The Casimir-Polder force: A manifestation of the QED vacuum

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Friedrich-Schiller-Universität Jena, Germany

Puzzle

What is greater than God,
more evil than the devil?
The poor have it,
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and if you eat it, you will die.

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Answer: Nothing

Content

- Introduction: Body-assisted QED vacuum
  - Quantization scheme
  - vacuum QED effects

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- Casimir-Polder force: Beyond perturbation theory
  - General theory
  - Dynamics in weak-coupling limit
  - Excited-state atom

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- Summary and outlook

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$\Delta A \Delta B \geq \frac{1}{2}|\langle[\hat{A}, \hat{B}]\rangle|$ (Heisenberg uncertainty relation)

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$\langle\hat{\mathbf{E}}(\mathbf{r})\rangle = 0$, $\langle\hat{\mathbf{B}}(\mathbf{r})\rangle = 0$ (no e.m. field *on average*)
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QED vacuum = vanishing of **average** electromagnetic fields

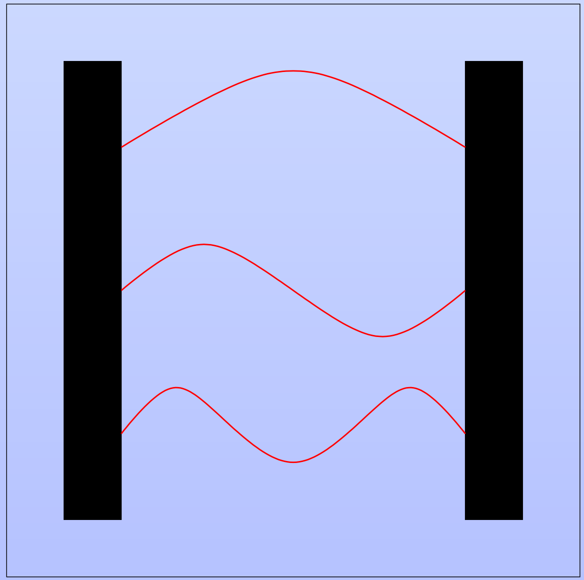
Normal-mode quantization

Quantized electric field:

$$\hat{\mathbf{E}}(\mathbf{r}) = \sum_k \mathbf{g}_k(\mathbf{r})\hat{a}_k + \text{H.c.}$$

$\mathbf{g}_k(\mathbf{r})$: normal modes

$\hat{a}_k^\dagger, \hat{a}_k$: creation, annihilation operators



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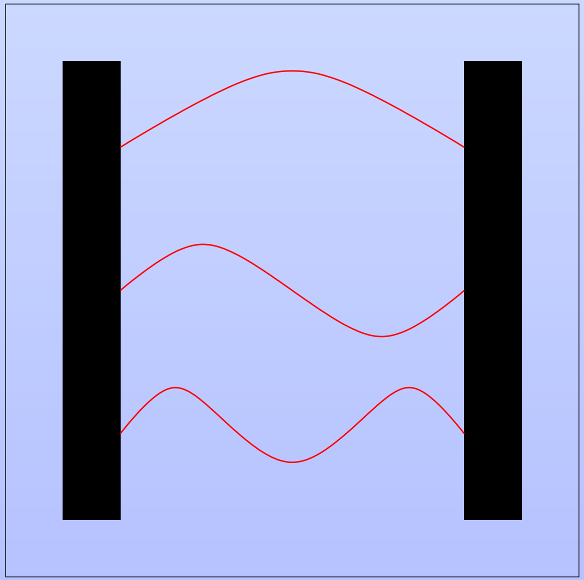

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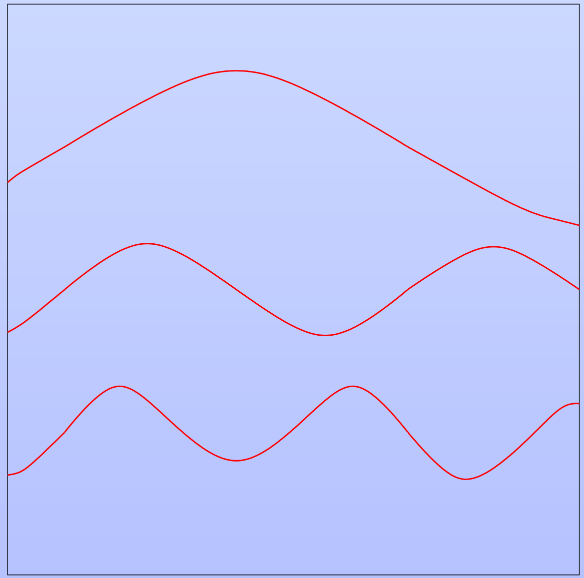

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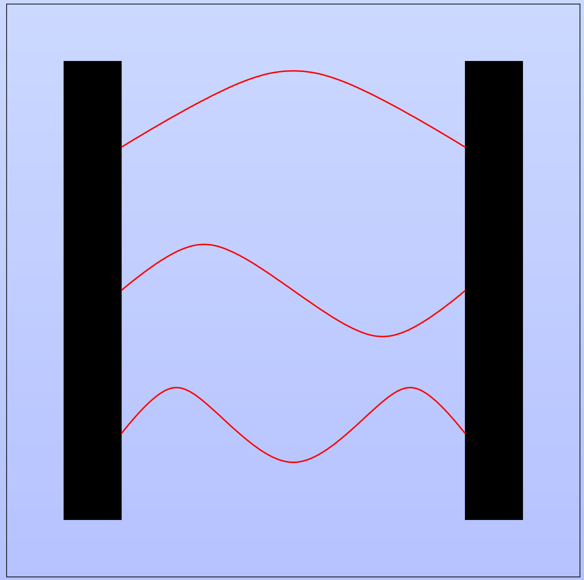

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- arbitrary arrangement of
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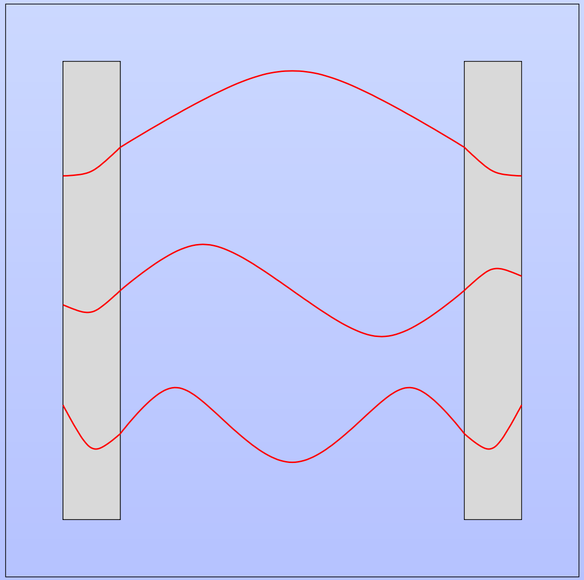

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Applicability:

- free space
- arbitrary arrangement of
  - perfectly reflecting bodies
  - nondispersive, nonabsorbing bodies

Not applicable to dispersing <u>and</u> absorbing bodies!

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3. **New materials:**
   artificial metamaterials with left-handed properties[1]
   $\rightarrow$ strongly dispersing and absorbing

[1] D. R. Smith *et. al.*, PRL **84**, 18, 4184 (2000)

Generalized quantization scheme

Normal-mode quantization of electric field:

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Quantized electric field in linear, causal media:

$$\hat{\mathbf{E}}(\mathbf{r}) = \int_0^\infty \mathrm{d}\omega \int \mathrm{d}^3r'\, i\sqrt{\frac{\hbar}{\pi\varepsilon_0}} \left\{ \frac{\omega^2}{c^2}\sqrt{\mathrm{Im}\,\varepsilon(\mathbf{r}',\omega)}\, \boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\hat{\mathbf{f}}_e(\mathbf{r}',\omega) + \frac{\omega}{c}\sqrt{-\mathrm{Im}\mu^{-1}(\mathbf{r}',\omega)}\left[\boldsymbol{\nabla}'\times\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\right]\hat{\mathbf{f}}_m(\mathbf{r}',\omega)\right\} + \text{H.c.}$$

Classical Green tensor

$$\left[\boldsymbol{\nabla}\times\mu^{-1}(\mathbf{r},\omega)\boldsymbol{\nabla}\times\ -\frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\right]\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)=\boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$$

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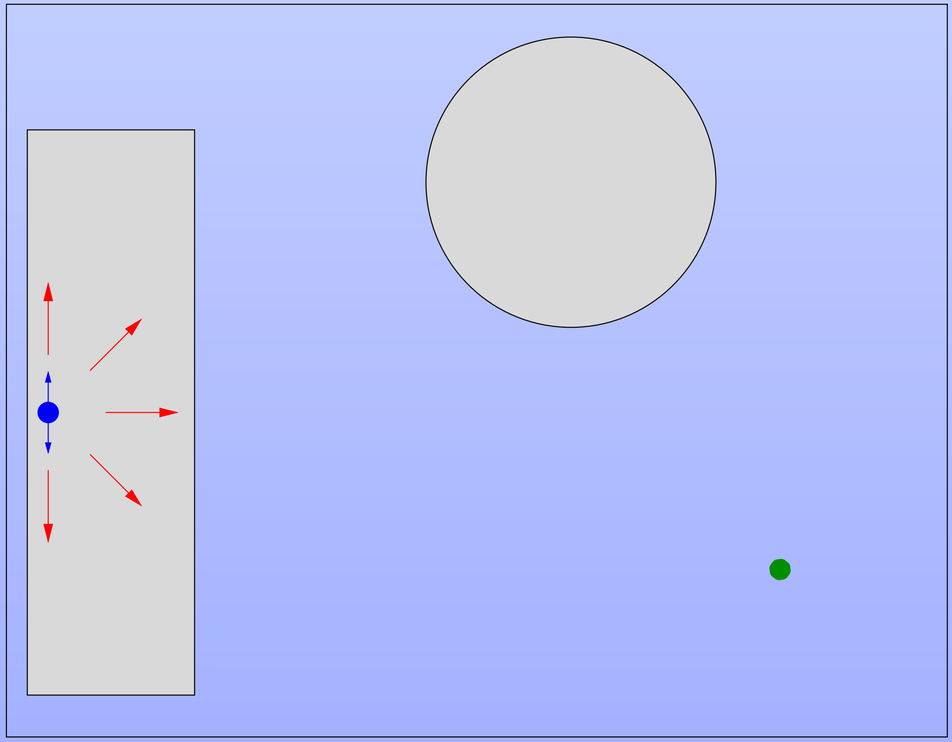
Physical interpretation:



Source at $\mathbf{r}$

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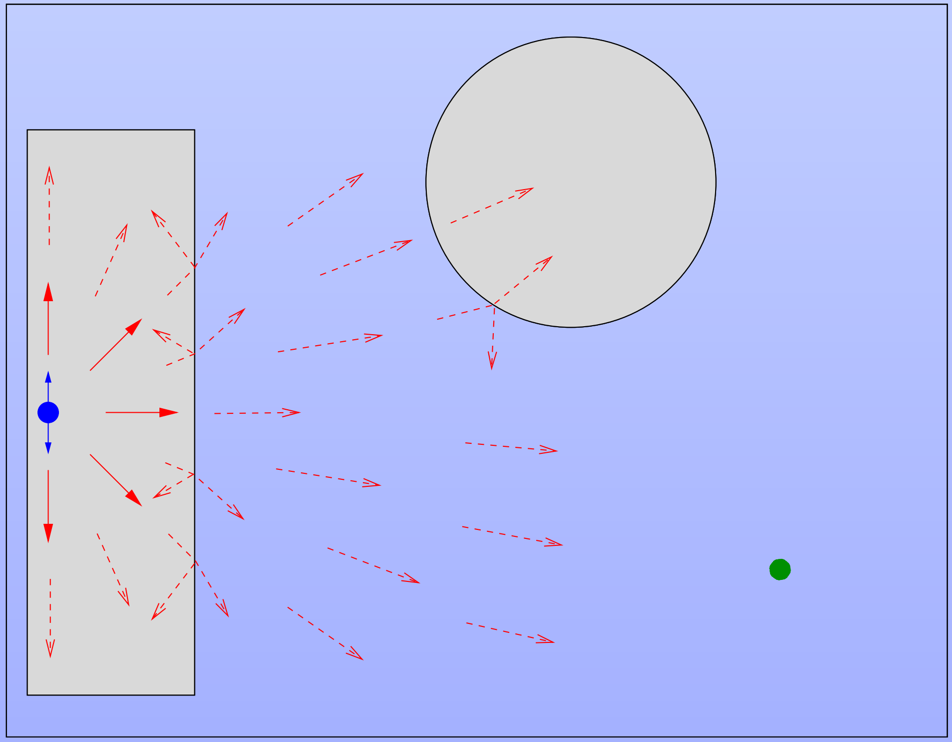

Source at $\mathbf{r}$ $\quad \boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$

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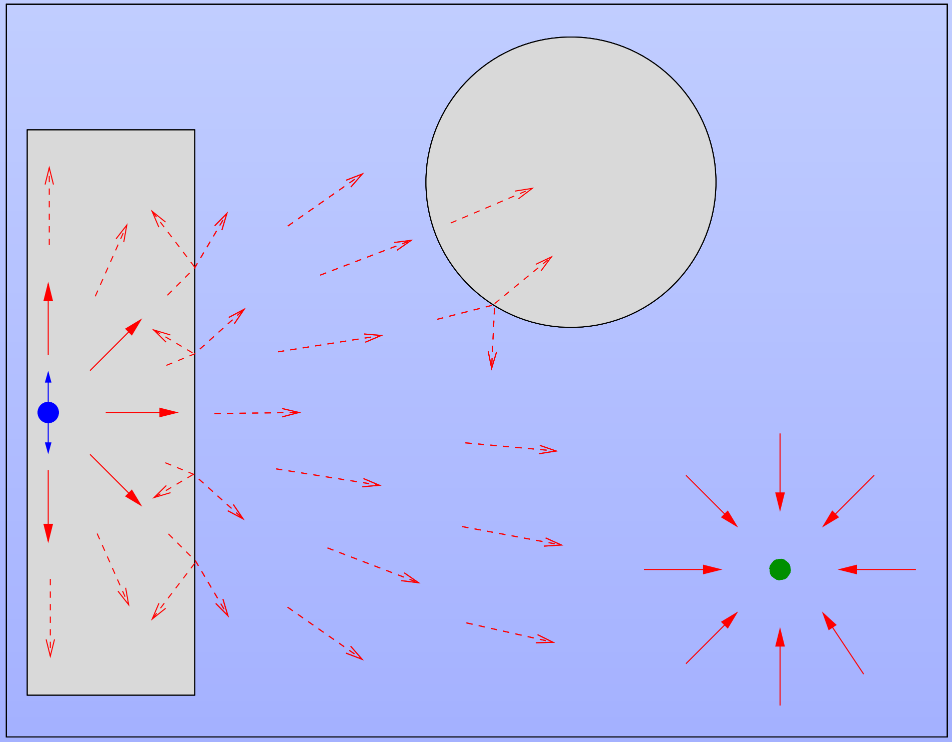

Source at $\mathbf{r}$ $\underrightarrow{\boldsymbol{G}(\mathbf{r},}\underrightarrow{\mathbf{r}',\omega)}$ Electric field at $\mathbf{r}'$

Creation/annihilation operators

Bosonic commutation relations:

$$\left[\hat{f}_{\lambda i}(\mathbf{r},\omega),\hat{f}^{\dagger}_{\lambda' j}(\mathbf{r}',\omega')\right]=\delta_{\lambda\lambda'}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')\quad(\lambda=e,m)$$

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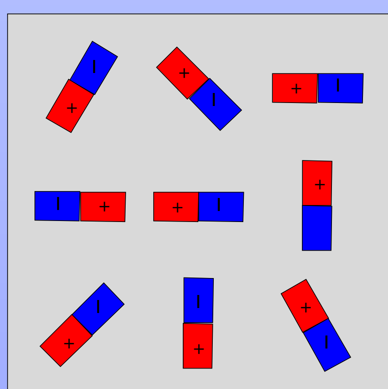
Physical interpretation:

Noise polarization:

$$\underline{\hat{\mathbf{P}}}_{\mathsf{N}}(\mathbf{r},\omega) = i\sqrt{\frac{\hbar\varepsilon_0}{\pi}} \times\sqrt{\mathrm{Im}\,\varepsilon(\mathbf{r},\omega)}\,\hat{\mathbf{f}}_e(\mathbf{r},\omega)$$



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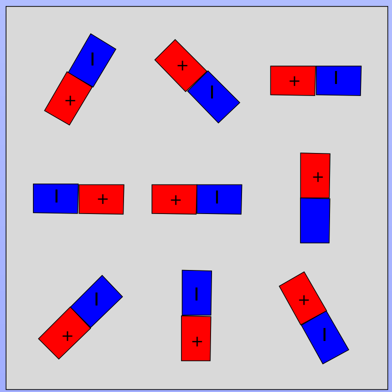
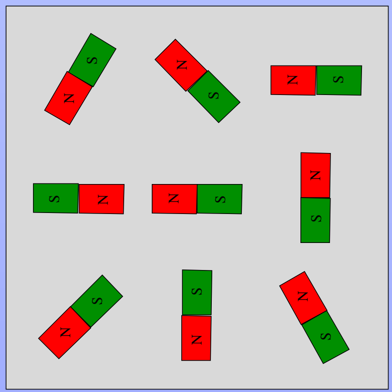


Noise magnetization:

$$\underline{\hat{\mathbf{M}}}_{\mathrm{N}}(\mathbf{r},\omega) = \sqrt{-\frac{\hbar\kappa_0}{\pi}} \times\sqrt{\mathrm{Im}\,\mu^{-1}(\mathbf{r},\omega)}\,\hat{\mathbf{f}}_m(\mathbf{r},\omega)$$



Atom-field dynamics

$$\hat{H} = \hat{H}_{\mathsf{MF}} + \hat{H}_{\mathsf{A}} + \hat{H}_{\mathsf{AMF}}$$

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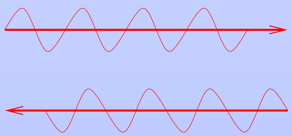
Medium-assisted field Hamiltonian:

$$\hat{H}_{\mathsf{MF}} = \sum_{\lambda=e,m} \int \mathrm{d}^3 r \int_0^\infty \mathrm{d}\omega\, \hbar\omega\, \hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r},\omega)\hat{\mathbf{f}}_\lambda(\mathbf{r},\omega)$$



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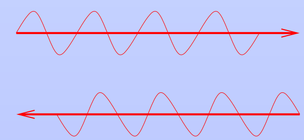
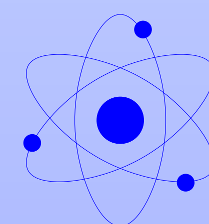


Atomic Hamiltonian:

$$\hat{H}_{\mathsf{A}} = \sum_\alpha \frac{\hat{\mathbf{p}}_\alpha{}^2}{2m_\alpha} + \frac{1}{2\varepsilon_0}\int \mathrm{d}^3 r \hat{\mathbf{P}}_{\mathsf{A}}{}^2(\mathbf{r})$$



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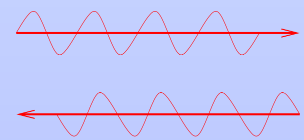

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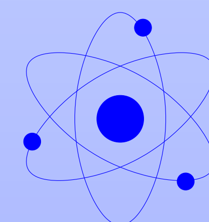
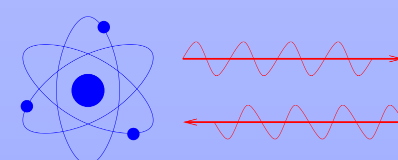


Electric dipole interaction:

$$\hat{H}_{\mathsf{AMF}} = -\hat{\mathbf{d}}\hat{\mathbf{E}}(\hat{\mathbf{r}}_{\mathsf{A}}) + \frac{1}{2m_{\mathsf{A}}}\left[\hat{\mathbf{p}}_{\mathsf{A}}, \hat{\mathbf{d}} \times \hat{\mathbf{B}}(\hat{\mathbf{r}}_{\mathsf{A}})\right]_+$$



QED vacuum in presence of linear, causal media

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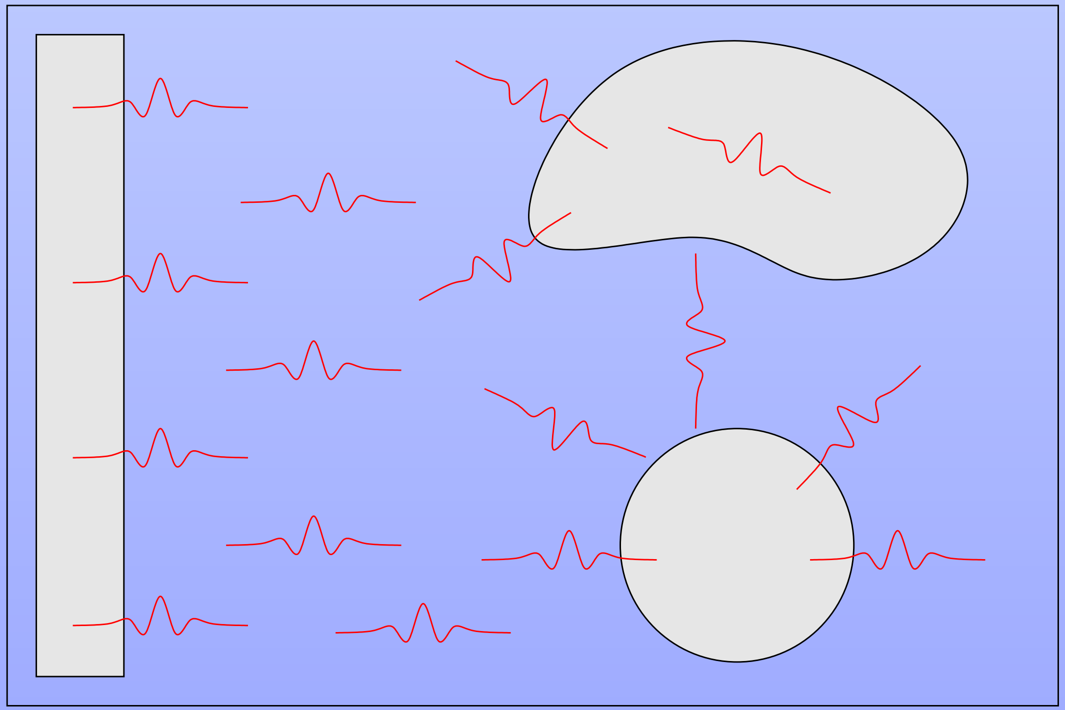
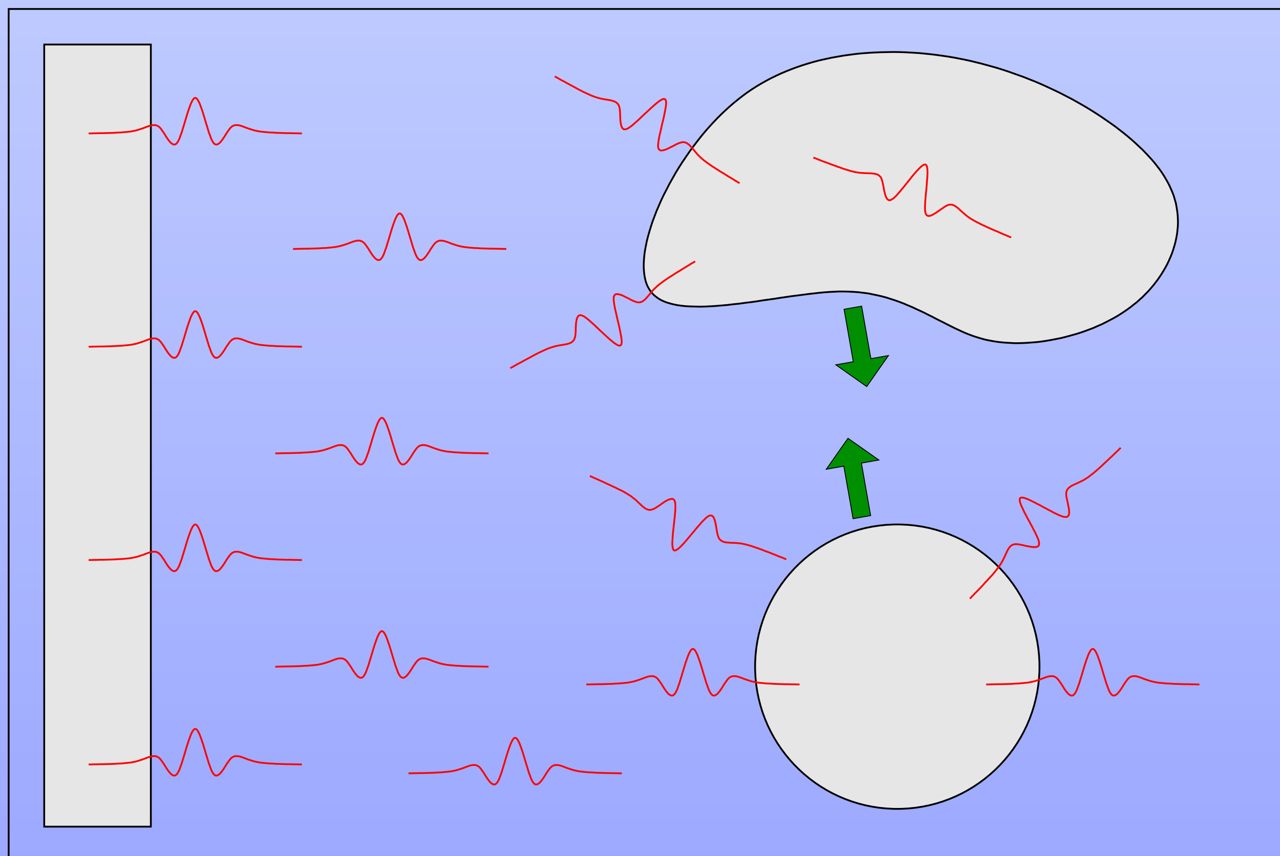
Highly structured fluctuations of the electromagnetic field!



Manifestations of the QED vacuum

- Casimir force [C. Raabe *et. al.*, PRA **68**, 033810 (2003)]



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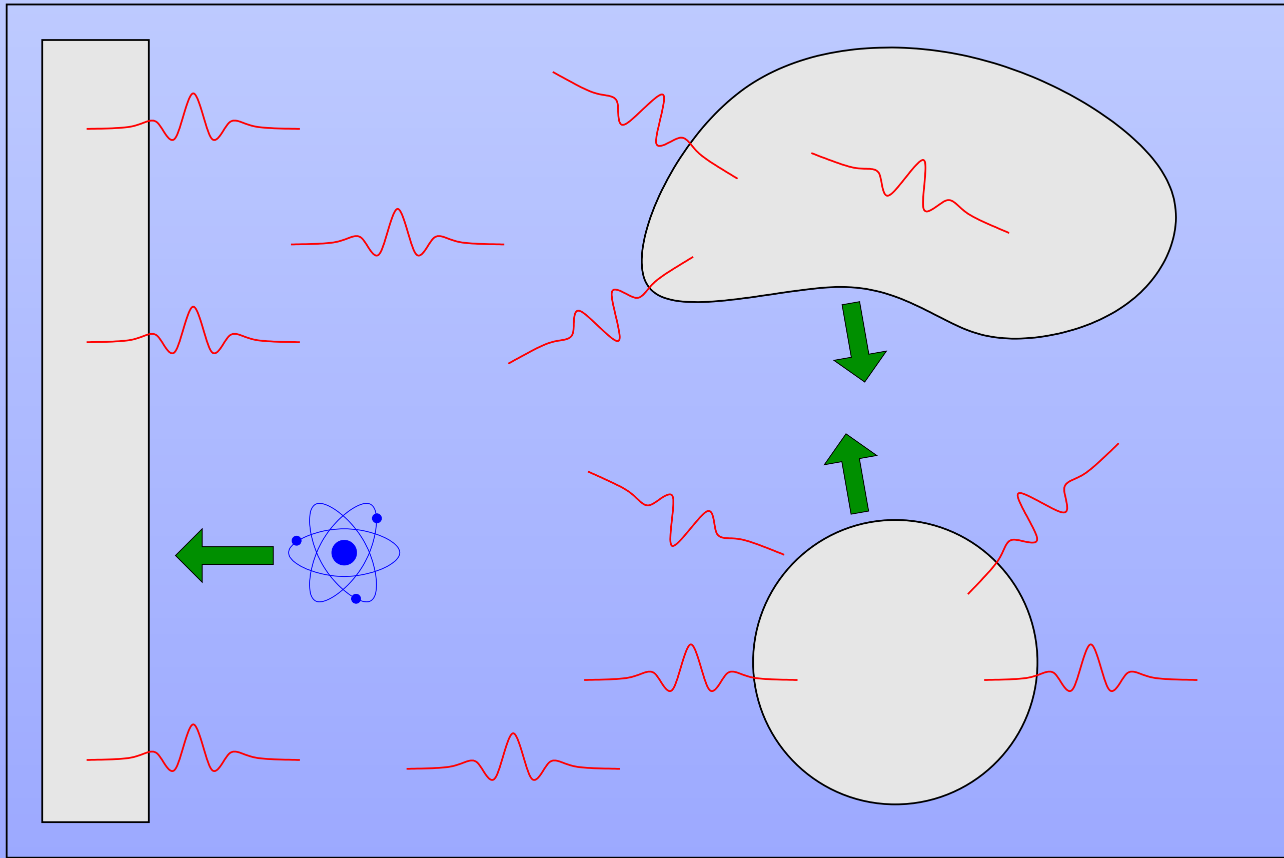

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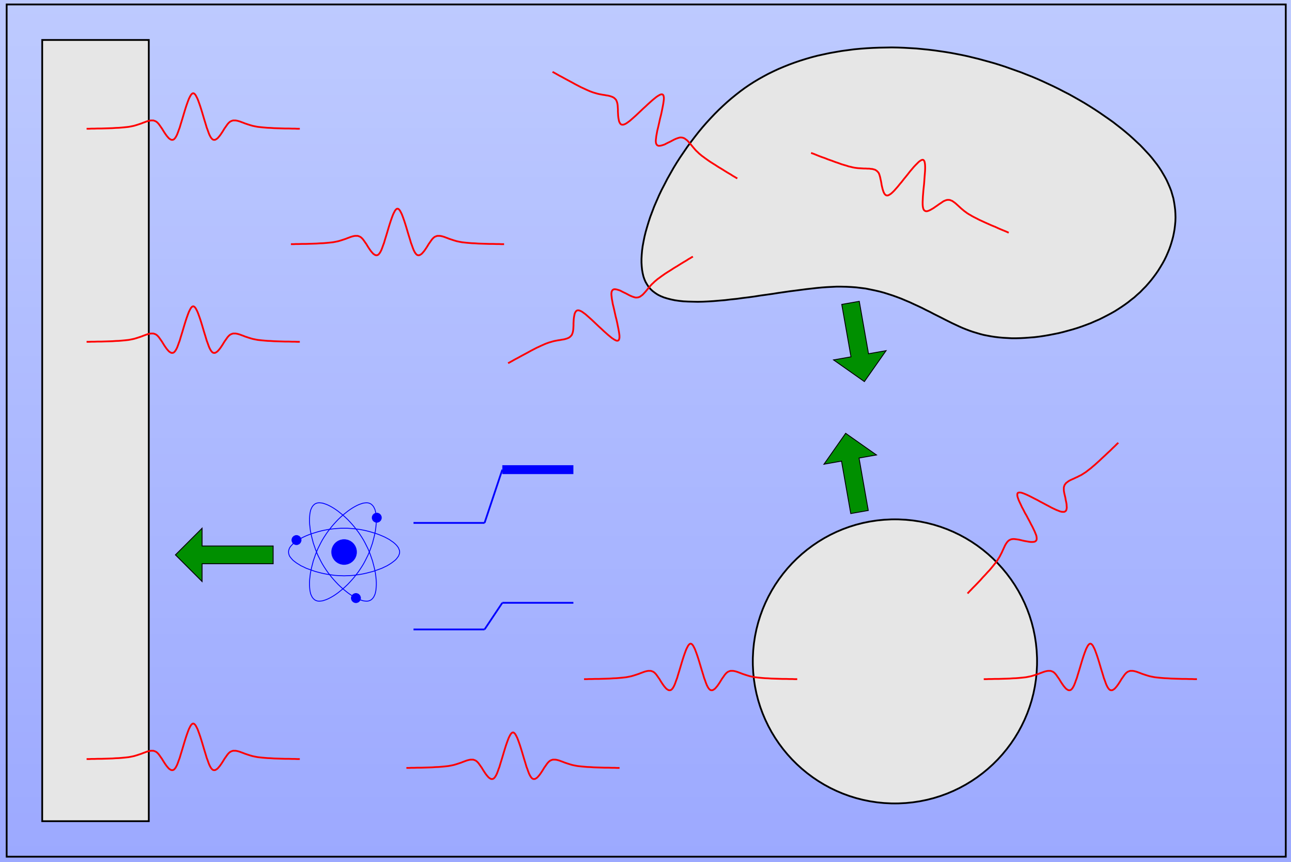

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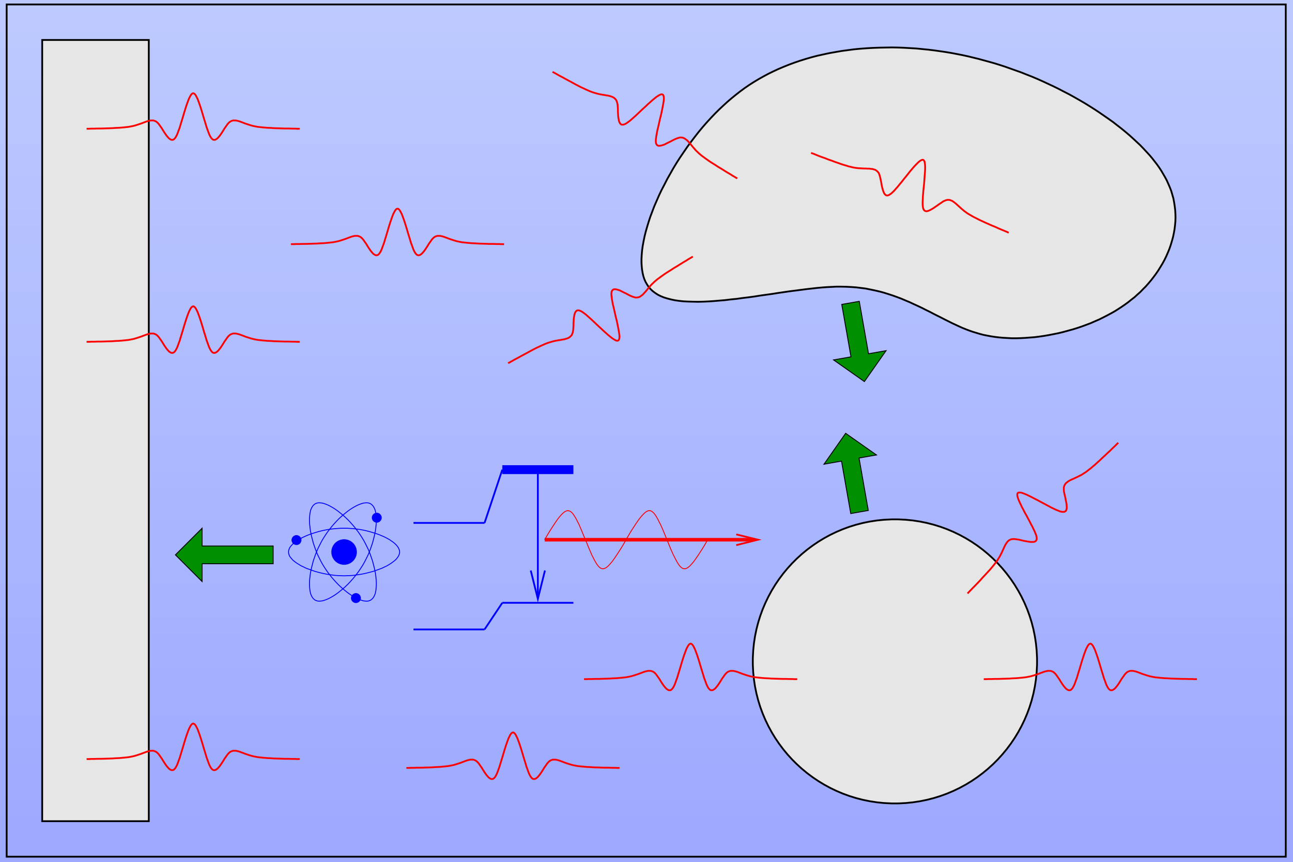
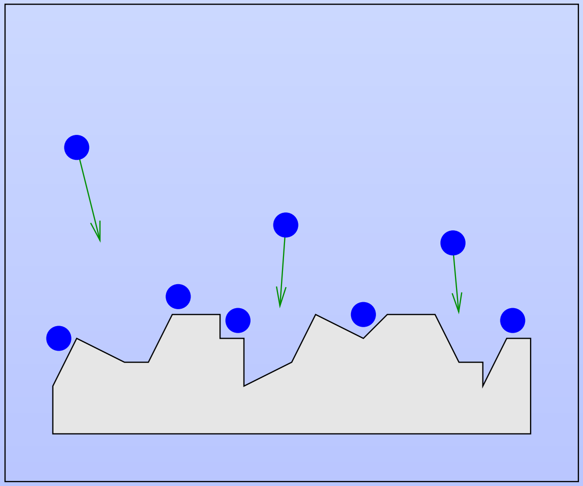
- Spontaneous decay [Ho *et. al.*, PRA **68**, 043816 (2003)]



Relevance of Casimir-Polder forces

- Adsorption of atoms/molecules to surfaces[1]



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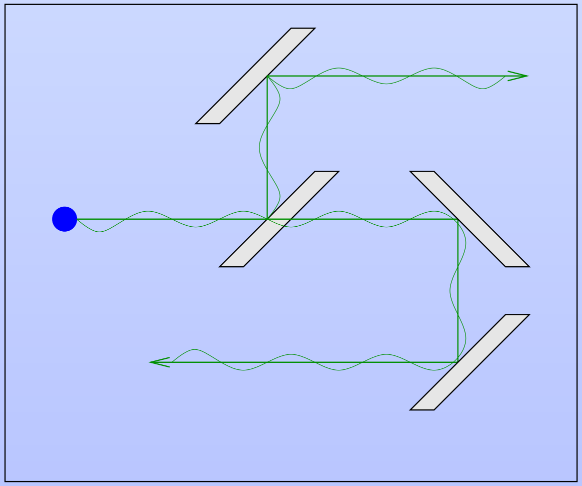
- Adsorption of atoms/molecules to surfaces[1]
- Atom optics[2]



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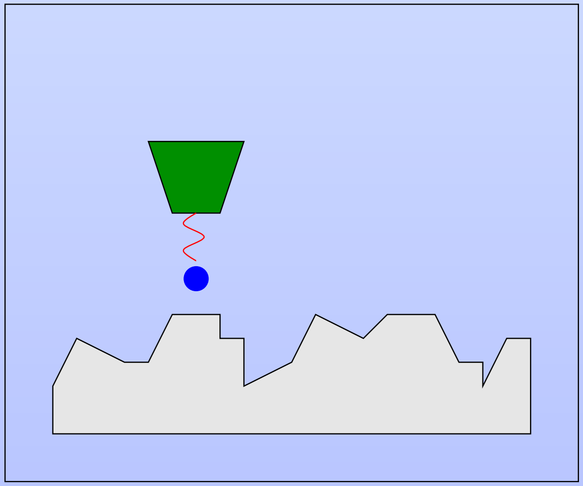
- Atomic-force microscopes[3]



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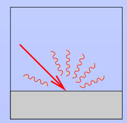
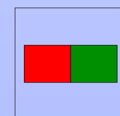
[3]G. Binnig *et. al.*, PRL **56**, 930 (1986)

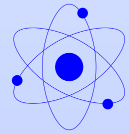
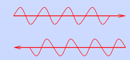
Casimir-Polder force:

Perturbative approach (ground-state atoms)

Open issues

- Role of material absorption 
- Influence of magnetic properties 

Van der Waals potential

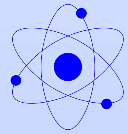
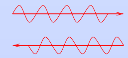
Idea: system in state $|0\rangle$  $\otimes$ $|\{0\}\rangle$ 

interaction $\hat{H}_{\mathsf{AMF}}$ $\Rightarrow$ energy shift ΔE_0

$\Rightarrow$ van-der-Waals potential $\Delta E_0 = \Delta E_0^{(0)} + U_0(\mathbf{r}_{\mathsf{A}})$

$\Rightarrow$ van-der-Waals force $\mathbf{F}_0(\mathbf{r}_{\mathsf{A}}) = -\boldsymbol{\nabla}_{\mathsf{A}} U_0(\mathbf{r}_{\mathsf{A}})$

Van der Waals potential

Idea: system in state $|0\rangle$  $\otimes$ $|\{0\}\rangle$ 

interaction $\hat{H}_{\mathrm{AMF}}$ $\Rightarrow$ energy shift ΔE_0

$\Rightarrow$ van-der-Waals potential $\Delta E_0 = \Delta E_0^{(0)} + U_0(\mathbf{r}_{\mathrm{A}})$

$\Rightarrow$ van-der-Waals force $\mathbf{F}_0(\mathbf{r}_{\mathrm{A}}) = -\boldsymbol{\nabla}_{\mathrm{A}} U_0(\mathbf{r}_{\mathrm{A}})$

2$^{\mathrm{nd}}$-order perturbation theory: $\Delta_2 E_0 = \sum_{\psi} \frac{|\langle 0|\langle\{0\}|\hat{H}_{\mathrm{AMF}}|\psi\rangle|^2}{E_0 - E_\psi}$

Van der Waals potential

Idea: system in state $|0\rangle \otimes |\{0\}\rangle$

interaction $\hat{H}_{\mathsf{AMF}} \Rightarrow$ energy shift ΔE_0

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$\Rightarrow$ van-der-Waals force $\mathbf{F}_0(\mathbf{r}_{\mathsf{A}}) = -\boldsymbol{\nabla}_{\mathsf{A}} U_0(\mathbf{r}_{\mathsf{A}})$

2nd-order perturbation theory: $\Delta_2 E_0 = \sum_\psi \frac{|\langle 0|\langle\{0\}|\hat{H}_{\mathsf{AMF}}|\psi\rangle|^2}{E_0 - E_\psi}$

Result:

$$U_0(\mathbf{r}_{\mathsf{A}}) = \frac{\hbar\mu_0}{2\pi}\int_0^\infty \mathrm{d}u\, u^2 \alpha_0^{(0)}(iu)\, \mathrm{Tr}\boldsymbol{G}^{(1)}(\mathbf{r}_{\mathsf{A}}, \mathbf{r}_{\mathsf{A}}, iu)$$

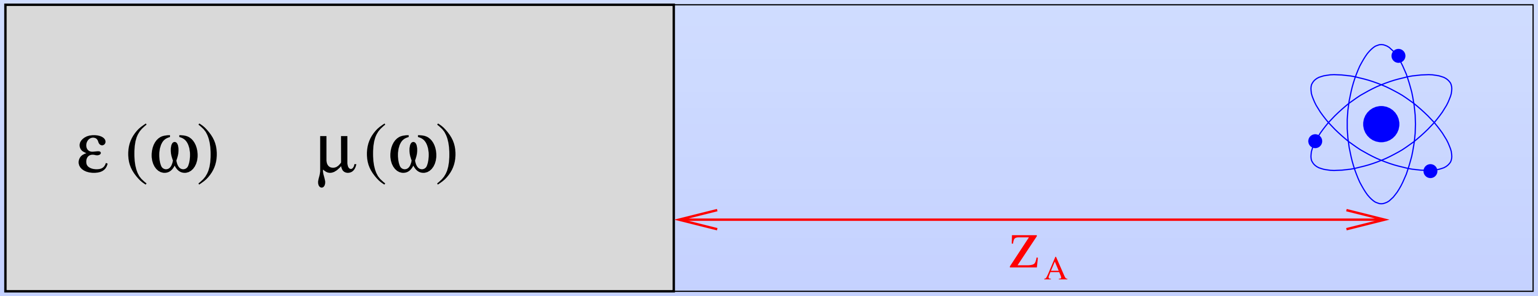
atomic polarizability:

$$\alpha_0^{(0)}(\omega) = \lim_{\epsilon\to 0} \frac{2}{3\hbar} \sum_k \frac{\omega_{k0}|\mathbf{d}_{0k}|^2}{\omega_{k0}^2 - \omega^2 - i\omega\epsilon}$$

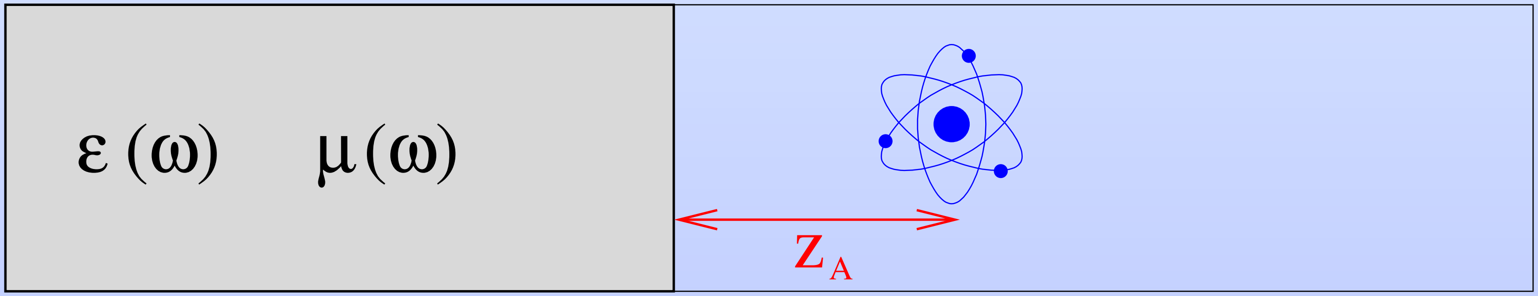
Scattering Green tensor:

$$\boldsymbol{G}^{(1)}(\mathbf{r}_{\mathsf{A}}, \mathbf{r}_{\mathsf{A}}, iu)$$

Atom near half space[1]

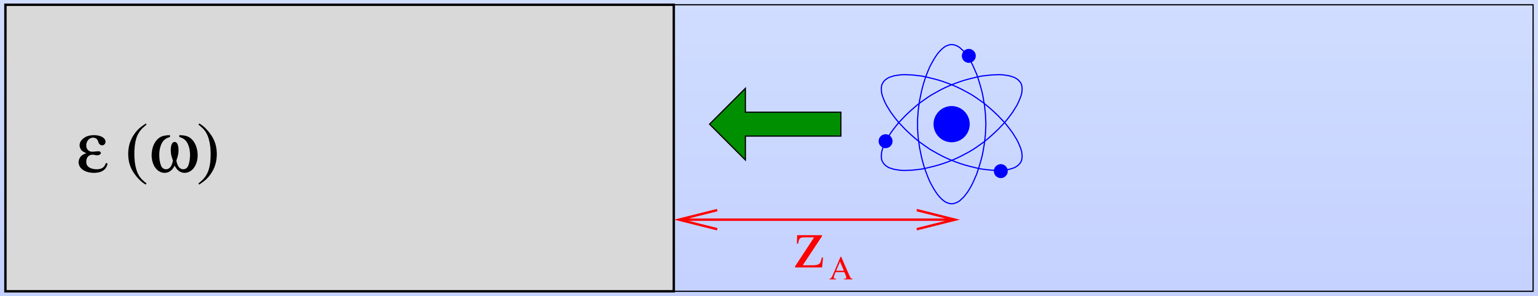


Atom near half space[1]



Nonretarded limit: $z_\mathsf{A} \ll c/\omega_t$

Atom near half space[1]

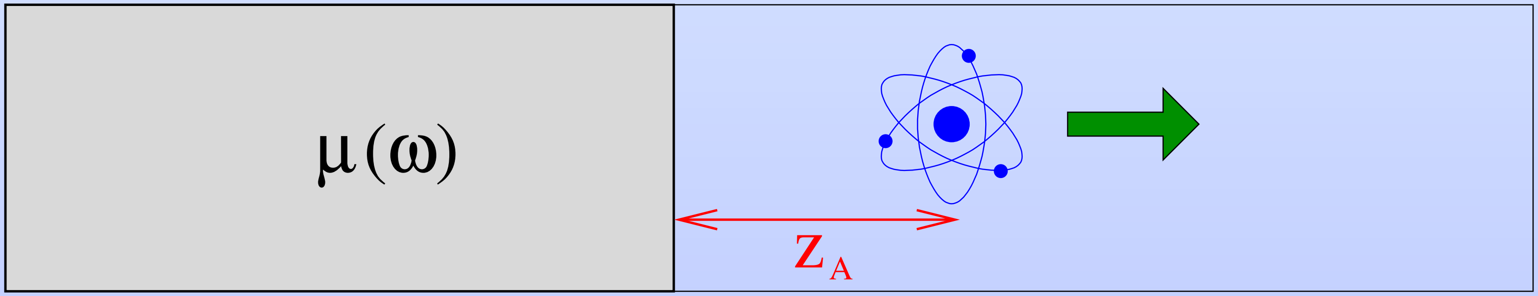


Nonretarded limit: $z_{\mathsf{A}} \ll c/\omega_t$

Purely dielectric half space: $$\boxed{U_0(z_{\mathsf{A}}) = -\frac{C_3}{z_{\mathsf{A}}^3}}$$

$$C_3 = \frac{\hbar}{16\pi^2\varepsilon_0}\int_0^\infty \mathrm{d}u\,\alpha_0^{(0)}(iu)\frac{\varepsilon(iu)-1}{\varepsilon(iu)+1} \geq 0$$

Atom near half space[1]



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Purely magnetic half space:
$$\boxed{U_0(z_{\mathsf{A}}) = +\frac{C_1}{z_{\mathsf{A}}^3}}$$

$$C_1 = \frac{\hbar}{16\pi^2\varepsilon_0}\int_0^\infty \mathrm{d}u\left(\frac{u}{c}\right)^2\alpha_0^{(0)}(iu)\left\{\frac{\mu(iu)-1}{\mu(iu)+1}+\frac{[\mu(iu)-1]}{2}\right\} \geq 0$$

[1]S. Y. Buhmann, T. Kampf, and D.-G. Welsch, in preparation

Atom near half space

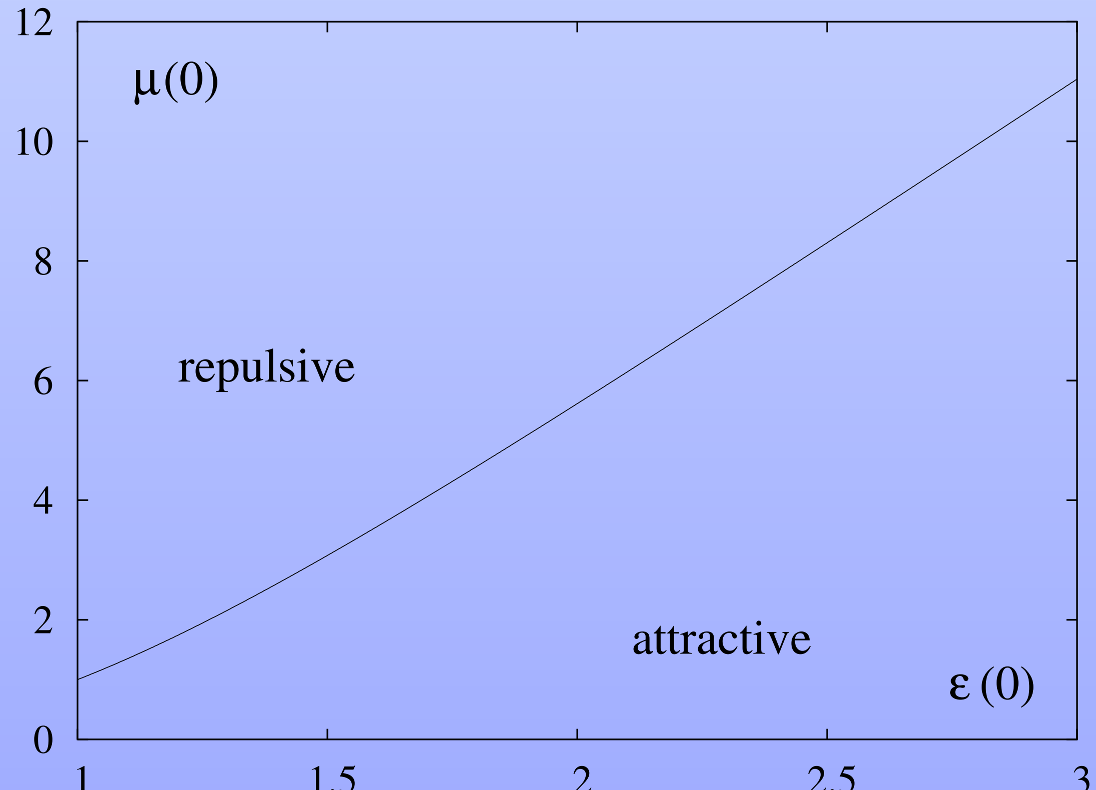
Retarded limit: $z_{\mathsf{A}} \gg c/\omega_r, c/\omega_{k0}$

$$\boxed{U_0(z_{\mathsf{A}}) = \frac{C_4}{z_{\mathsf{A}}^4}} \qquad C_4 = C_4[\alpha(0), \varepsilon(0), \mu(0)] \gtreqless 0$$

Atom near half space

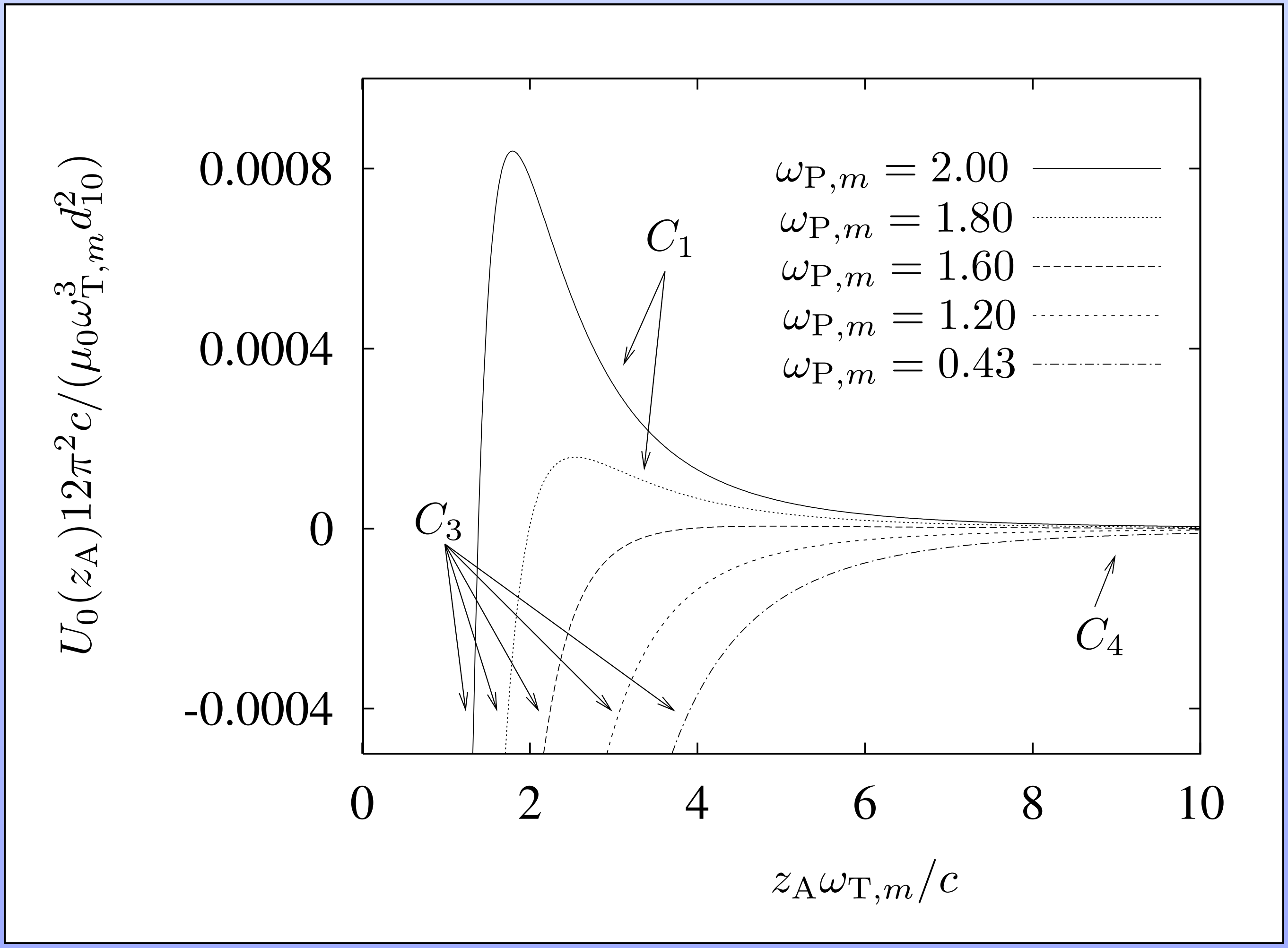
Retarded limit: $z_\mathrm{A} \gg c/\omega_r, c/\omega_{k0}$

$$U_0(z_\mathrm{A}) = \frac{C_4}{z_\mathrm{A}^4} \qquad C_4 = C_4[\alpha(0), \varepsilon(0), \mu(0)] \gtreqless 0$$

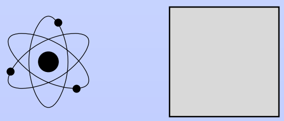


Atom near half space

Numerical results: two-level atom, Drude-Lorentz model



Comparison of different forces

distance	nonretarded		retarded	
objects	$e \leftrightarrow e$	$e \leftrightarrow m$	$e \leftrightarrow e$	$e \leftrightarrow m$
	$U \propto + \frac{1}{z^3}$	$U \propto - \frac{1}{z}$	$U \propto + \frac{1}{z^4}$	$U \propto - \frac{1}{z^4}$

Comparison of different forces

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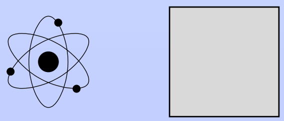
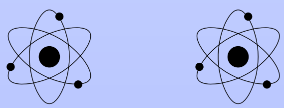
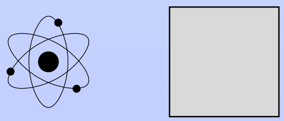
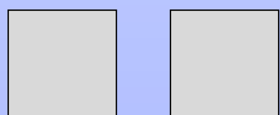
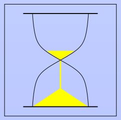
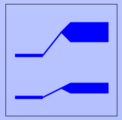
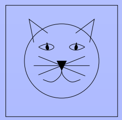
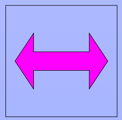
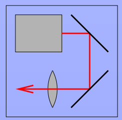
	$U \propto +\frac{1}{z^3}$	$U \propto -\frac{1}{z}$	$U \propto +\frac{1}{z^4}$	$U \propto -\frac{1}{z^4}$
	$U \propto +\frac{1}{z^6}$	$U \propto -\frac{1}{z^4}$	$U \propto +\frac{1}{z^7}$	$U \propto -\frac{1}{z^7}$

Comparison of different forces

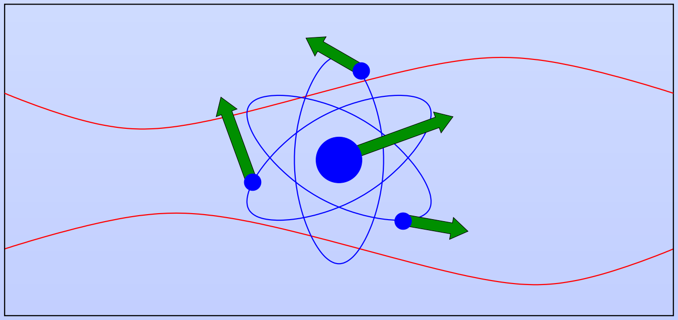
distance	nonretarded		retarded	
objects	$e \leftrightarrow e$	$e \leftrightarrow m$	$e \leftrightarrow e$	$e \leftrightarrow m$
	$U \propto + \frac{1}{z^3}$	$U \propto - \frac{1}{z}$	$U \propto + \frac{1}{z^4}$	$U \propto - \frac{1}{z^4}$
	$U \propto + \frac{1}{z^6}$	$U \propto - \frac{1}{z^4}$	$U \propto + \frac{1}{z^7}$	$U \propto - \frac{1}{z^7}$
	$F \propto + \frac{1}{z^3}$	$F \propto - \frac{1}{z}$	$F \propto + \frac{1}{z^4}$	$F \propto - \frac{1}{z^4}$

Casimir-Polder force: Beyond perturbation theory

Open issues

- Temporal evolution of the force 
- Influence of body-induced shifting and broadening of atomic transition lines 
- Force for arbitrary atomic states 
- Force in case of strong atom-field coupling 
- Force for arbitrary field states 

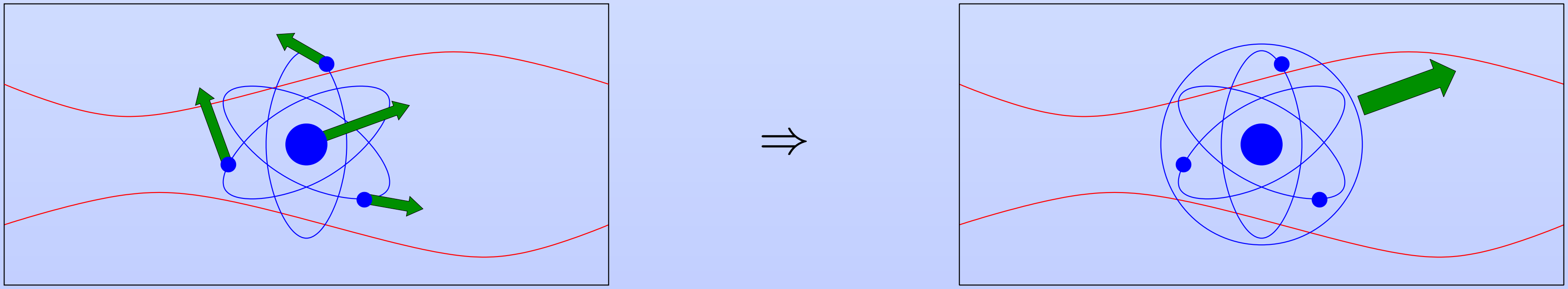
General theory



Lorentz force on charged particles (electric dipole app.):

$$\hat{\mathbf{f}}_\alpha = q_\alpha \Big\{ \hat{\mathbf{E}}(\mathbf{r}_\mathsf{A}) - \boldsymbol{\nabla}\hat{\varphi}_\mathsf{A}(\hat{\mathbf{r}}_\mathsf{A}) + \tfrac{1}{2}\Big[\dot{\hat{\mathbf{r}}}_\alpha \times \hat{\mathbf{B}}(\hat{\mathbf{r}}_\mathsf{A}) - \hat{\mathbf{B}}(\hat{\mathbf{r}}_\mathsf{A}) \times \dot{\hat{\mathbf{r}}}_\alpha\Big]\Big\}$$

General theory



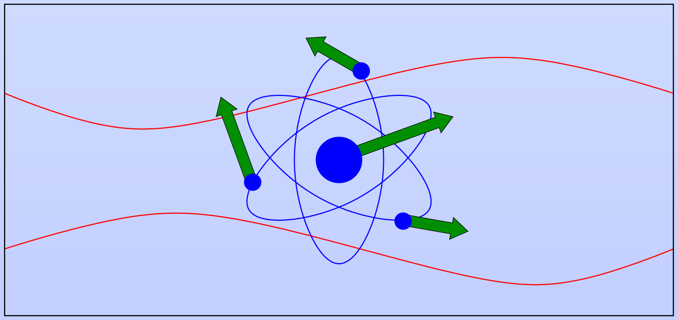
Lorentz force on charged particles (electric dipole app.):

$$
\begin{aligned}
\hat{\mathbf{f}}_\alpha &= q_\alpha \Big\{ \hat{\mathbf{E}}(\mathbf{r}_\mathsf{A}) - \boldsymbol{\nabla}\hat{\varphi}_\mathsf{A}(\hat{\mathbf{r}}_\mathsf{A}) \\
&\quad + \tfrac{1}{2}\Big[\dot{\hat{\mathbf{r}}}_\alpha \times \hat{\mathbf{B}}(\hat{\mathbf{r}}_\mathsf{A}) - \hat{\mathbf{B}}(\hat{\mathbf{r}}_\mathsf{A}) \times \dot{\hat{\mathbf{r}}}_\alpha\Big]\Big\}
\end{aligned}
$$

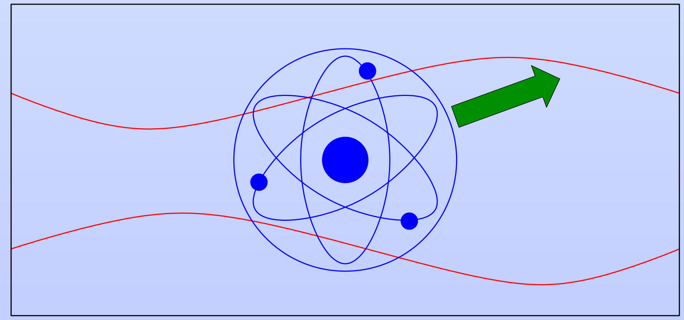
Lorentz force on an atom:

$$
\boxed{\left\langle \hat{\mathbf{F}} \right\rangle_\mathsf{AMF} = \left\{ \boldsymbol{\nabla} \left\langle \hat{\mathbf{d}}\hat{\mathbf{E}}(\mathbf{r}) \right\rangle_\mathsf{AMF} + \frac{\mathsf{d}}{\mathsf{d}t} \left\langle \hat{\mathbf{d}} \times \hat{\mathbf{B}}(\mathbf{r}) \right\rangle_\mathsf{AMF} \right\}_{\mathbf{r}=\hat{\mathbf{r}}_\mathsf{A}}}
$$

General theory



$\Rightarrow$



Lorentz force on charged particles (electric dipole app.):

$$\hat{\mathbf{f}}_\alpha = q_\alpha\Big\{\hat{\mathbf{E}}(\mathbf{r}_\mathsf{A}) - \boldsymbol{\nabla}\hat{\varphi}_\mathsf{A}(\hat{\mathbf{r}}_\mathsf{A}) + \tfrac{1}{2}\left[\dot{\hat{\mathbf{r}}}_\alpha \times \hat{\mathbf{B}}(\hat{\mathbf{r}}_\mathsf{A}) - \hat{\mathbf{B}}(\hat{\mathbf{r}}_\mathsf{A}) \times \dot{\hat{\mathbf{r}}}_\alpha\right]\Big\}$$

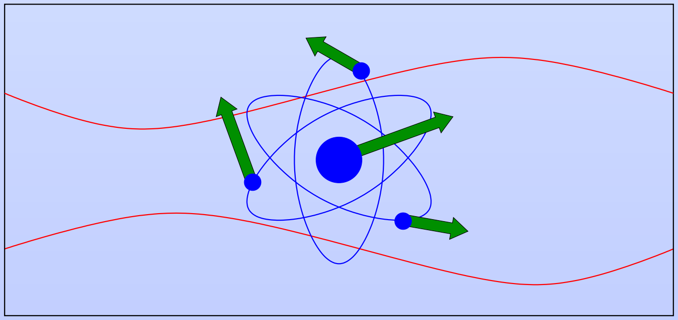
Lorentz force on an atom:

$$\left\langle\hat{\mathbf{F}}\right\rangle_\mathsf{AMF} = \left\{\boldsymbol{\nabla}\left\langle\hat{\mathbf{d}}\hat{\mathbf{E}}(\mathbf{r})\right\rangle_\mathsf{AMF} + \frac{\mathrm{d}}{\mathrm{d}t}\left\langle\hat{\mathbf{d}}\times\hat{\mathbf{B}}(\mathbf{r})\right\rangle_\mathsf{AMF}\right\}_{\mathbf{r}=\hat{\mathbf{r}}_\mathsf{A}}$$

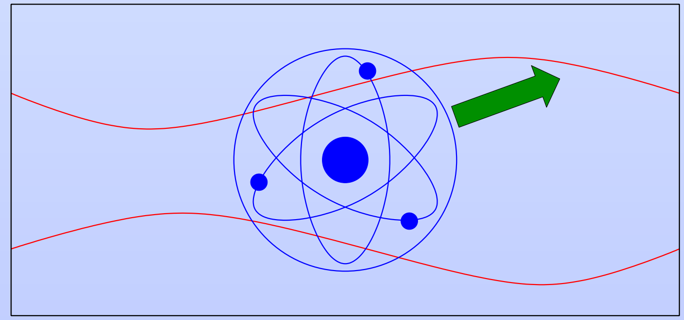
Applicability:

- *Field state:* arbitrary
- *Atomic state:* arbitrary
- *Coupling:* strong/weak

General theory



$\Rightarrow$



Lorentz force on charged particles (electric dipole app.):

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Applicability:

- *Field state:* arbitrary
- *Atomic state:* arbitrary
- *Coupling:* strong/weak

In the following:

$\rightarrow$ vacuum: $\hat{\rho}_\mathsf{MF} = |\{0\}\rangle\langle\{0\}|$

$\rightarrow$ weak coupling

CP force: Weak-coupling limit

Remaining task: Solving the dynamics (Markov approximation)

$$\hat{\mathbf{E}}(\mathbf{r}) = \hat{\mathbf{E}}(\mathbf{r},t) = ?, \quad \hat{\mathbf{d}} = \hat{\mathbf{d}}(t) = ?$$

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$$\hat{\mathbf{E}}(\mathbf{r}) = \hat{\mathbf{E}}(\mathbf{r}, t) = ?, \quad \hat{\mathbf{d}} = \hat{\mathbf{d}}(t) = ?$$

Casimir-Polder force:

$$\boxed{\mathbf{F}(\mathbf{r}_\mathsf{A}, t) = \sum_{m,n} \sigma_{nm}(t) \mathbf{F}_{mn}(\mathbf{r}_\mathsf{A})}$$

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Casimir-Polder force:

$$\mathbf{F}(\mathbf{r}_\mathsf{A},t) = \sum_{m,n} \sigma_{nm}(t)\mathbf{F}_{mn}(\mathbf{r}_\mathsf{A})$$

⇑ ⇑

Atomic density matrix elements:

$\sigma_{nm}(t)$

Associated force components:

$\mathbf{F}_{mn}(\mathbf{r}_\mathsf{A})$

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⇑ ⇑

Atomic density matrix elements:

$$\sigma_{nm}(t)$$

Associated force components:

$$\mathbf{F}_{mn}(\mathbf{r}_\mathsf{A})$$

⇑ ⇑

Body-induced change of atomic level structure:

$$\omega_{nm} \to \tilde{\omega}_{nm}(\mathbf{r}_\mathsf{A}), \quad \Gamma_n(\mathbf{r}_\mathsf{A})$$

Change of atomic level structure

Shift of atomic transition frequencies:

$$\tilde{\omega}_{nm}(\mathbf{r}_{\mathsf{A}}) = \omega_{nm} + \delta\omega_n(\mathbf{r}_{\mathsf{A}}) - \delta\omega_m(\mathbf{r}_{\mathsf{A}})$$

$$\delta\omega_n(\mathbf{r}_{\mathsf{A}}) = \sum_k \frac{\mu_0}{\pi\hbar}\mathcal{P}\int_0^\infty \mathrm{d}\omega\,\omega^2 \frac{\mathbf{d}_{nk}\mathrm{Im}\boldsymbol{G}^{(1)}(\mathbf{r}_{\mathsf{A}},\mathbf{r}_{\mathsf{A}},\omega)\mathbf{d}_{kn}}{\tilde{\omega}_{nk}-\omega}$$

Change of atomic level structure

Shift of atomic transition frequencies:

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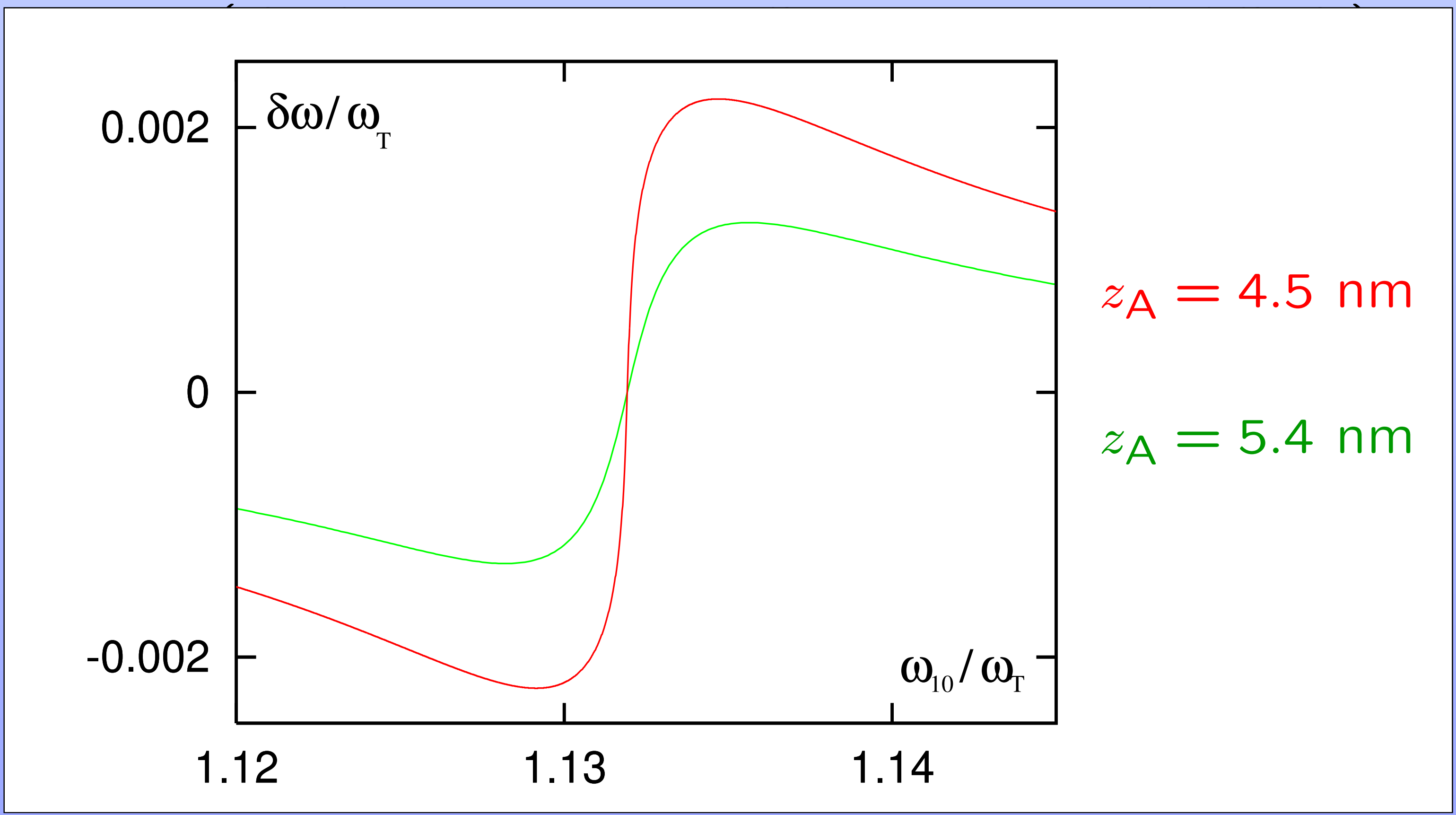
Example: Two-level atom near dielectric half space



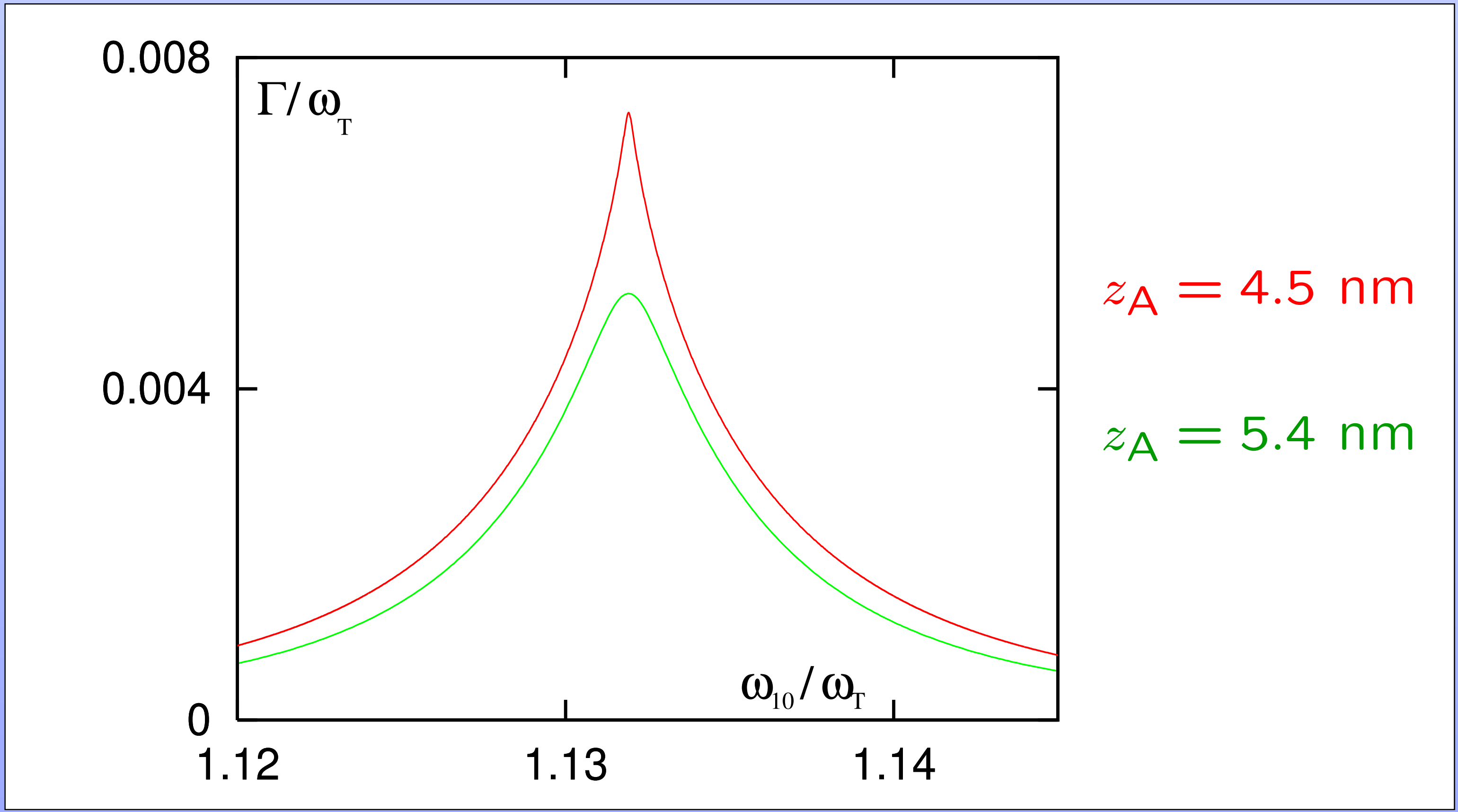
Decay-induced broadening of atomic levels:

$$\begin{aligned}\Gamma_n(\mathbf{r}_\mathsf{A}) &= \sum_k \Theta[\tilde{\omega}_{nk}(\mathbf{r}_\mathsf{A})]\Gamma_{nk}(\mathbf{r}_\mathsf{A}) \\ &= \frac{2\mu_0}{\hbar}\Theta[\tilde{\omega}_{nk}(\mathbf{r}_\mathsf{A})]\tilde{\omega}^2_{nk}(\mathbf{r}_\mathsf{A})\mathbf{d}_{nk}\mathrm{Im}\boldsymbol{G}\left[\mathbf{r}_\mathsf{A},\mathbf{r}_\mathsf{A},\tilde{\omega}_{nk}(\mathbf{r}_\mathsf{A})\right]\mathbf{d}_{kn}\end{aligned}$$

Decay-induced broadening of atomic levels:

$$\begin{aligned}\Gamma_n(\mathbf{r}_A) &= \sum_k \Theta[\tilde{\omega}_{nk}(\mathbf{r}_A)]\Gamma_{nk}(\mathbf{r}_A) \\ &= \frac{2\mu_0}{\hbar}\Theta[\tilde{\omega}_{nk}(\mathbf{r}_A)]\tilde{\omega}^2_{nk}(\mathbf{r}_A)\mathbf{d}_{nk}\mathrm{Im}\boldsymbol{G}\,[\mathbf{r}_A,\mathbf{r}_A,\tilde{\omega}_{nk}(\mathbf{r}_A)]\mathbf{d}_{kn}\end{aligned}$$

Example: Two-level atom near dielectric half space (single-resonance medium, nonretarded limit)



Excited-(eigen)state force

Dominant contribution to the force:

$$\mathbf{F}^{\mathsf{r}}_{nn}(\mathbf{r}_{\mathsf{A}}) = \frac{\mu_0}{2}\sum_k \Theta[\tilde{\omega}_{nk}(\mathbf{r}_{\mathsf{A}})]\Omega^2_{nk}(\mathbf{r}_{\mathsf{A}}) \left\{\boldsymbol{\nabla} \otimes \mathbf{d}_{nk}\boldsymbol{G}^{(1)}[\mathbf{r},\mathbf{r},\Omega_{nk}(\mathbf{r}_{\mathsf{A}})]\mathbf{d}_{kn}\right\}_{\mathbf{r}=\mathbf{r}_{\mathsf{A}}} + \text{H.c.}$$

$$\Omega_{nk}(\mathbf{r}_{\mathsf{A}}) = \tilde{\omega}_{nk}(\mathbf{r}_{\mathsf{A}}) + i[\Gamma_n(\mathbf{r}_{\mathsf{A}}) + \Gamma_k(\mathbf{r}_{\mathsf{A}})]/2$$

$\rightarrow$ **Influenced by level shifting and broadening!**

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$\rightarrow$ **Influenced by level shifting and broadening!**

Example: Two-level atom near dielectric half space
(single-resonance medium, nonretarded limit)

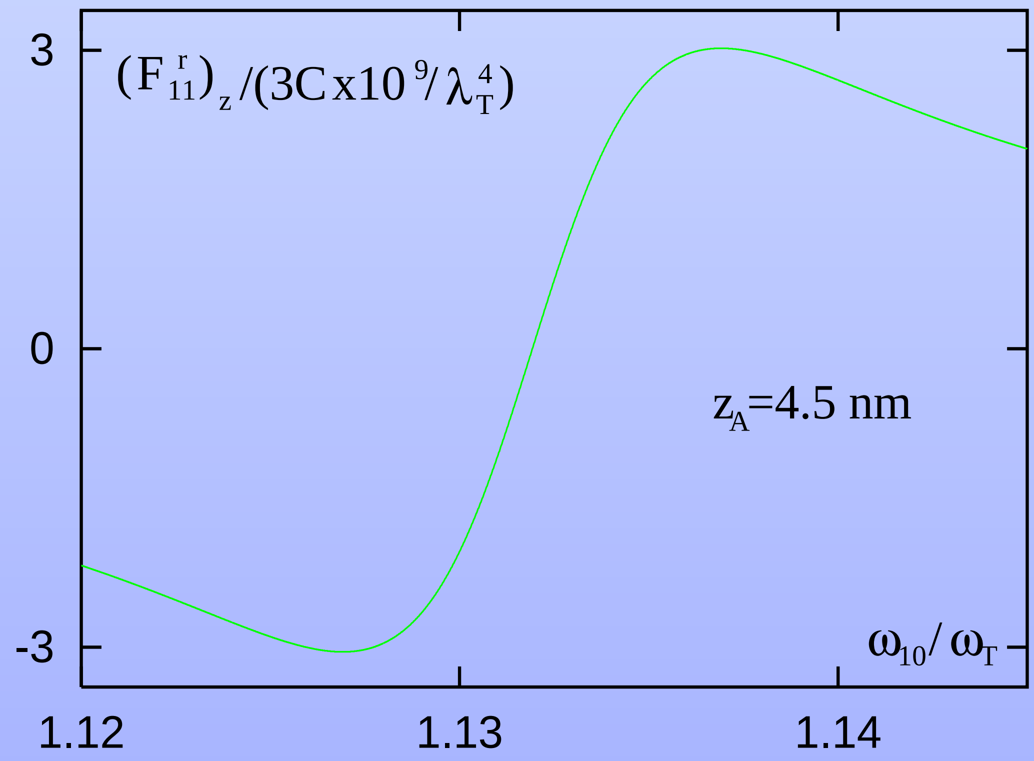
$$F_{11}(z_{\mathrm{A}}) = -\frac{3(d_x^2+d_y^2+2d_z^2)}{32\pi\varepsilon_0 z_{\mathrm{A}}^4}\frac{|\varepsilon[\Omega_{10}(z_{\mathrm{A}})]|^2-1}{|\varepsilon[\Omega_{10}(z_{\mathrm{A}})]+1|^2}$$

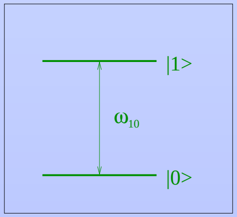
$$\varepsilon[\Omega_{10}(z_{\mathrm{A}})] = 1 + \frac{\omega_{\mathrm{P}}^2}{\omega_{\mathrm{T}}^2 - \tilde{\omega}_{10}^2(z_{\mathrm{A}}) - i[\Gamma(z_{\mathrm{A}})+\gamma]\tilde{\omega}_{10}(z_{\mathrm{A}})}$$

$\rightarrow \Gamma(z_{\mathrm{A}}) + \gamma$ **plays the role of total absorption parameter!**

Excited-(eigen)state force

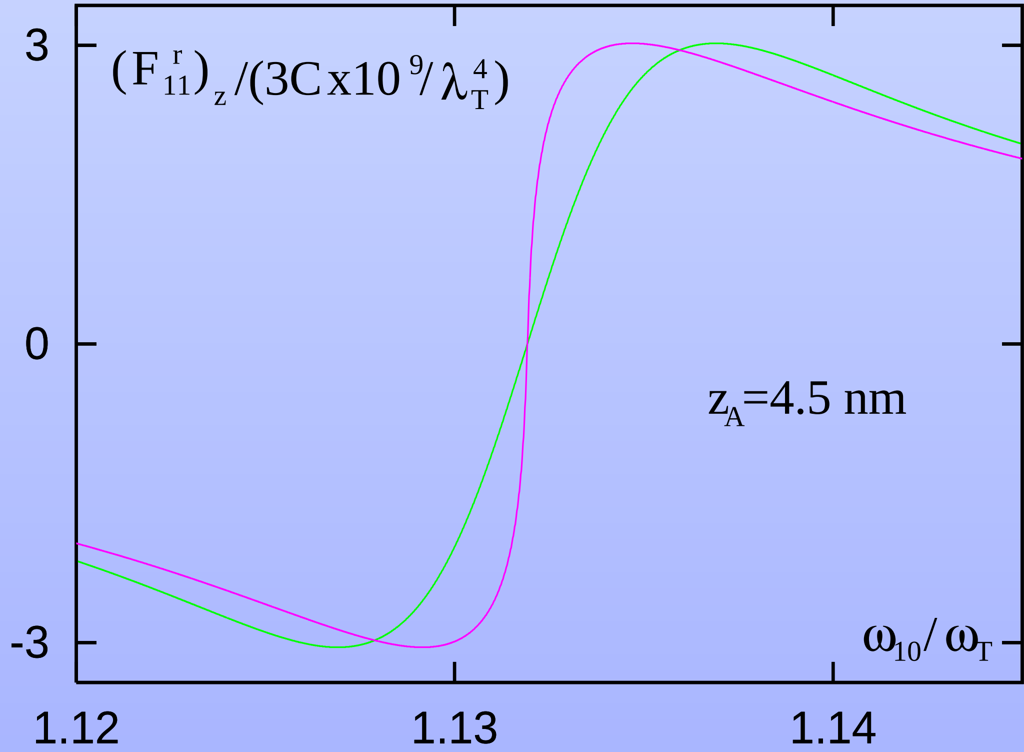
Example: Two-level atom near dielectric half space (single-resonance medium, nonretarded limit)

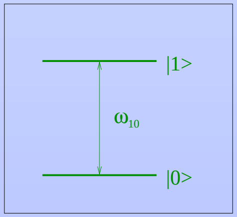


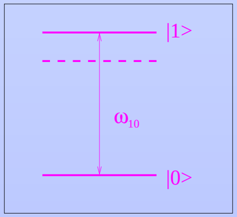


Excited-(eigen)state force

Example: Two-level atom near dielectric half space
(single-resonance medium, nonretarded limit)

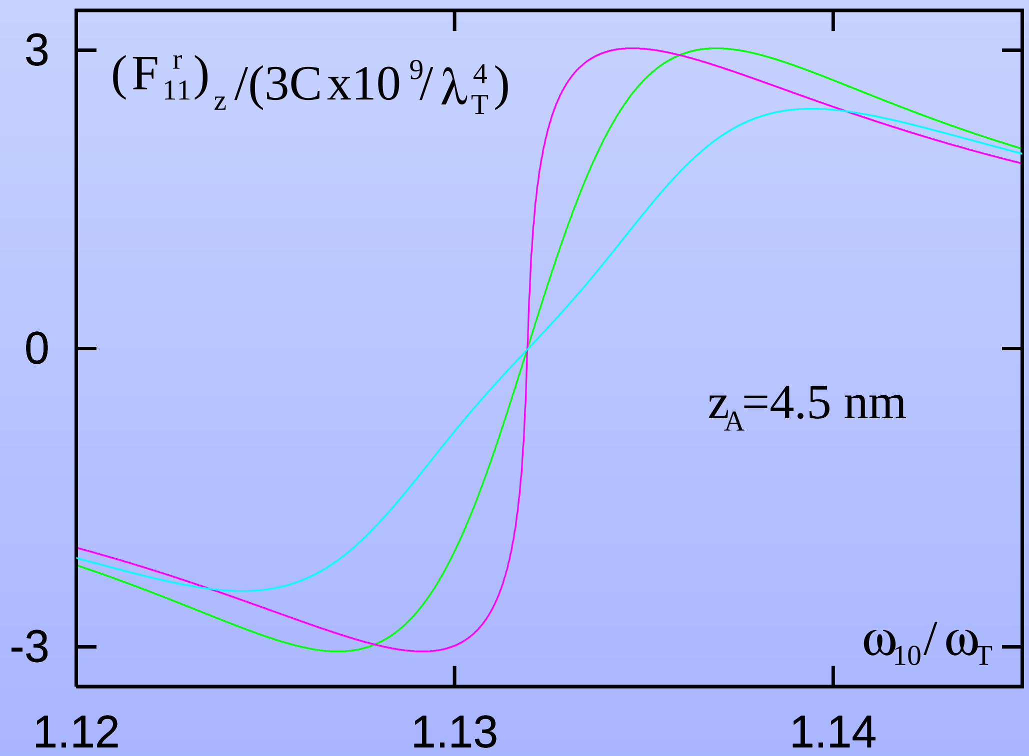


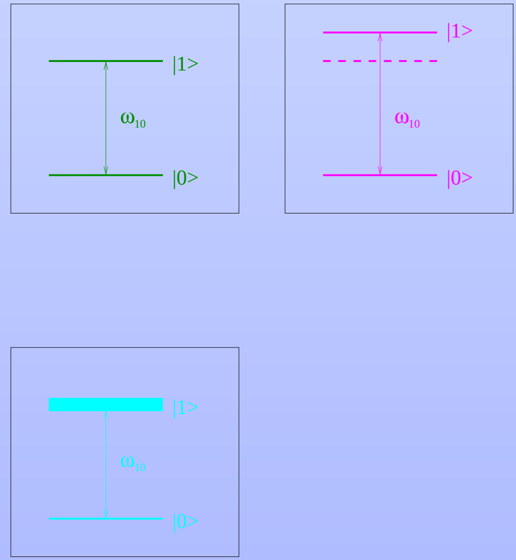




Excited-(eigen)state force

Example: Two-level atom near dielectric half space
(single-resonance medium, nonretarded limit)





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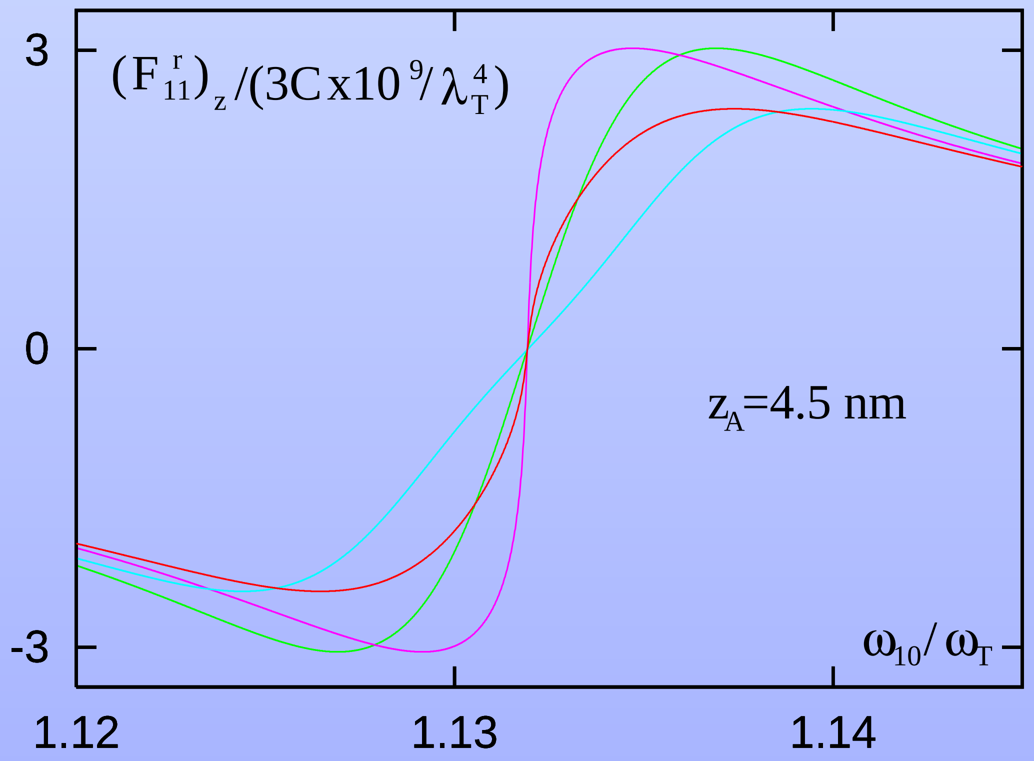
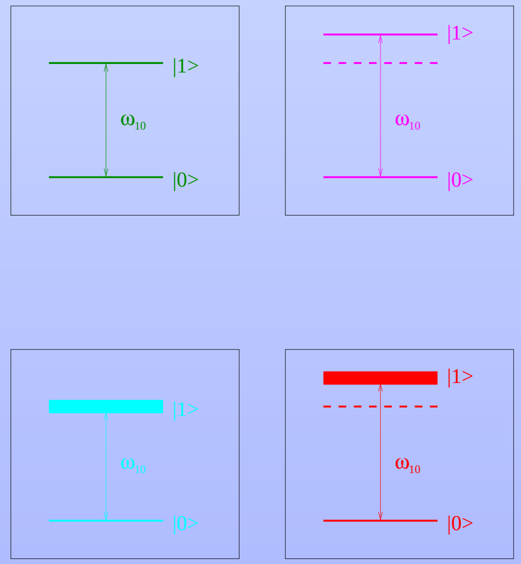
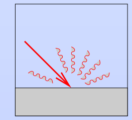
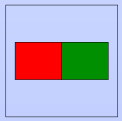
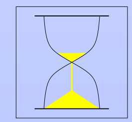
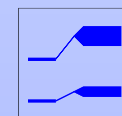




Casimir-Polder force: Summary

- Absorption: included in Green tensor formalism 
- Magnetic bodies → attractive vs repulsive van der Waals potentials 
- Spontaneous decay → temporal evolution of the force 
- Shifting and broadening of atomic transition lines
  → noticeable influence in nonretarded limit 

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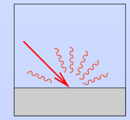
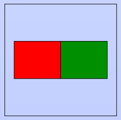
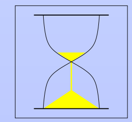
- Absorption: included in Green tensor formalism 
- Magnetic bodies → attractive vs repulsive van der Waals potentials 
- Spontaneous decay → temporal evolution of the force 
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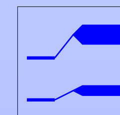
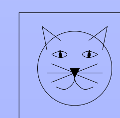
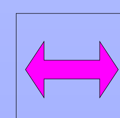
  → noticeable influence in nonretarded limit 

Outlook

- Closer investigation of off-diagonal force components 
- Force in case of strong atom-field coupling 
- Force for arbitrary field states