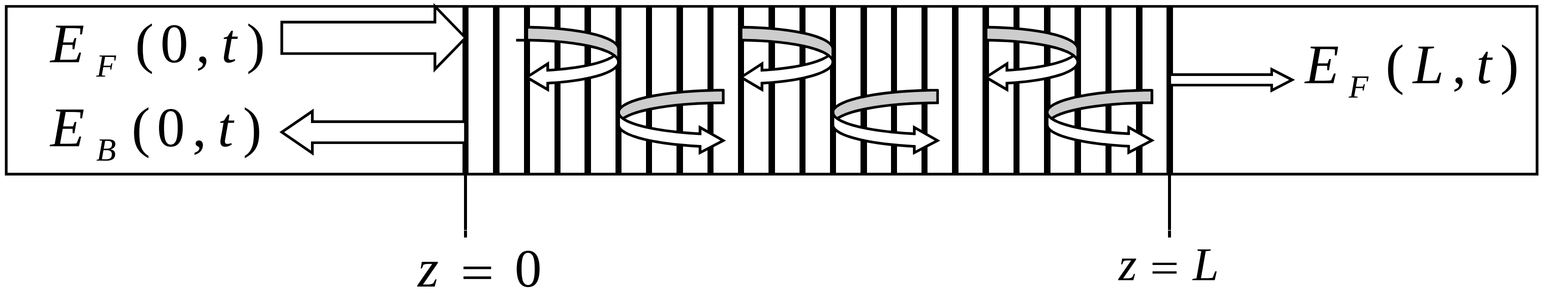
Waveguide with undersized region



Frustrated total internal reflection



1-D Photonic Bandgap Structure



Grating coupling constant $\kappa = \pi \Delta n / \lambda_B$

Bragg wavelength λ_B

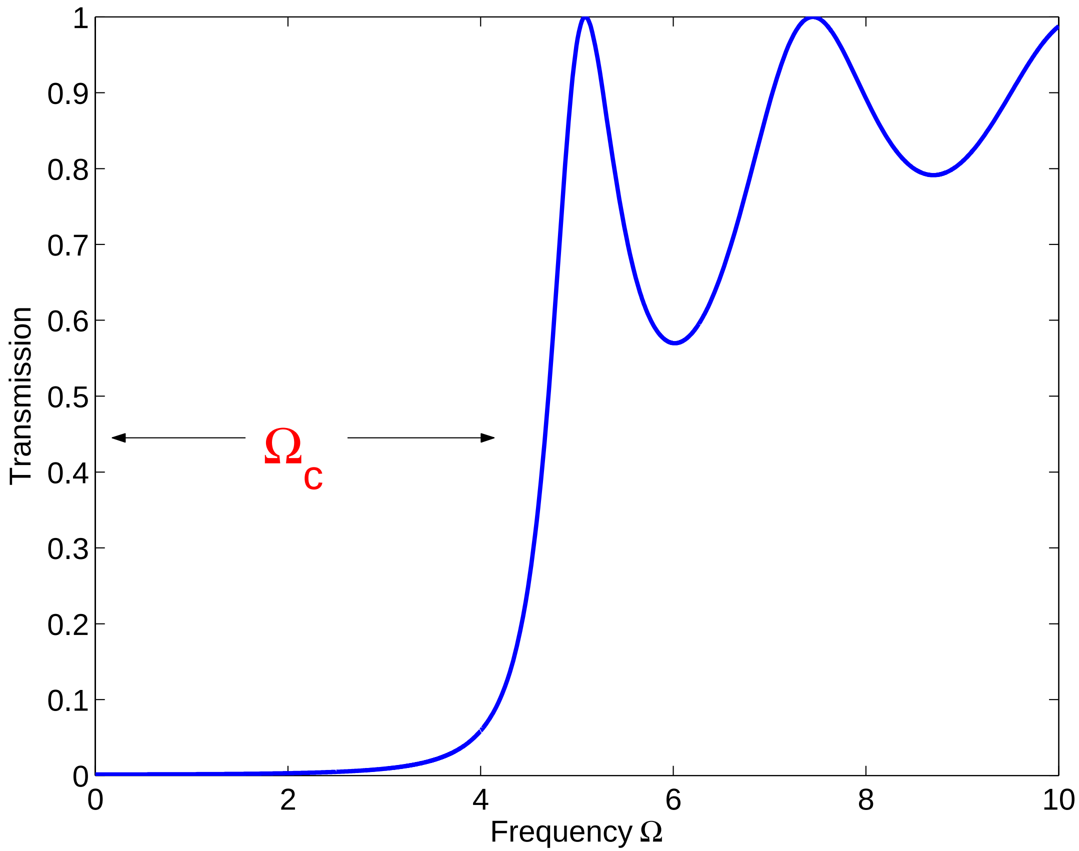
Bragg frequency ω_B

Detuning $\Omega = \omega - \omega_B$

Validity of Classical EM Approach

- The "single-photon" aspect of this experiment only plays a role in the detection process.
- The tunneling part of the experiment is completely describable by the classical Maxwell equations since it is a linear process. "Propagation effects are then governed by the classical wave equations, and quantization merely affects detection *statistics* and higher-order effects." (R.Y. Chiao and A.M. Steinberg, *Progress in Optics*)

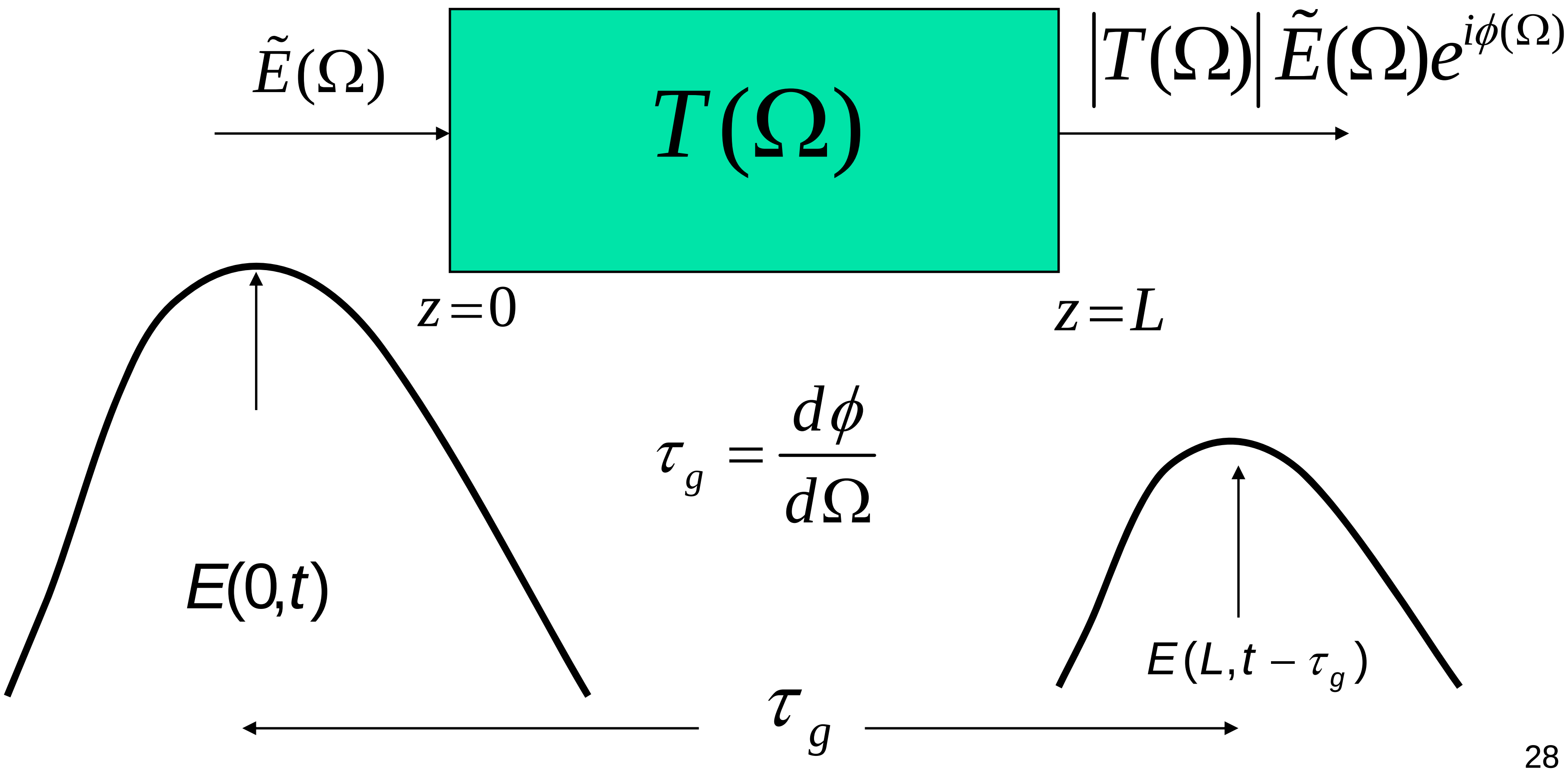
Electromagnetic Barrier Transmission



$$\Omega_c = \kappa v$$

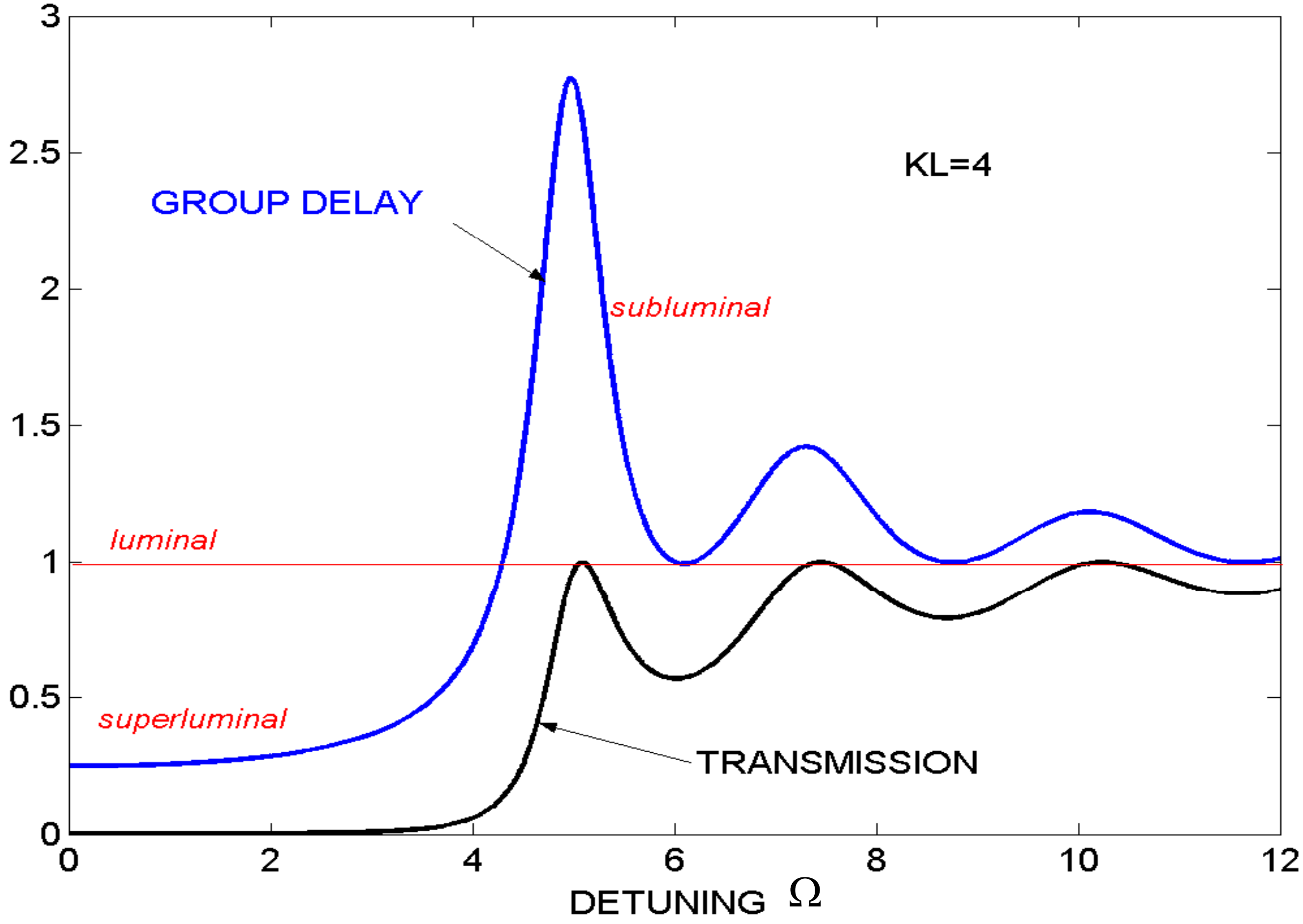
where v = c/n

Group delay τ_g



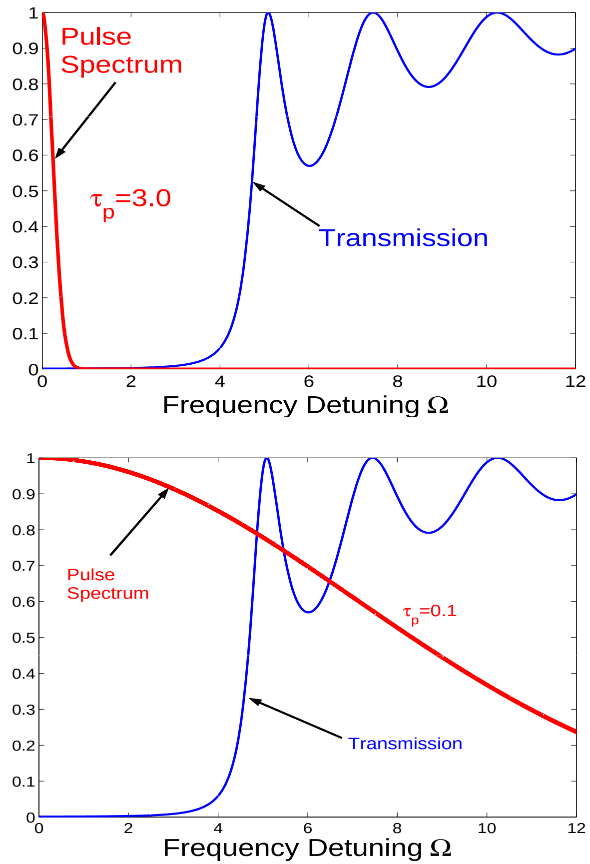
$$\tau_g = \frac{d\phi}{d\Omega}$$

Transmission and Group Delay



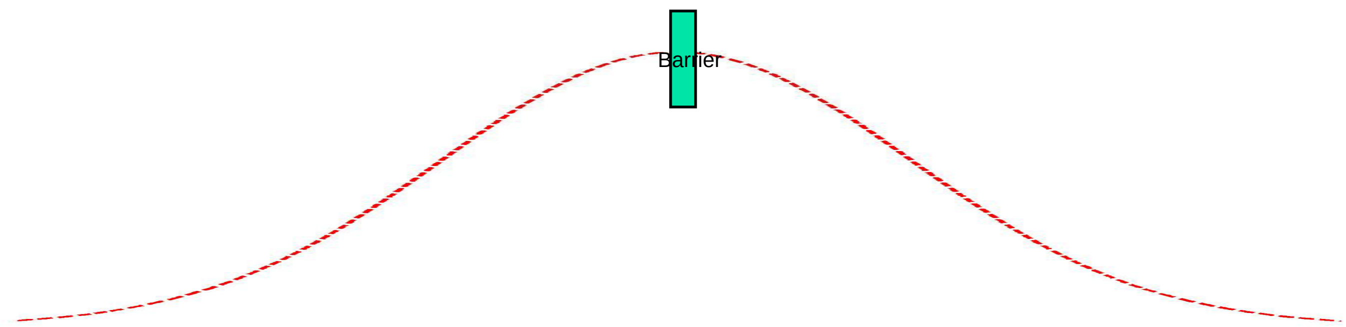
Distortionless Tunneling

- Tunneling without distortion requires that the pulse bandwidth be narrow compared to stopband.
- Broadband pulses have significant spectral content outside stopband.



Tunneling is Quasi-static

- In all tunneling time experiments, pulse length greatly exceeds device length.
- At any instant, field distribution in barrier is approximately the steady state distribution.



Requirement: Pulse bandwidth $1/\tau_p << \Omega_c = \kappa v$

$$\Rightarrow v\tau_p >> 1/\kappa$$

Since $\kappa L \sim 1$, this implies $v\tau_p >> L$

Tunneling time experiments

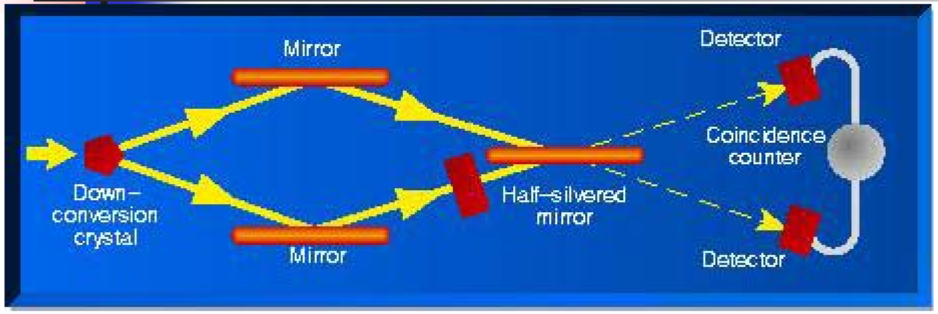
- Enders and Nimtz (1992): Electromagnetic measurements show “zero time” tunneling
- Steinberg, Kwiat, and Chiao, (1993): Measurements show superluminal group velocities in photon tunneling (~*1.7c*)
- Spielmann, et al (1994): Photonic experiments confirm Hartman’s predictions
- Longhi, et al (2002): Photonic experiments show no reshaping of tunneled pulses.

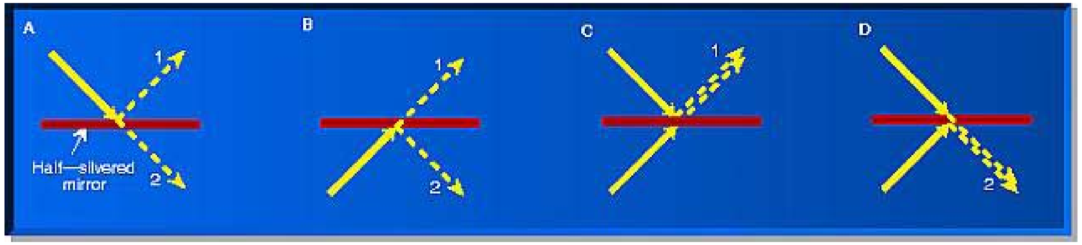
Experimental confirmation of Hartman effect

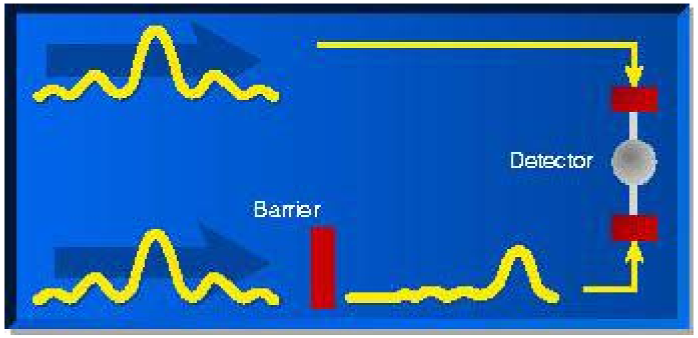
Balcou and Dutriaux, PRL, 78, 852 (1997)

QuickTime™ and a
decompressor
are needed to see this picture.

The SKC Experiment





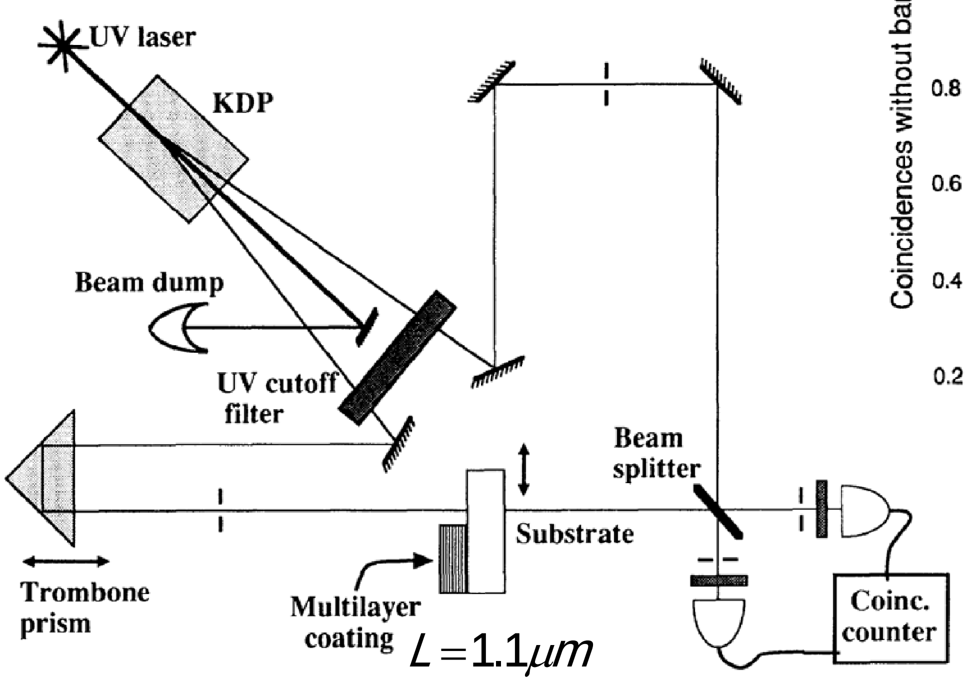


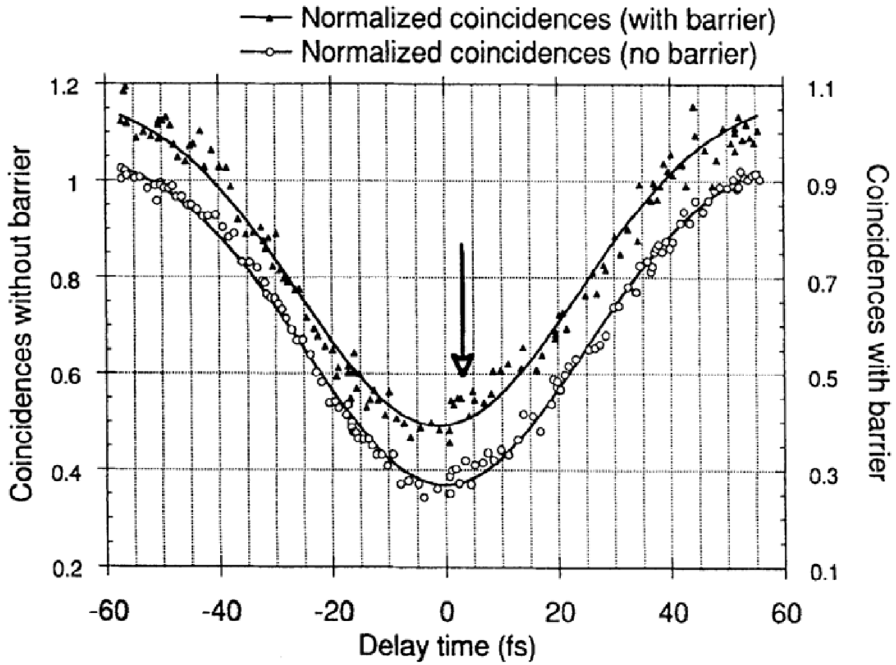
Steinberg, Kwiat, and Chiao, *PRL 71, 708 (1993)*

VOLUME 71, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1993

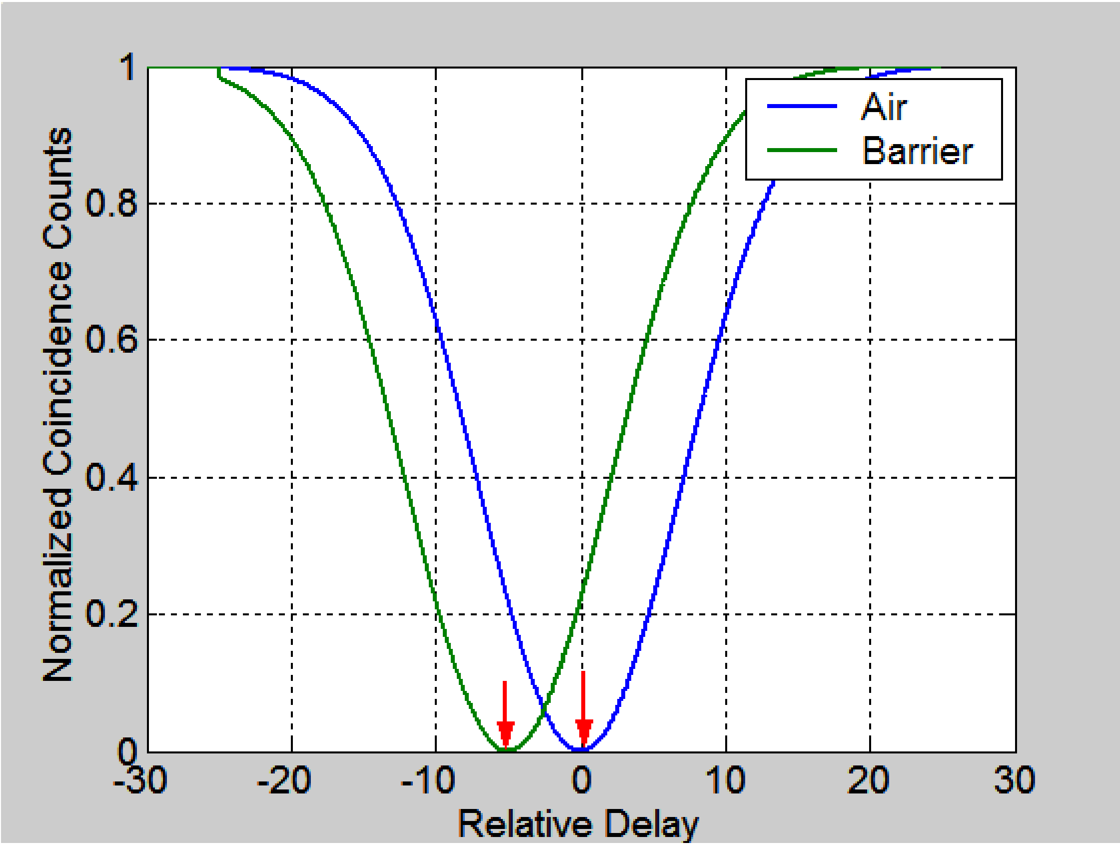
Measurement of the Single-Photon Tunneling Time

A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao
Department of Physics, University of California, Berkeley, California 94720





Free space delay=3.6 fs
Measured group delay=2.13 fs
Inferred group velocity=1.7*c*



Count rate ~

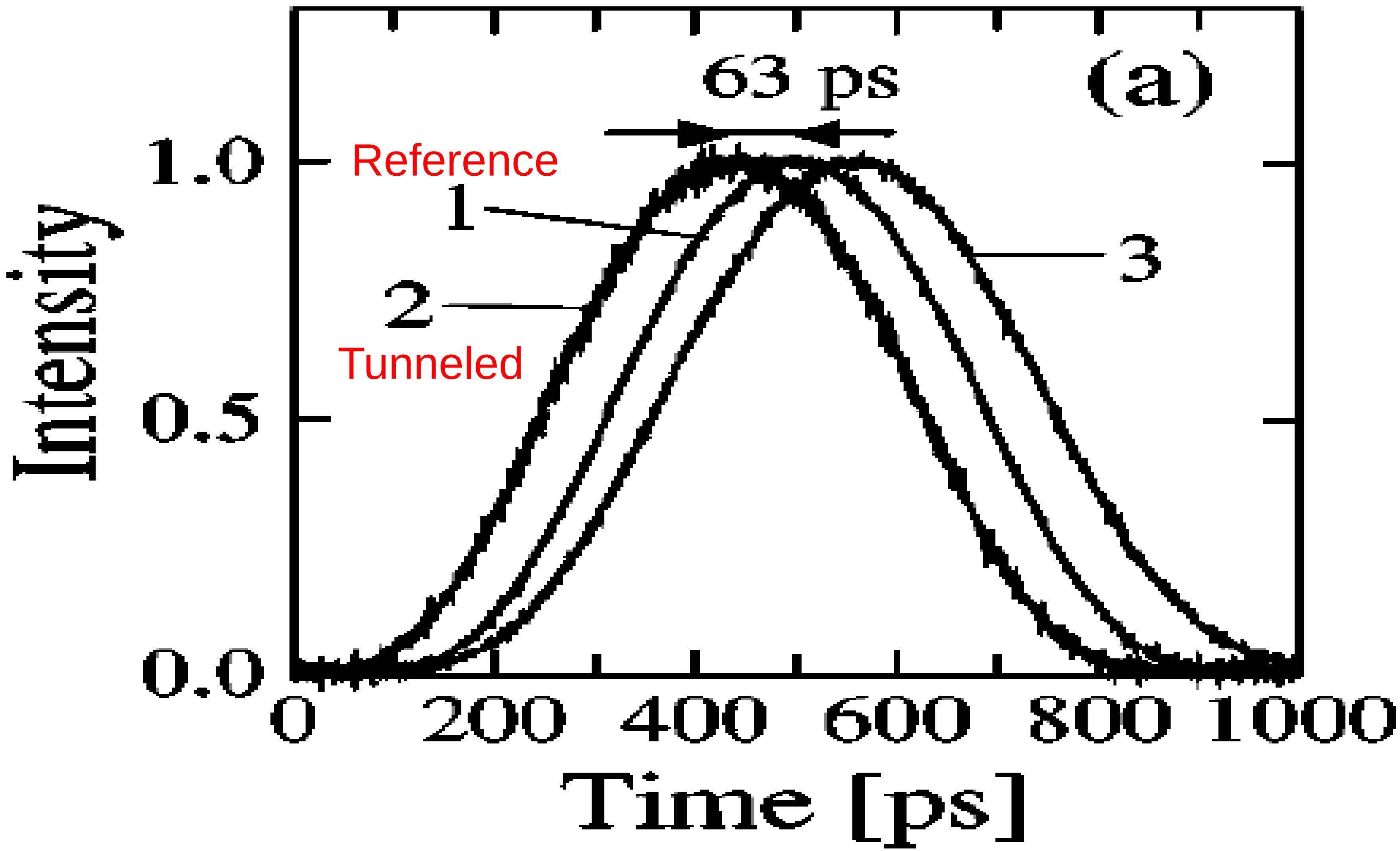
$$[1 - e^{-(\Delta x / c\tau_p)^2}]$$

Results of Steinberg, Kwiat, Chiao (SKC) Experiment

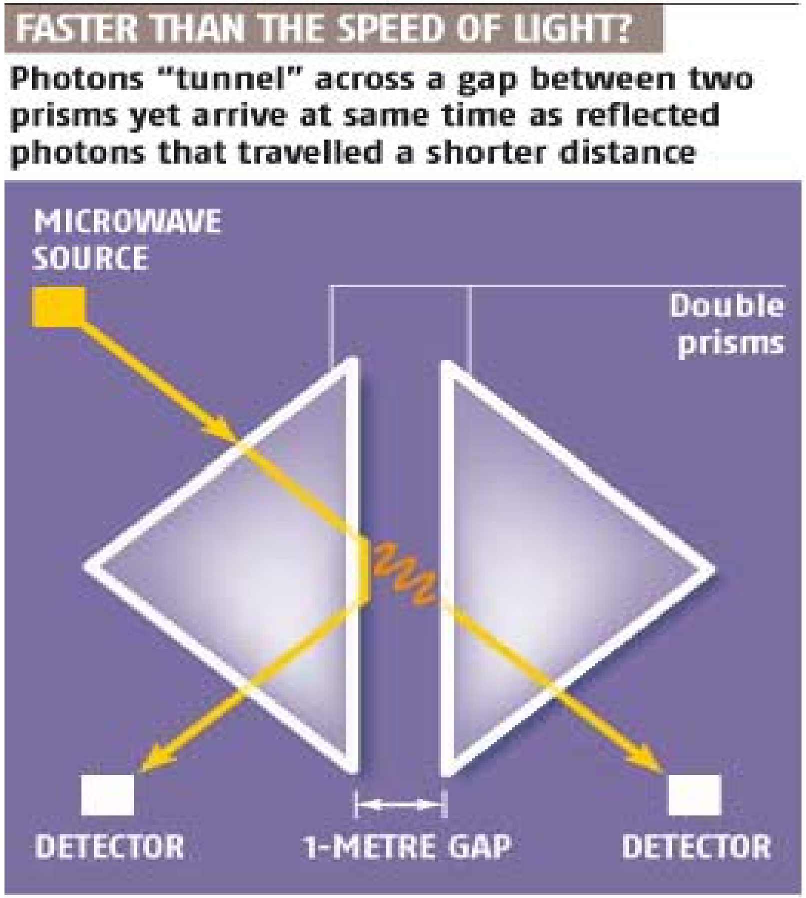
- Measured group delay of tunneling single photon ~ 2.13 fs. The delay is less than one optical cycle (2.34 fs) at 702 nm!
- From this delay a "group velocity" of 1.7*c* was inferred.
- Reasonable agreement with group delay prediction based on Maxwell's equations.
- Conclusion is that tunneling is superluminal and that single photons tunnel faster than light.

Another Experimental Result

(S. Longhi, et al, Phys. Rev. E, 2001)



Frustrated total internal reflection



New Scientist, August 17, 2007

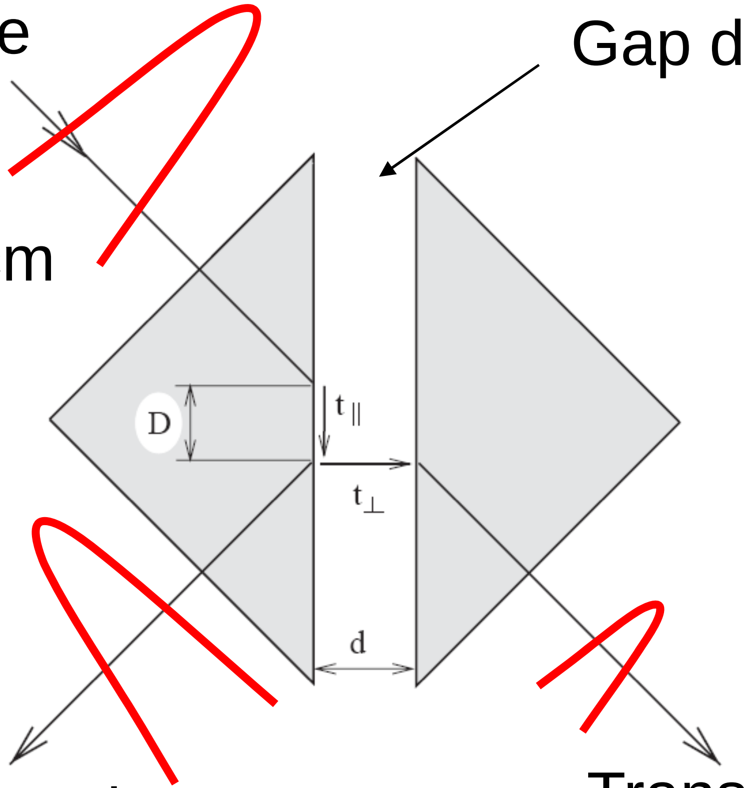
"Nimtz and Stahlhofen said the reflected photons and the tunneled photons both arrived at their respective photodetectors at the same time, leading them to conclude that some of the microwaves traveled faster than the speed of light. They also found that the tunneling time didn't change on a distance of up to three feet."

The Experiment

Incident Microwave Pulse
16 ns wide

Gap d~5 cm

Wavelength=3.28 cm



(*The pulses are much longer than shown here.)

Reflected
Delay=100 ps

Transmitted
Delay=100 ps

Klein-Gordon Equation for Tunneling Pulses

$$\frac{\partial^2 E_F}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E_F}{\partial t^2} = \kappa^2 E_F$$

$$E_F \sim \cos(Kz - \Omega t)$$

Dispersion relation: $K^2 = \Omega^2 / v^2 - \kappa^2$

Cutoff $\Omega_c = \kappa v$

Evanescent Waves (1)

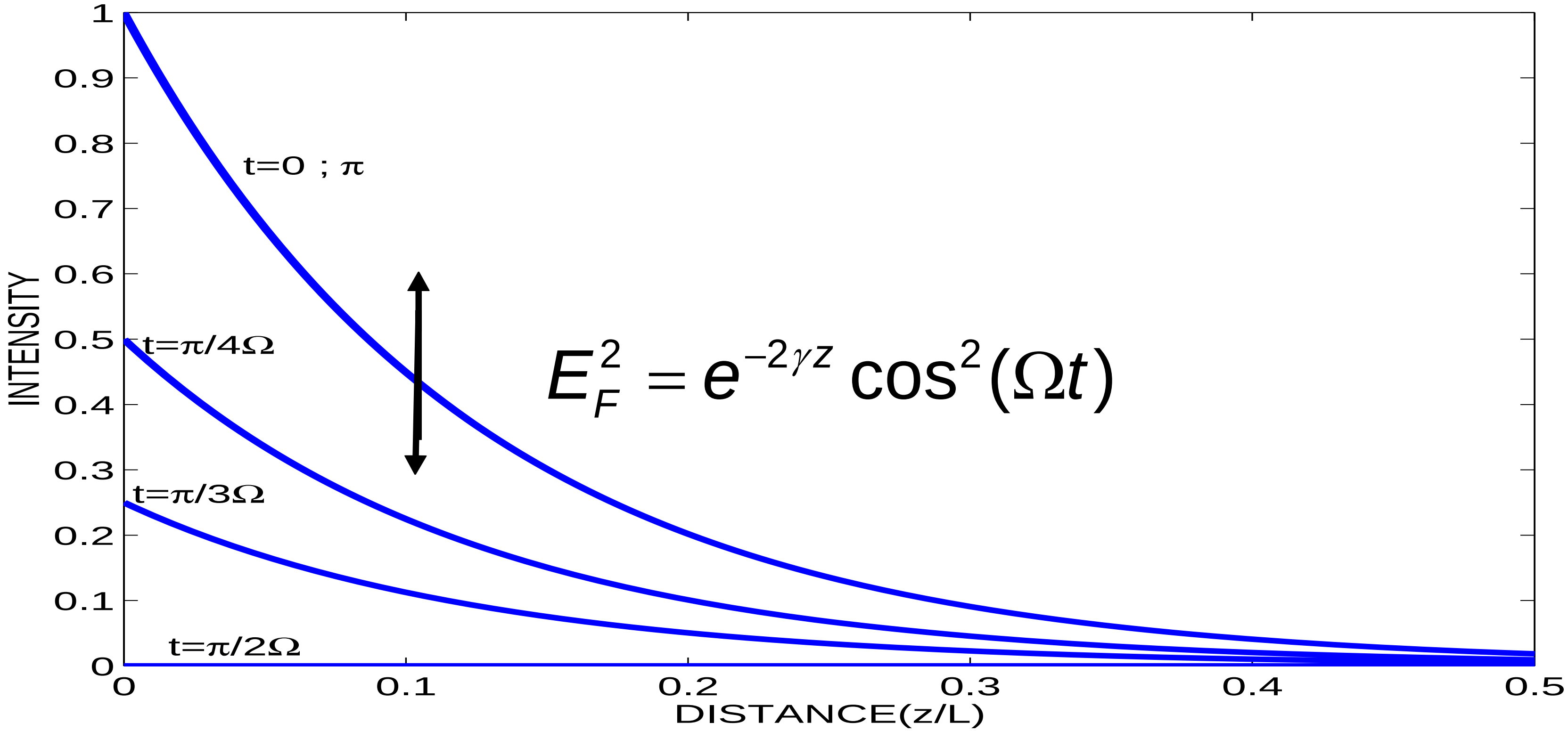
- For frequencies below cutoff frequency, we have exponentially attenuating standing "waves"

$$E_F \sim \exp(-\gamma z)\cos(\Omega t)$$

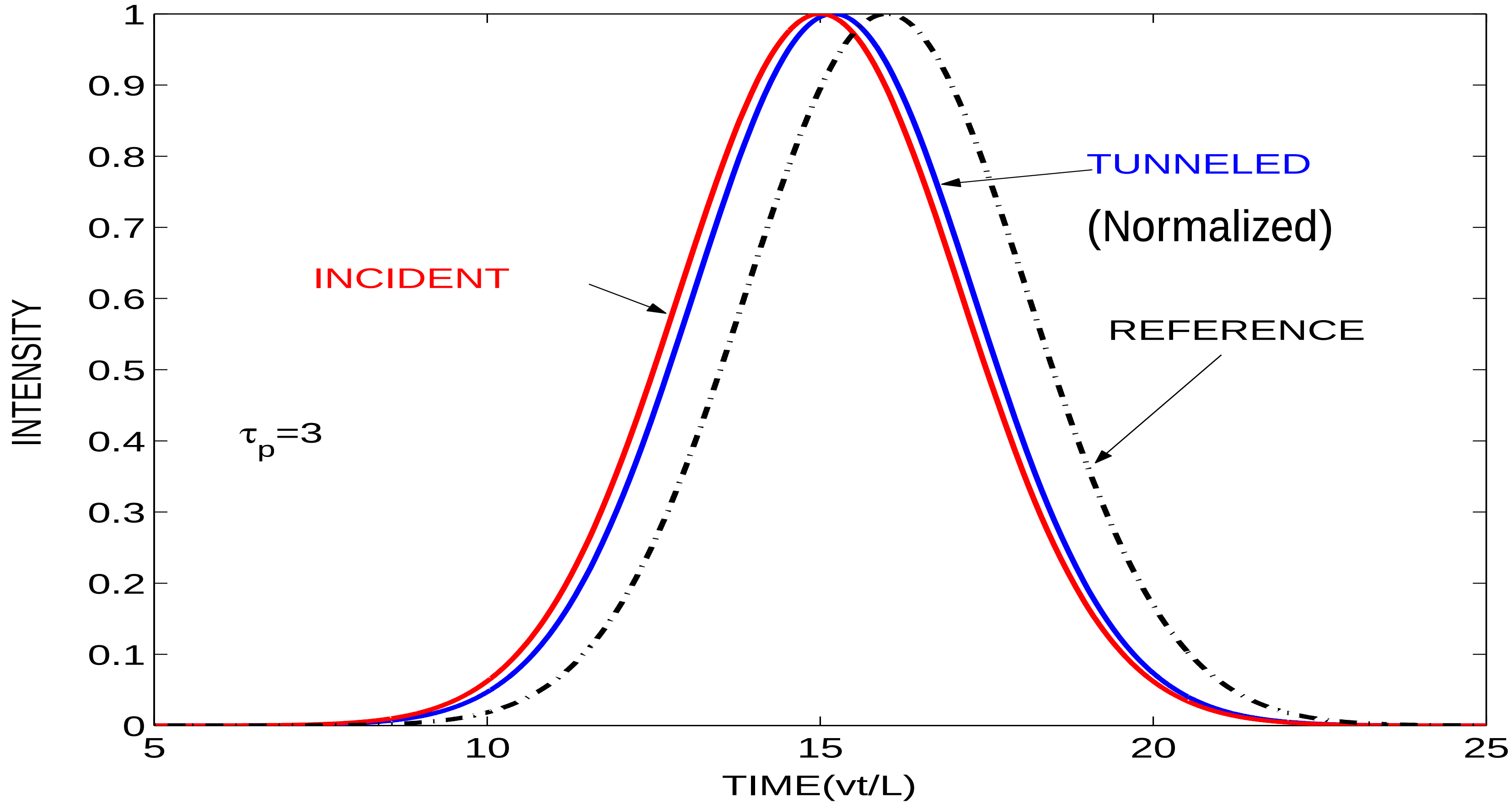
Attenuation constant $\gamma = \sqrt{(\Omega_c^2 - \Omega^2)/v^2}$

Evanescent Waves (2)

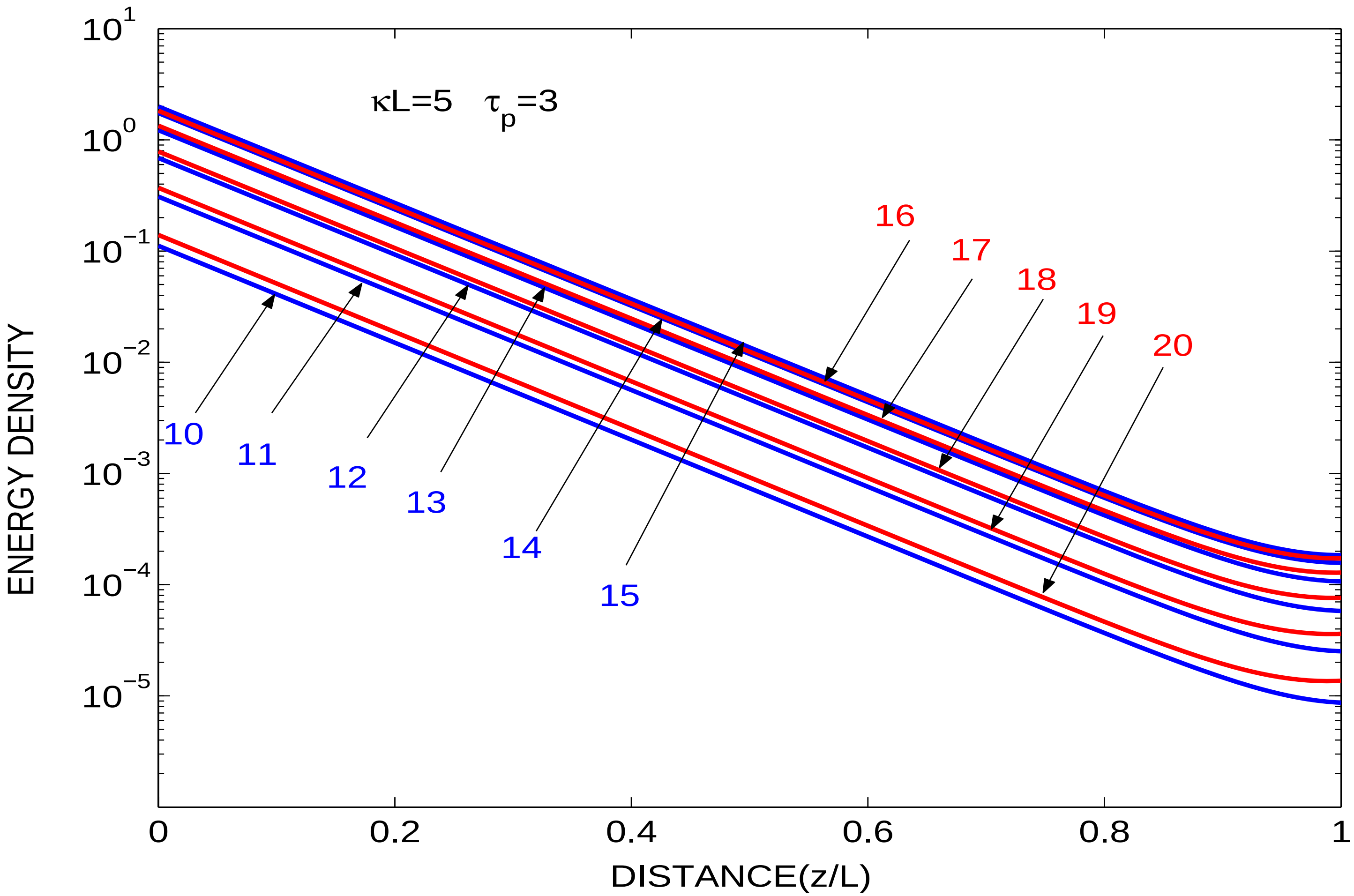
- Evanescent waves do not go anywhere. They merely stand and wave, every part in phase.



Tunneling simulation: $\tau_p = 3$



Long Pulse Tunneling: $\tau_P = 3$



Tunneling long pulse

Energy
Density

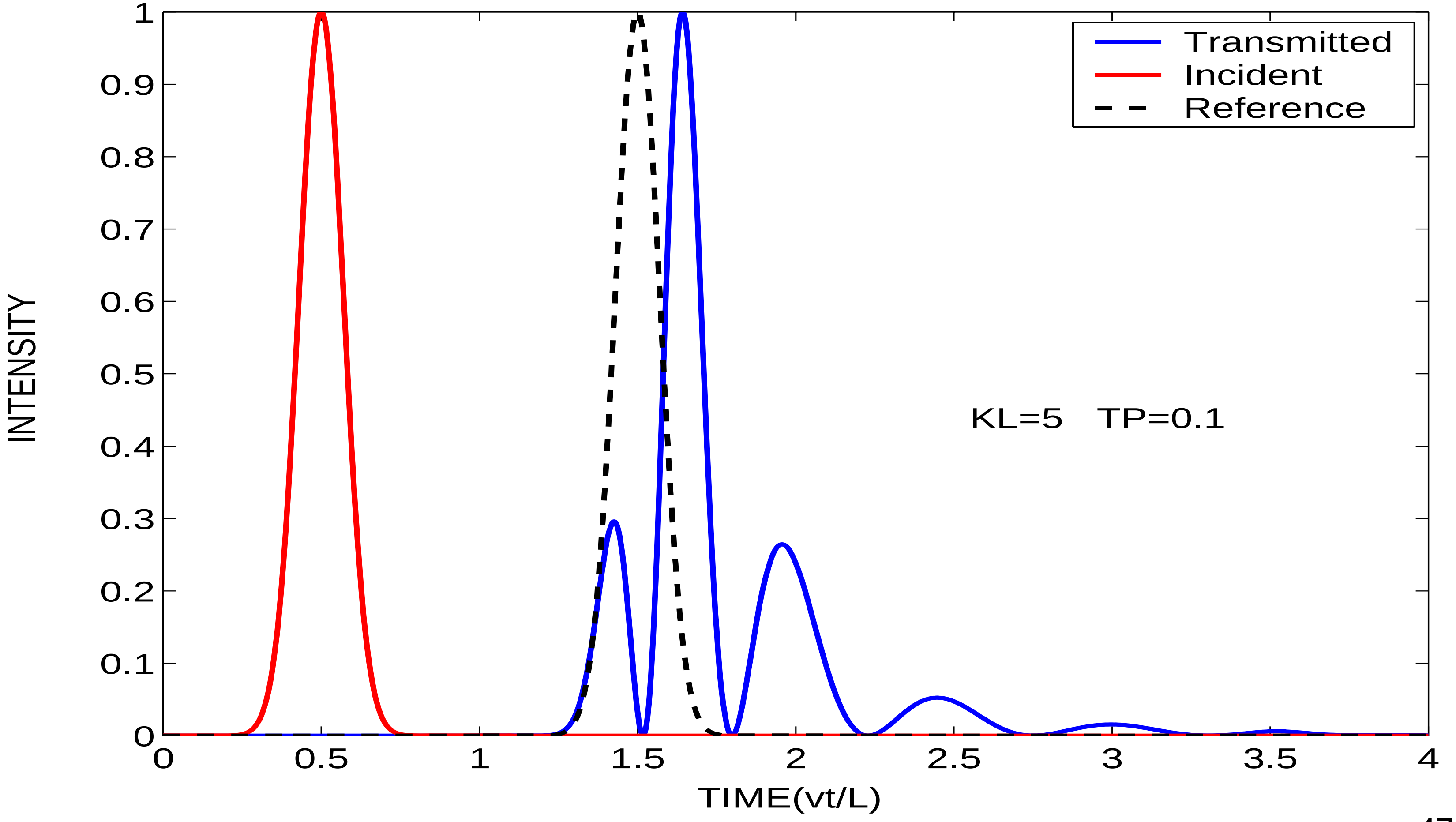
QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

0

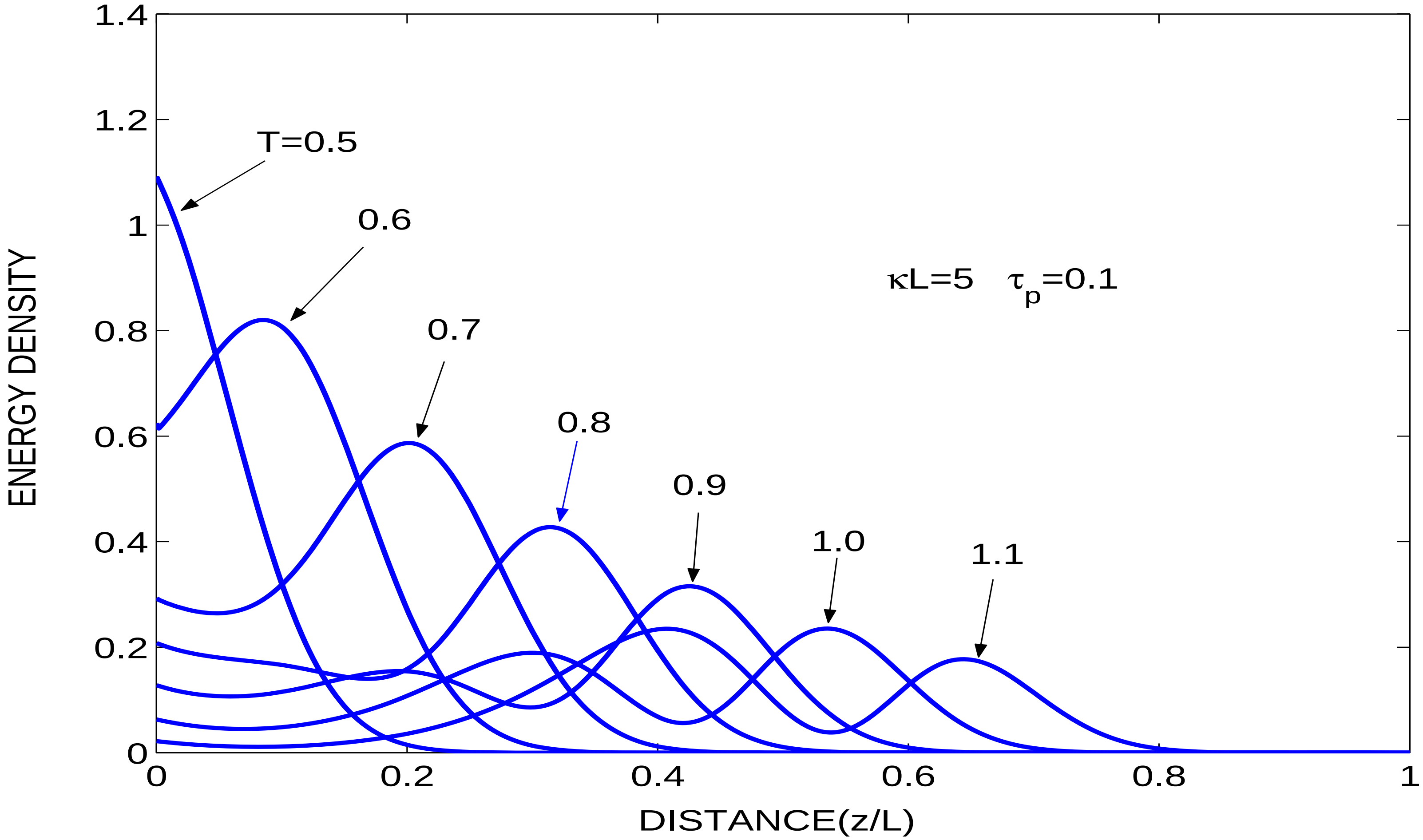
L

Distance along Barrier

Short pulse tunneling: $\tau_p = 0.1$

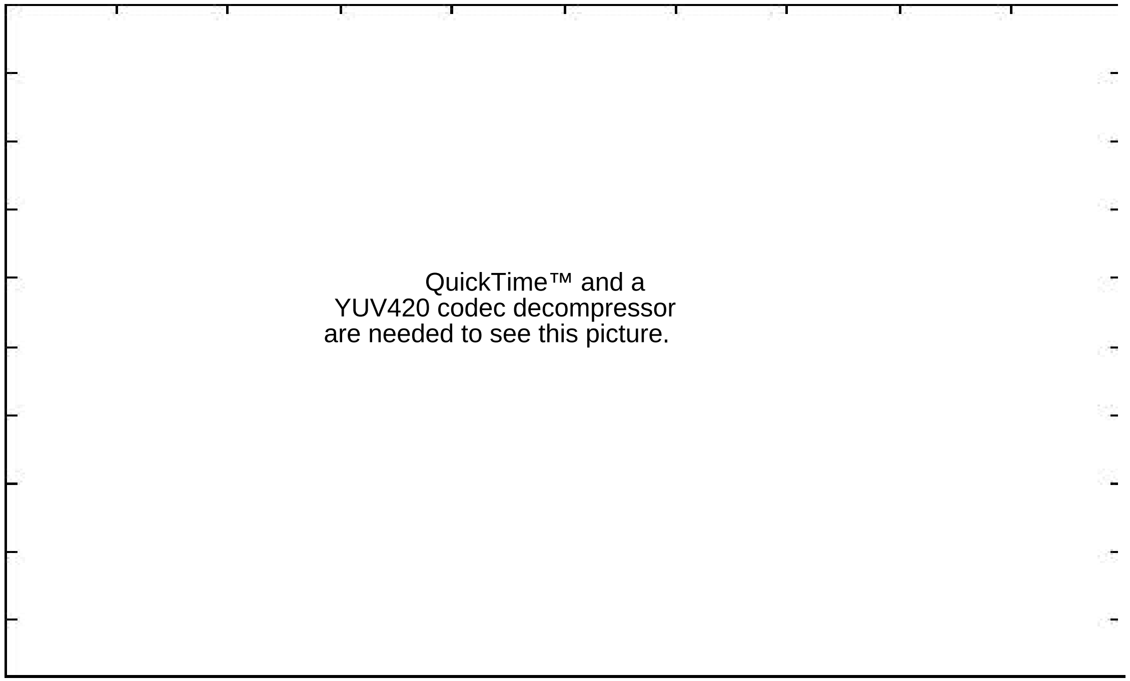


Short pulse tunneling: $\tau_p = 0.1$



“Tunneling” short pulse

Energy Density



0 L

Distance along Barrier

Group delay from transmission phase shift

Phase shift: $\phi = \tan^{-1}\left[\left(\Omega/\gamma v\right)\tanh\gamma L\right]$

Group delay : $\tau_g = d\phi \ / d\Omega$

$$\tau_g = \frac{1}{v}\left[\frac{\kappa^2}{\gamma^2}\frac{\tanh\gamma L}{\gamma} - L\left(\frac{\Omega}{\gamma v}\right)^2 \mathrm{sech}^2\gamma L\right]\cos^2\phi$$

Note that the quantity in blue $= v\tau_g$

Dwell Time τ_d

- Dwell time=Stored Energy/Input Power
- A property of an entire wave function with reflected and transmitted components.
- Does not differentiate between reflection and transmission channels.
- It is not a transit time if pulse is mostly reflected.

Time-average stored energy

$$\langle U \rangle = \frac{1}{4}\int_{vol} (\varepsilon \mathbf{E}\cdot\mathbf{E}^* + \mu \mathbf{H}\cdot\mathbf{H}^*)\, dv$$

$$\langle U \rangle = \left(\frac{1}{2}\varepsilon E_0^2 A\right)\left[\frac{\kappa^2}{\gamma^2}\frac{\tanh \gamma L}{\gamma} - L\left(\frac{\Omega}{\gamma v}\right)^2 \operatorname{sech}^2 \gamma L\right]\cos^2\phi$$

Dwell time: lifetime of stored energy escaping through both ends

$$\tau_d = \frac{\langle U \rangle}{P_{in}}$$

Dwell time is identical to group delay!

$$\tau_d = \frac{1}{v}\left[\frac{\kappa^2}{\gamma^2}\frac{\tanh\gamma L}{\gamma} - L\left(\frac{\Omega}{\gamma v}\right)^2 \operatorname{sech}^2\gamma L\right]\cos^2\phi$$

$$= \tau_g$$

And both are proportional to stored energy in the barrier.

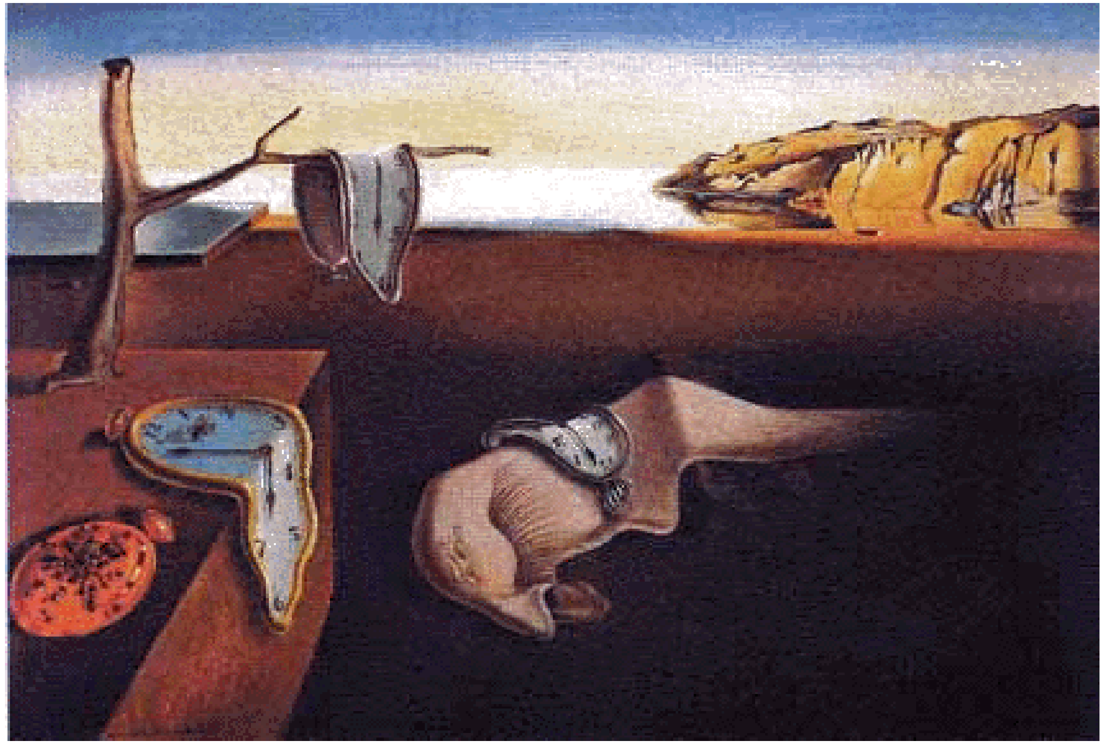
Winful, Optics Express, 2002; PRL 2003

For PBG's dwell time is identical to group delay!

$$\tau_d = \frac{\langle U \rangle}{P_{in}} = \frac{\partial \phi}{\partial \Omega} = \tau_g = \frac{L}{v}\left[\frac{\tanh \kappa L}{\kappa L}\right]$$

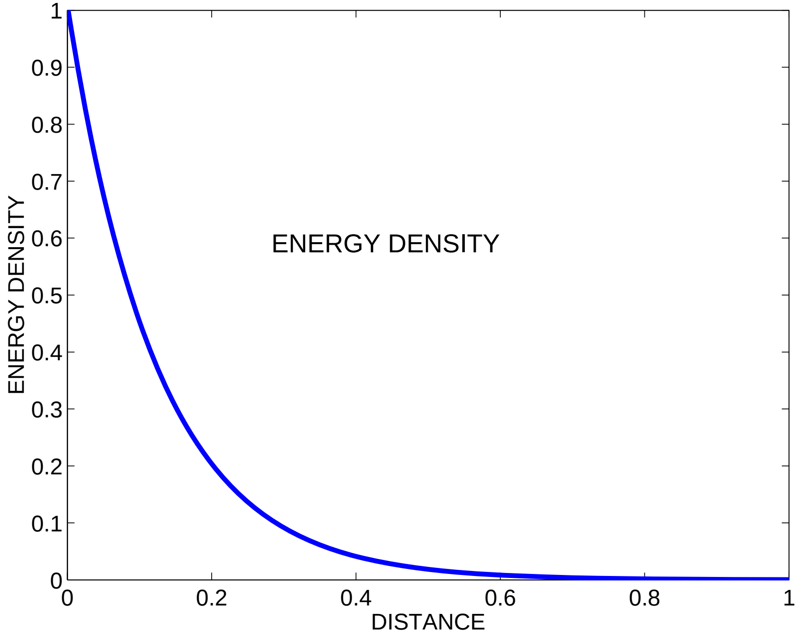
And both are proportional to stored energy in the barrier.

Origin of the Hartman effect...or why does time appear to stand still?



Saturation of stored energy with barrier length

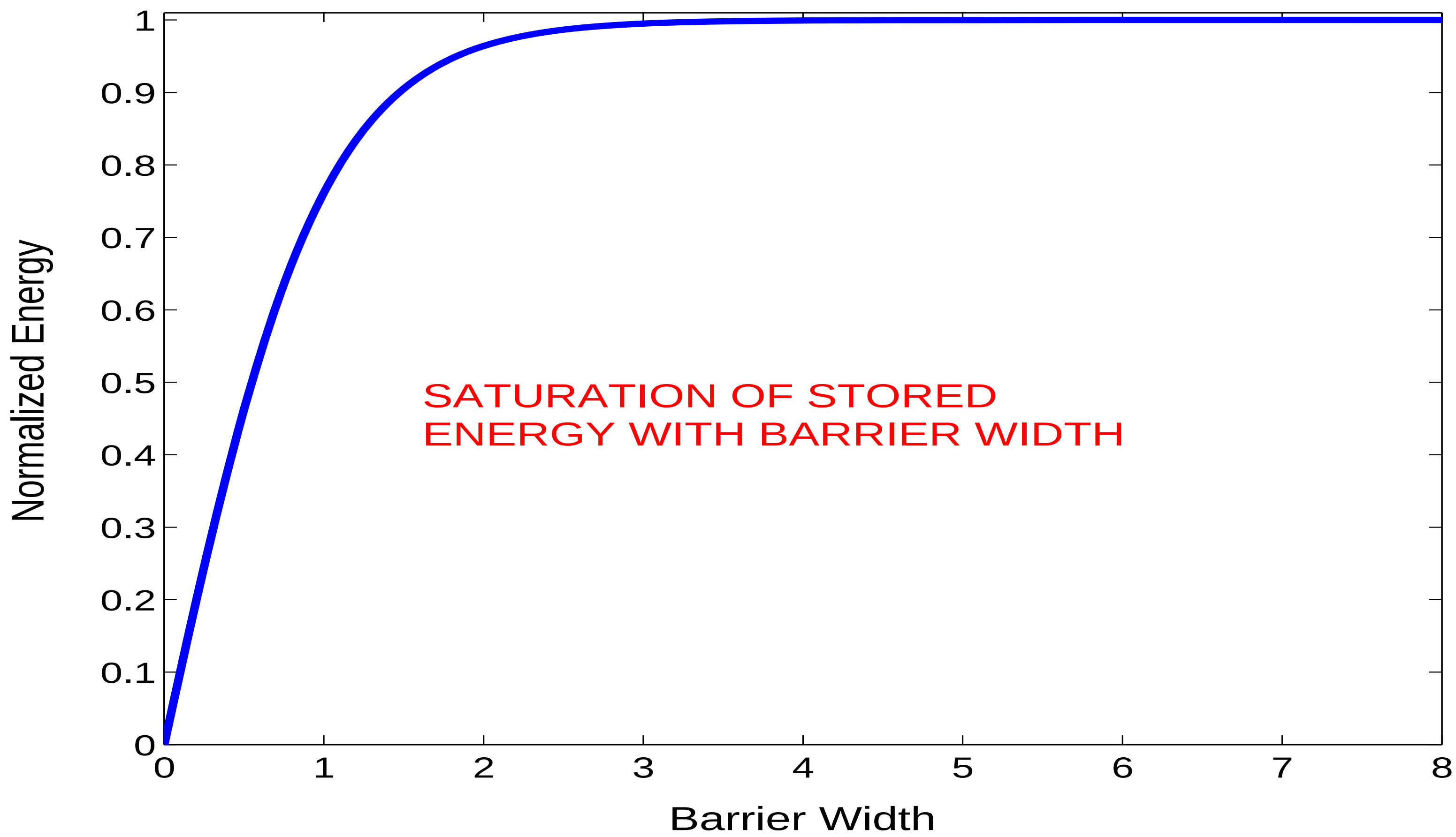
$$\lim_{L\to\infty}\langle U\rangle = \frac{1}{2}\varepsilon E_0^2 A/\kappa = P_{in}/\kappa v$$



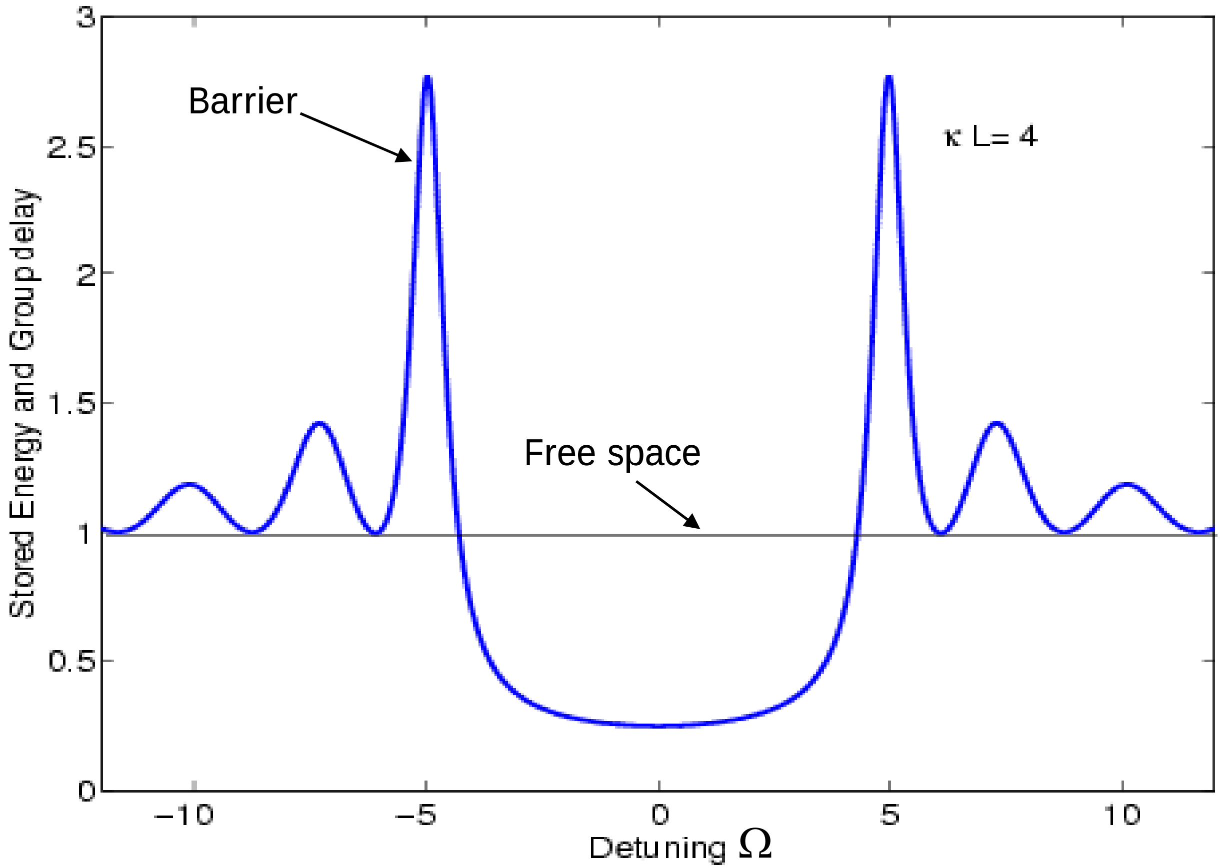
$$\Rightarrow \lim_{L\to\infty}\tau_g = 1/\kappa v$$

Group delay saturates because stored Energy saturates.

Origin of Hartman Effect

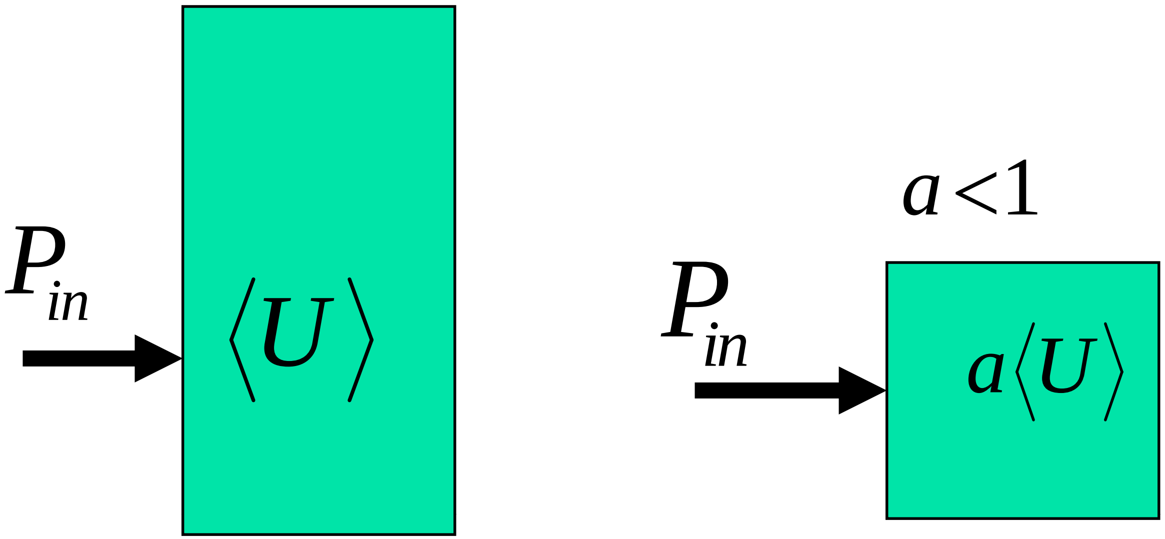


Stored Energy and Group Delay

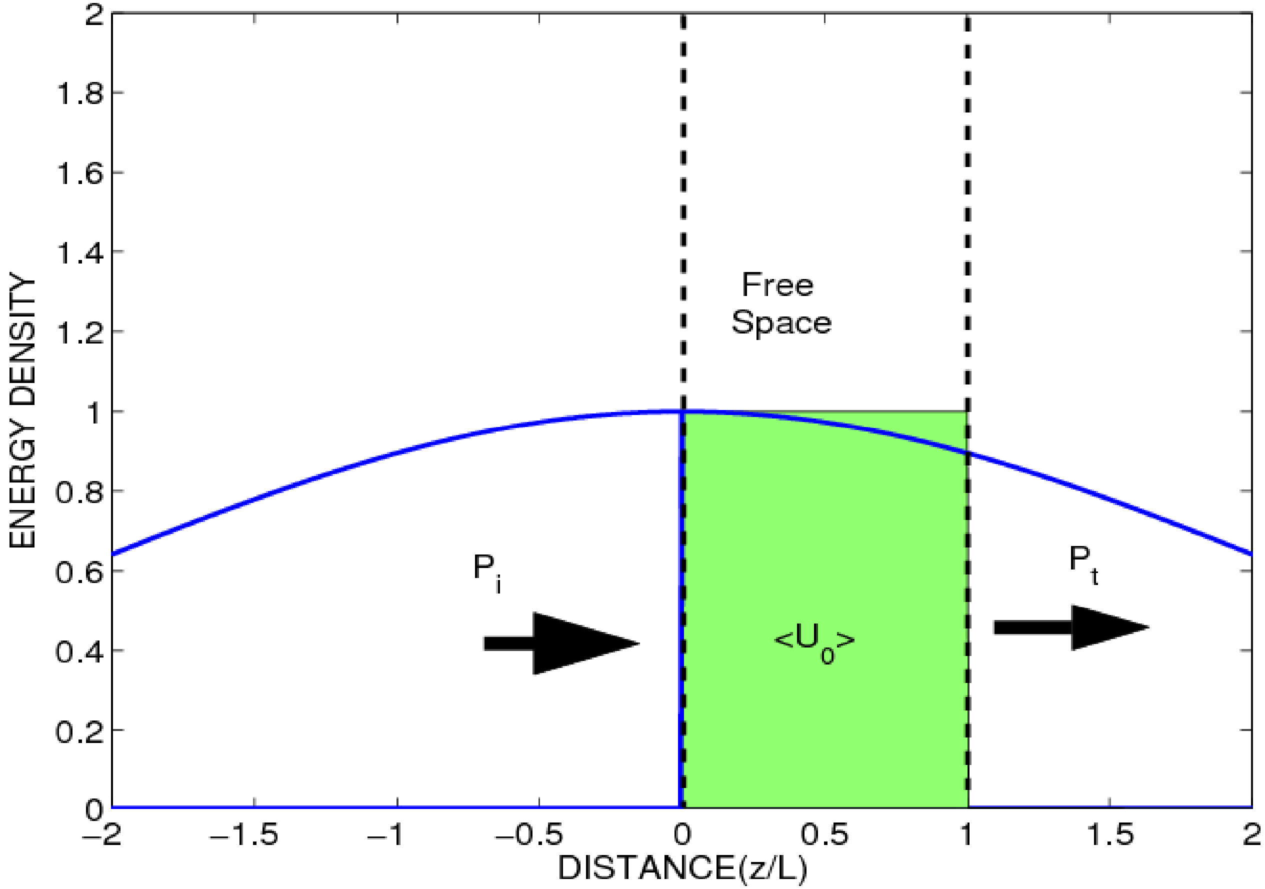


Origin of “superluminality”

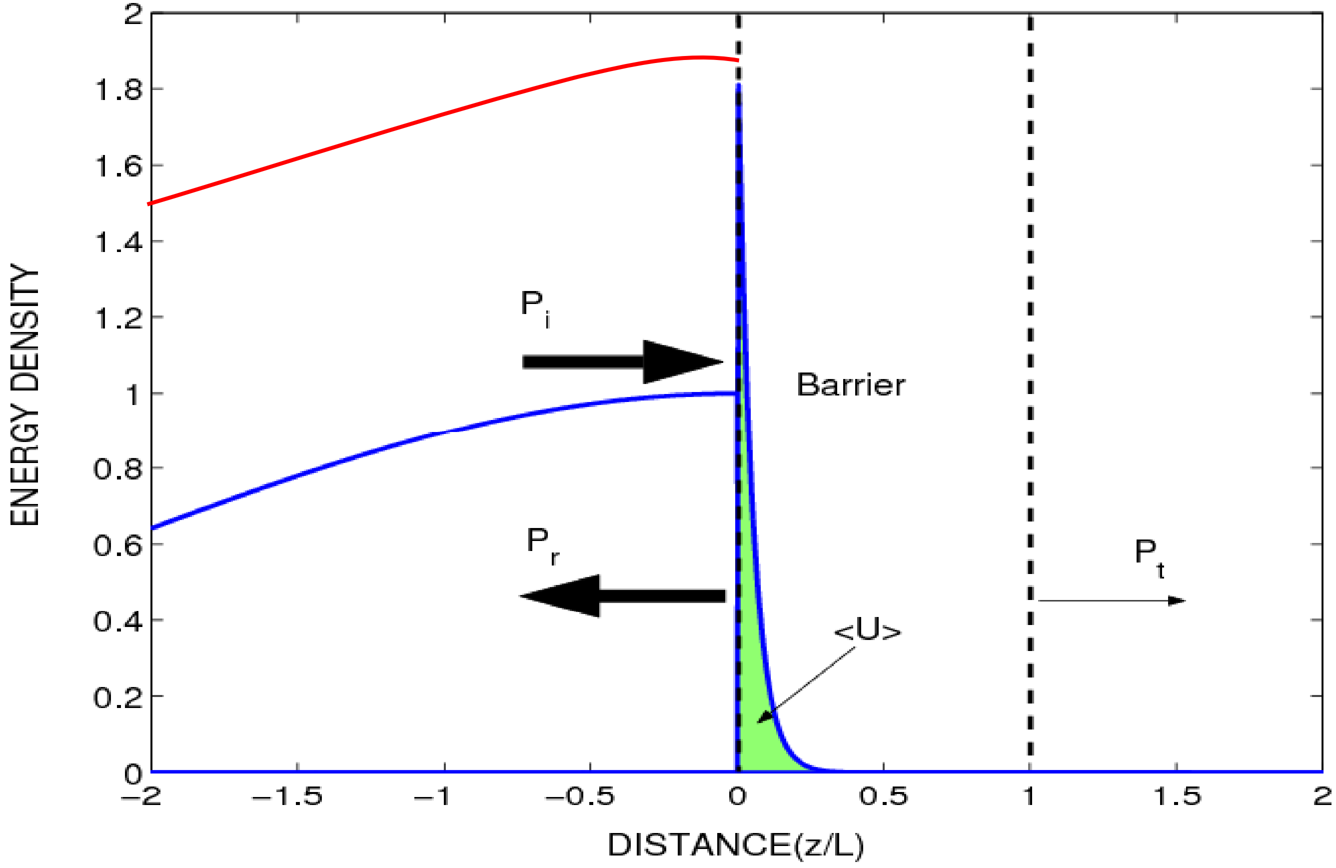
- For the same input power, the less energy stored, the smaller the delay



Energy Density in Air



Energy Density in Barrier

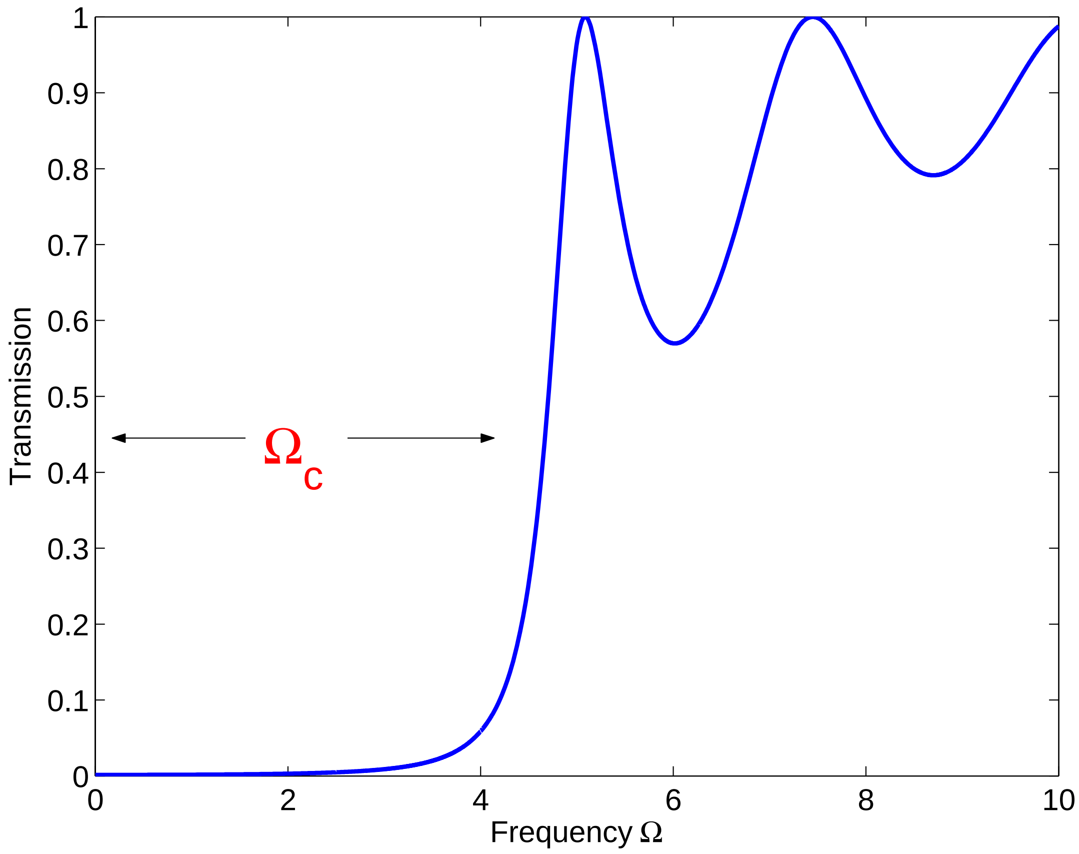


Limiting group delay

At midband (Ω=0) group delay is just the inverse of cutoff frequency (filter bandwidth)

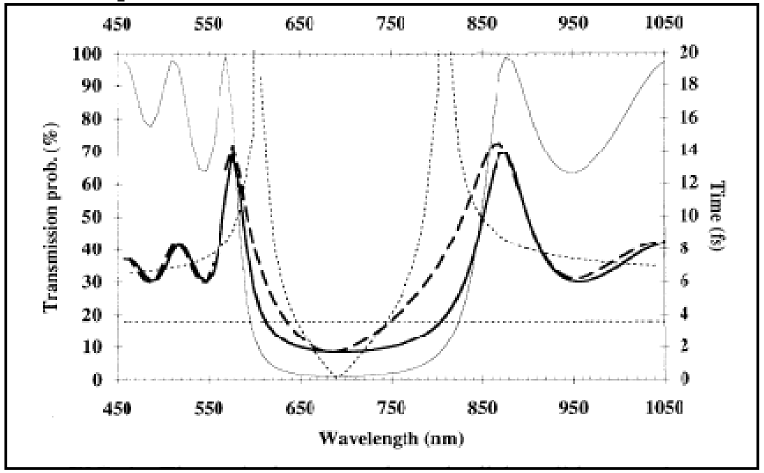
$$\lim_{L\to\infty} \tau_g = 1/\kappa\nu = \frac{1}{\Omega_c}$$

Relation between group delay and cutoff frequency



$$\tau_g = \frac{1}{\Omega_c}$$

Test of the Limiting Group Delay



SKC experiment

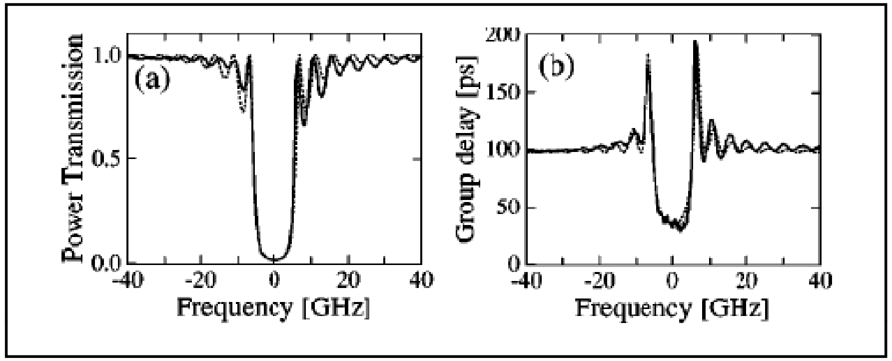
Full width of stop band $\Delta\lambda = 250\,\text{nm}$

Center wavelength $\lambda_0 = 702$ nm

$\Delta f = c\Delta\lambda / \lambda_0^2$

Predicted: $\tau_{g\infty} = 1/\pi\Delta f = 2.1$ fs

Measured group delay=2.13 fs



Longhi, et al experiment

Bandwidth $BW = 9.2\,\text{GHz}$

Limiting group delay $\tau_{g\infty} = 1/\kappa v = 34.6\,\text{ps}$

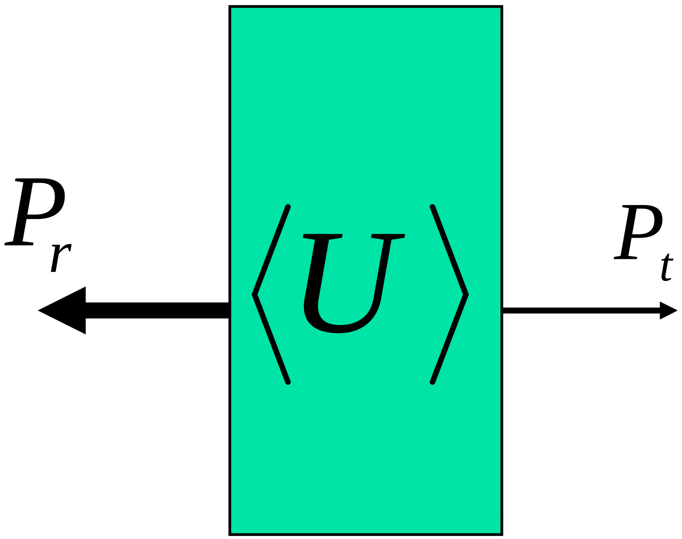
Measured group delay=34 ps

Meaning of Group Delay

- Lifetime of stored energy escaping through both ends

$$\tau_g = \frac{\langle U \rangle}{P_{in}}$$

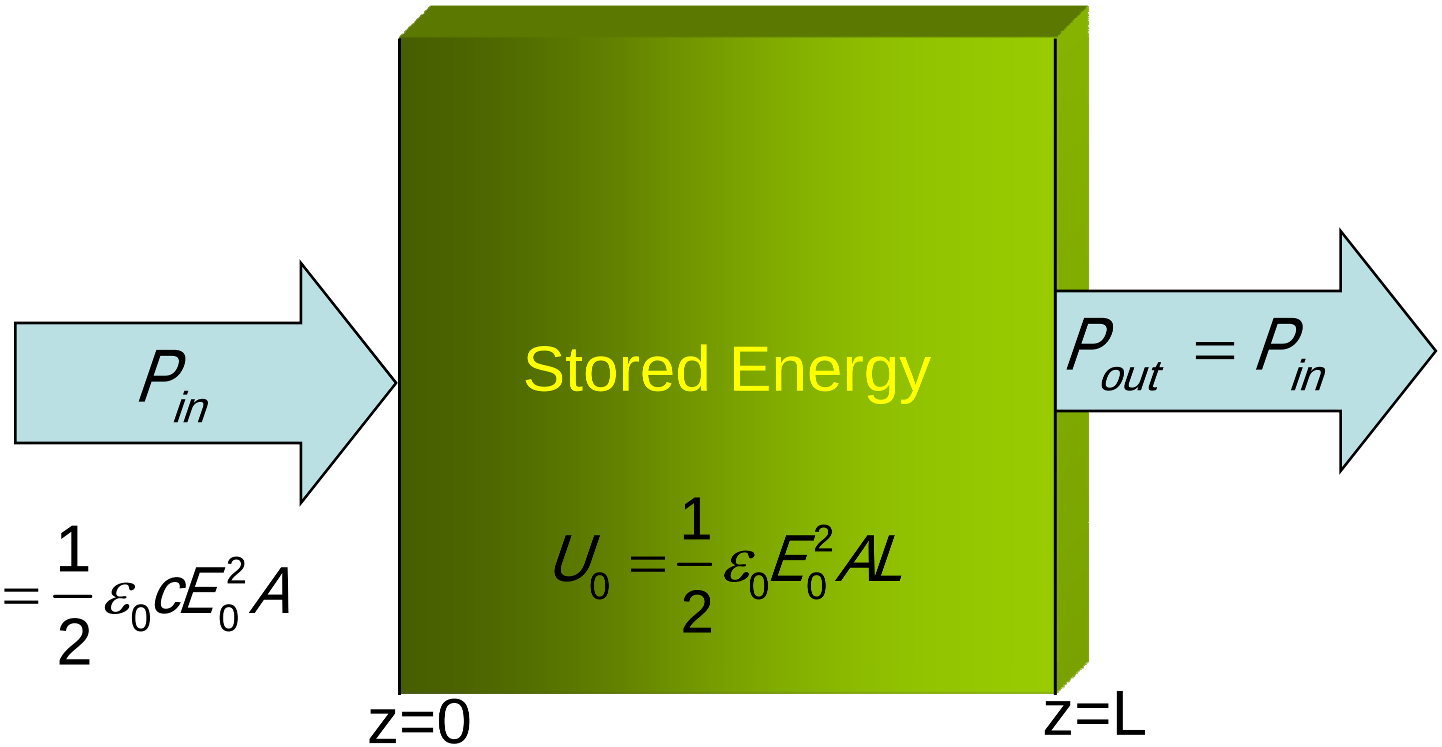
$$P_{in} = P_t + P_r$$



$$\frac{1}{\tau_g} = \frac{P_{in}}{\langle U \rangle} = \frac{P_t}{\langle U \rangle} + \frac{P_r}{\langle U \rangle}$$

$$\frac{1}{\tau_g} = \frac{1}{\tau_t} + \frac{1}{\tau_r}$$

Group delay = lifetime of stored energy

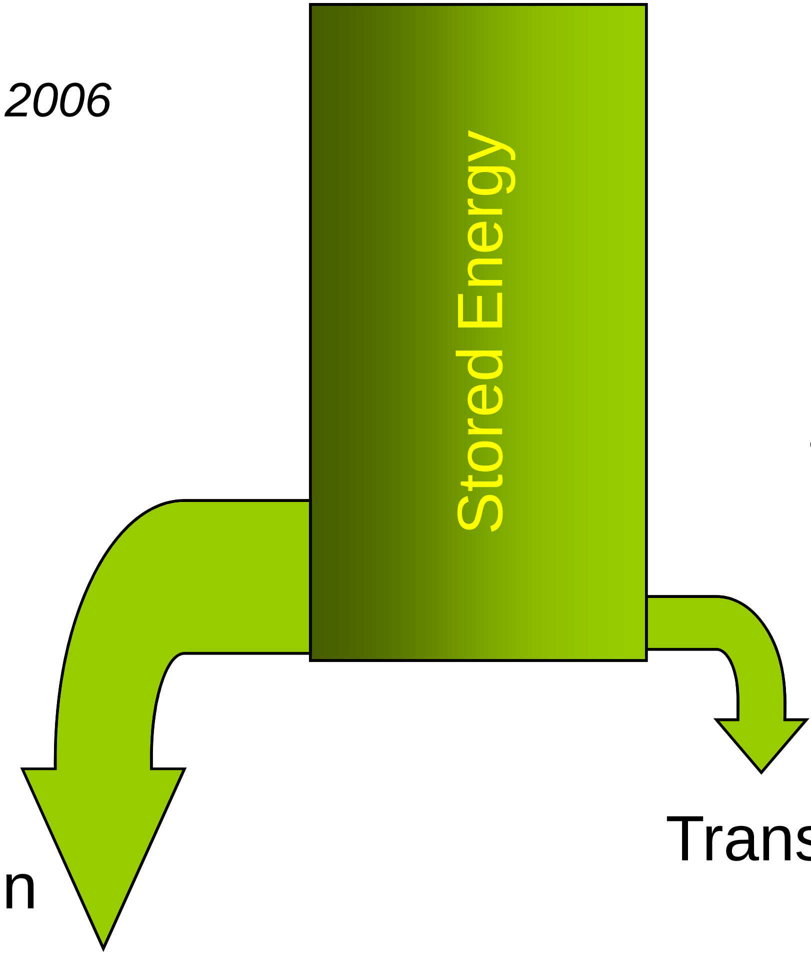


$$P_{in} = \frac{1}{2}\varepsilon_0 c E_0^2 A$$

Free space group delay $\tau_g = \frac{U_0}{P_{in}} = \frac{L}{c}$

Group delay = time to empty tank from both sides simultaneously

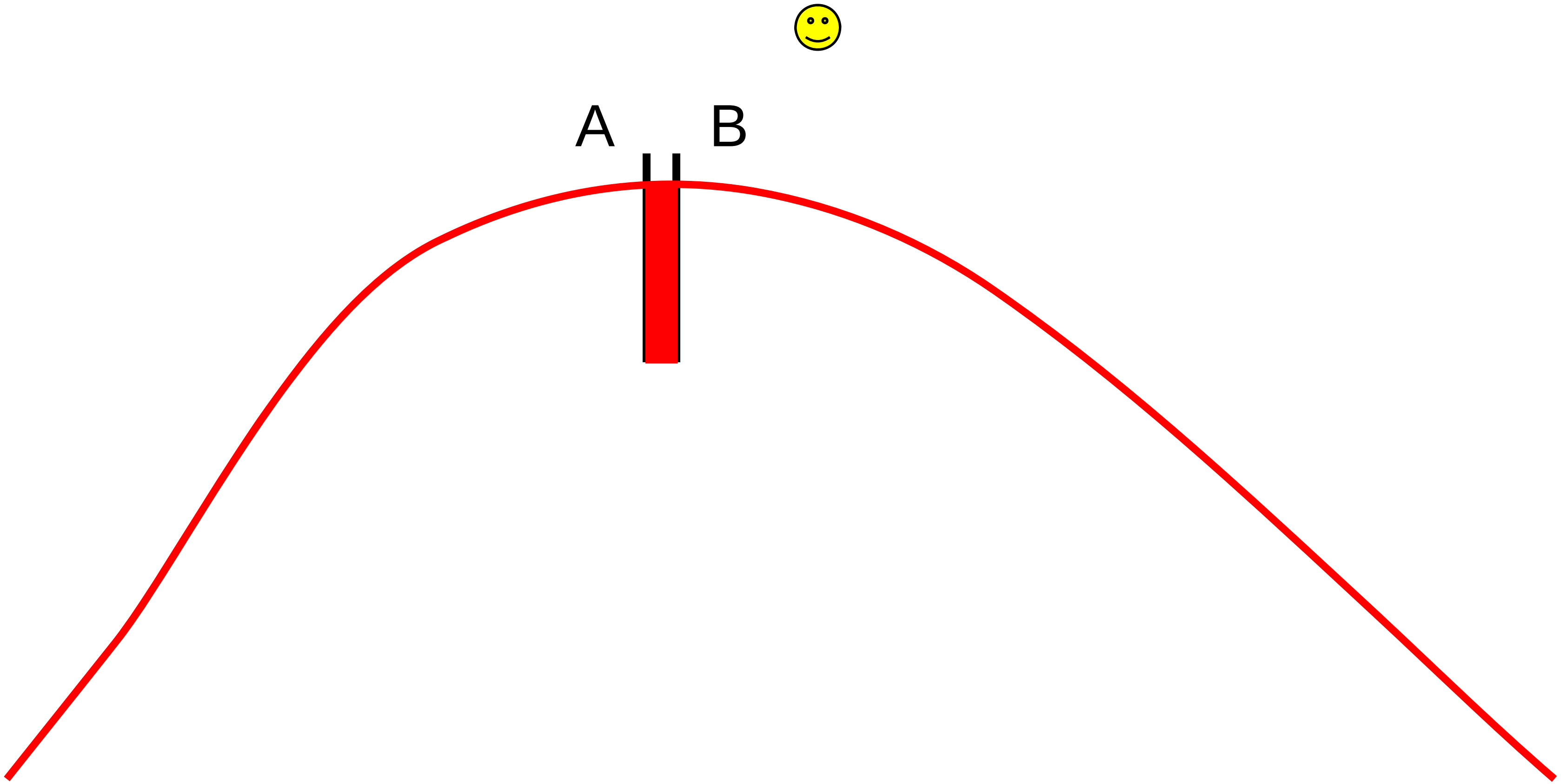
H.G. Winful, NJP, 2006



For symmetric barrier, delay is the same whether monitored on reflection or transmission side

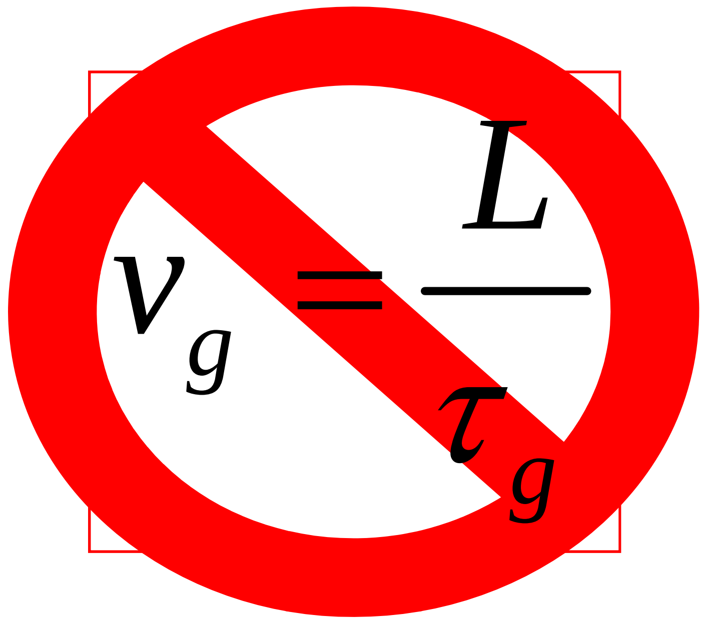
Classical Notion of Transit Time

Classical Notion of Transit Time

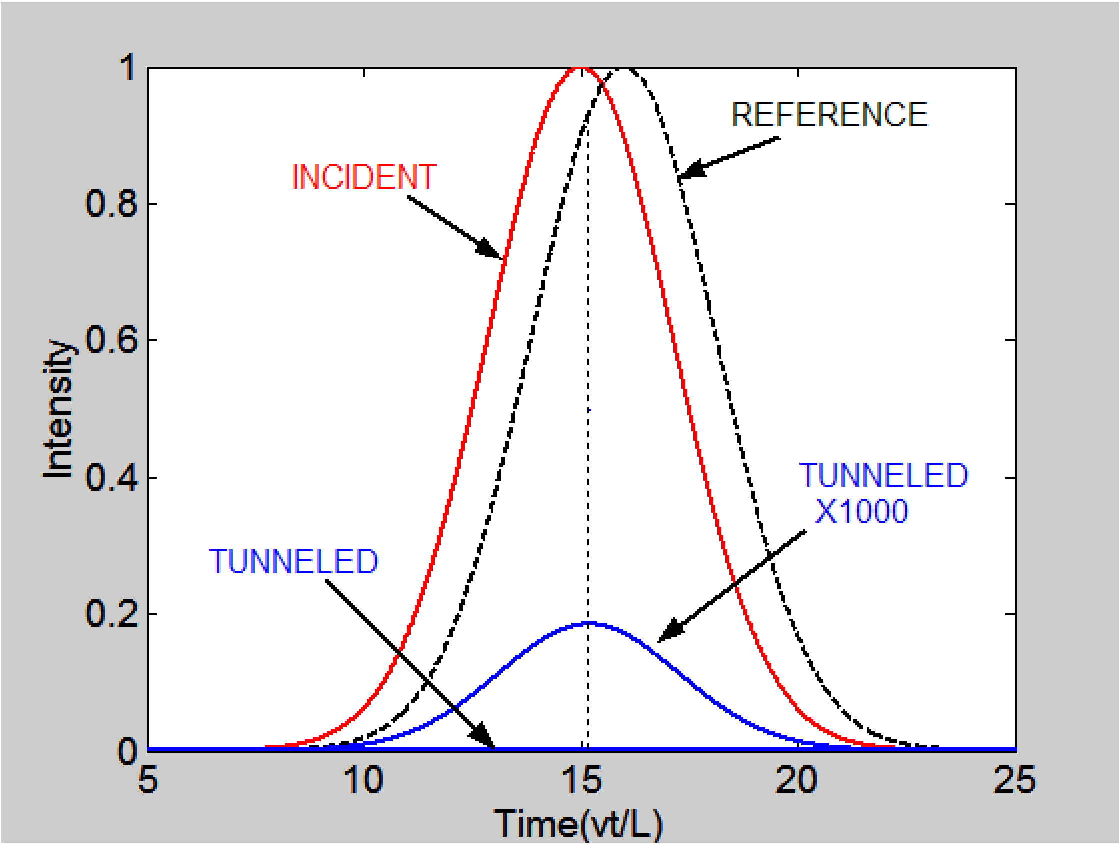


Not a propagation delay!!!

- It has been assumed for years that the group delay in tunneling is a propagation delay.
- Based on that assumption, superluminal group velocities have been inferred.

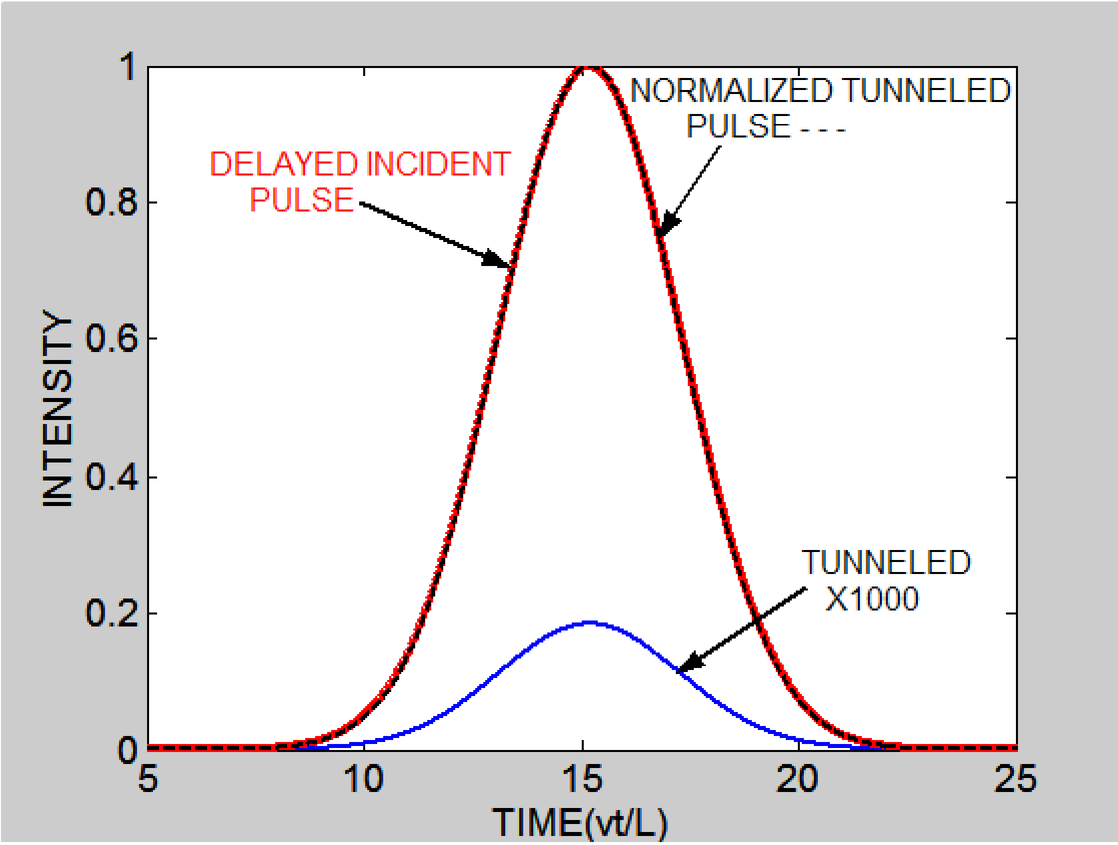


Tunneled Intensity Negligible Compared to Reference



- Even at its peak the tunneled pulse is orders of magnitude smaller than the reference pulse.

There is no Pulse Shortening!!!



Looking inside a barrier

Input pulse

QuickTime™ and a
decompressor
are needed to see this picture.

Output pulse

QuickTime™ and a
decompressor
are needed to see this picture.

QuickTime™ and a
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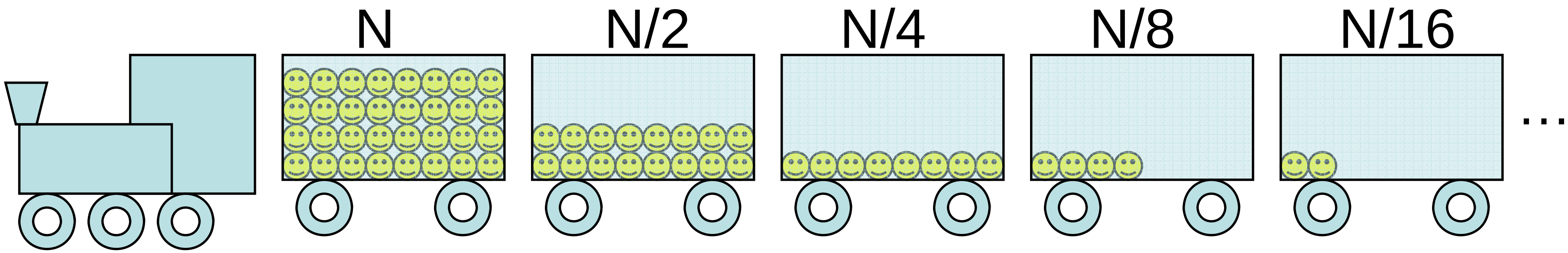
Train Analogy to Hartman Effect

Each successive car carries half the number of passengers in previous car:

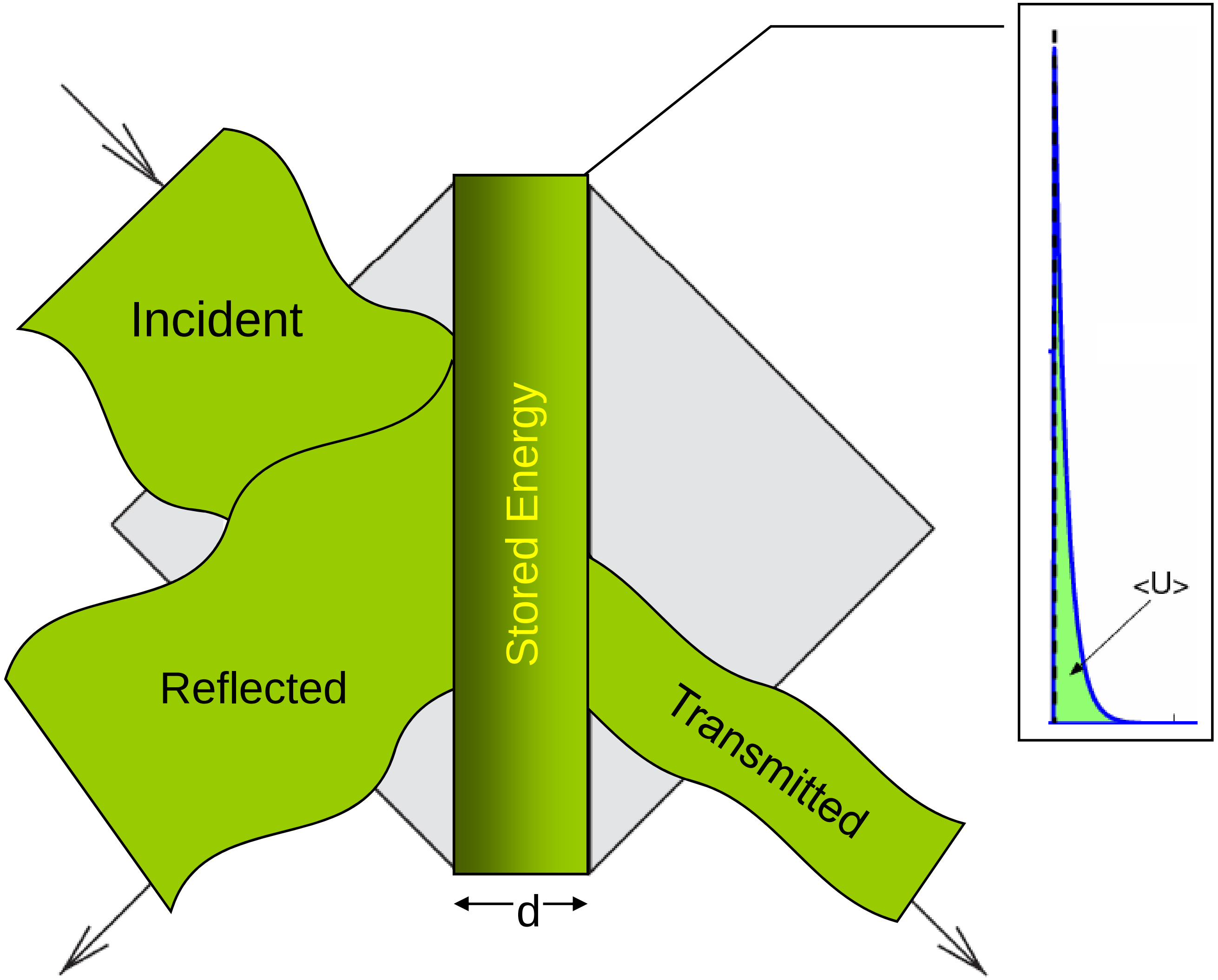


The maximum number of passengers is 2N no matter how long the train is.

Group delay = time it takes to empty the train, proportional to number of passengers on train.

If the number of passengers becomes independent of length, so does the group delay.

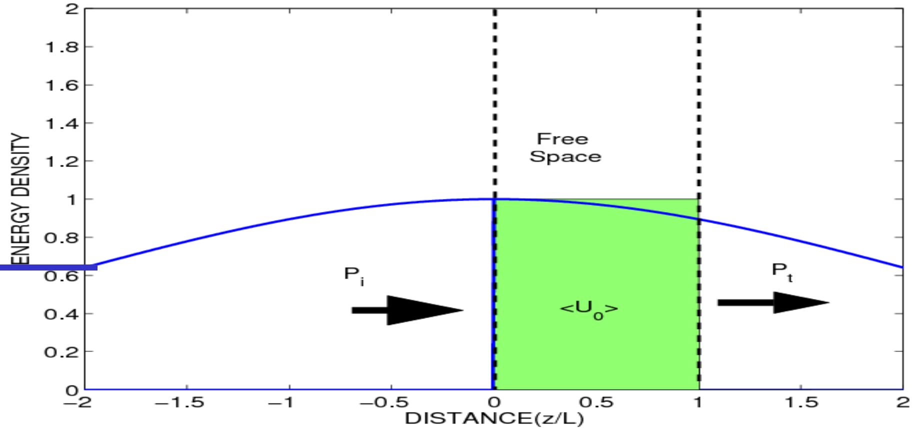
Stored energy in cavity (tank)

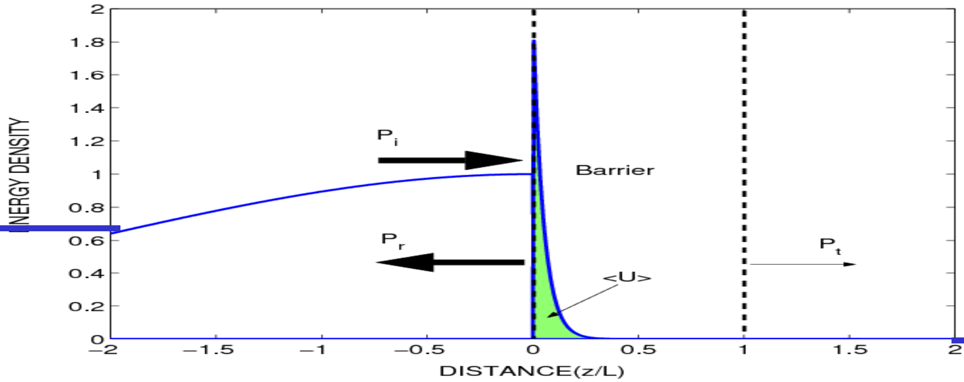


Effect of barrier:

- Reduces stored energy below free space value.
- Creates partial standing wave in barrier.
- Reduces accumulated phase below the propagation phase shift.
- Delays output peak by lifetime of stored energy.

Free space delay $\tau_0 = L/v$





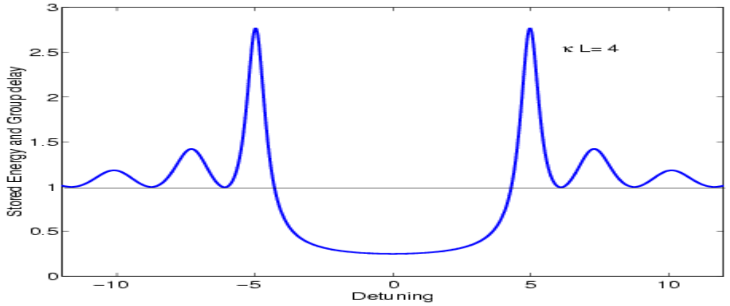
Add propagation delay to equalize paths

ΔL

Barrier delay $\tau_g = \tau_0 (\tanh \kappa L) / \kappa L$

Absence of Dispersion

- No dispersion in the middle of stop band.



- Phase delay equals group delay if *U* does not depend on Ω

$$\frac{\partial \phi}{\partial \Omega} = \frac{U}{P_{in}} \Rightarrow \phi = \frac{U}{P_{in}} \Omega$$

- Phase shift and phase delay simply proportional to stored energy.

Reinterpretation of SKR Experiment

- Since measured delay is less than one optical cycle, best to describe as phase shift.
- Signal and idler photons are simply independent modes of the electromagnetic field.
- Barrier reduces energy stored in region occupied by signal photon.
- Phase shift of signal mode reduced because of reduced stored energy.
- To equalize paths, need to add propagation delay to signal path.

Reinterpretation of Tunneling Experiments (1) Longhi et al

- Parameters: L=2cm, $\kappa L = 2.8$

$$\frac{\tau_g}{\tau_0} = \frac{U}{U_0} = \frac{\tanh \kappa L}{\kappa L} = 0.35$$

Hence barrier stores 35% of the energy stored in barrier-free region.
Group delay due to barrier should be 35% of barrier value.

$$\tau_g = .35\tau_0 = 34 \text{ ps}$$

$$\tau_0 = L/v = 97 \text{ ps}$$

Experimental group delay: 97 ps – 63 ps=34 ps

$$L = 2 \text{ cm}$$

$$v = 2.065 \times 10^8 \text{ m/s}$$

Relation top Quantum Tunneling

- Hartman effect due to saturation of number of particles under barrier.
- Group delay is lifetime of transient state.

Conclusions

- Group delay in tunneling is not a transit time but the lifetime of stored energy leaking out of both ends. [*New J. Phys., June 2006*]
- Group delay due to barrier is shorter than for equal length of free space because barrier stores less energy for same input power.
- Hartman effect is due to saturation of stored energy with barrier length.
- Short group *delay* does not imply superluminal *velocity.*

References

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- H.G. Winful, *Phys. Rev. Lett.,* **90,** 032901 (2003)
- H.G. Winful, *IEEE JSTQE,* **9,** 17 (2003)
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