Nonlinear optics of atoms and molecules

Chris Search (→ Stevens Inst. Of Technology)
Markku Jääskeläinen
Takahiko Miyakawa

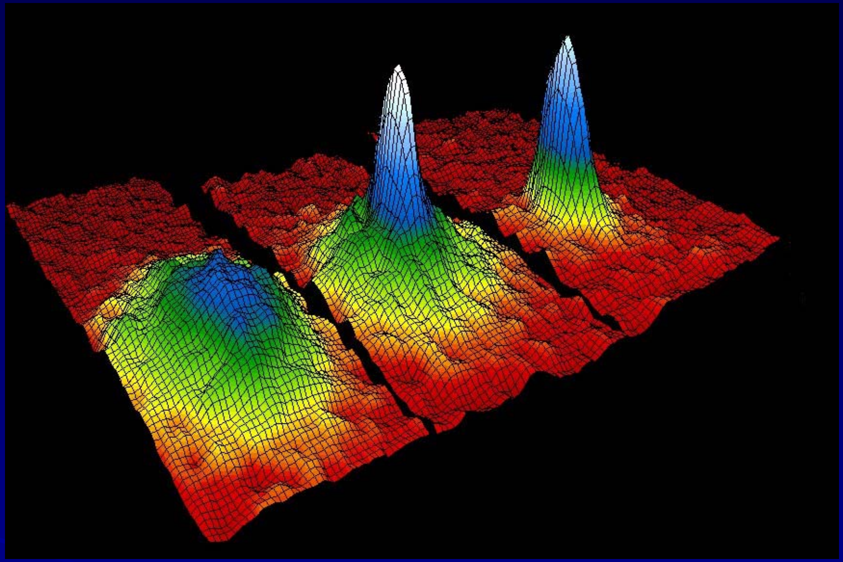
Omjyoti Dutta
Dominic Meiser
Hermann Uys

Pierre Meystre

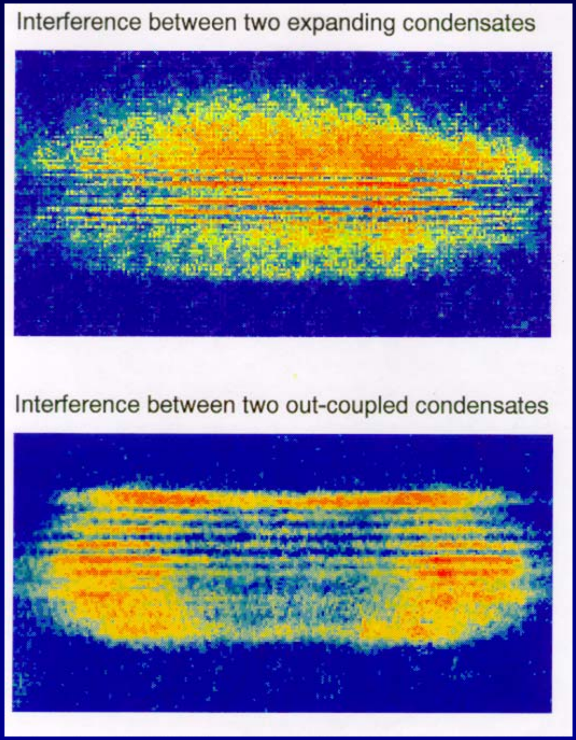
Optical Sciences Center
The University of Arizona

ARO NASA NSF ONR

Matter-wave field



(Courtesy E. Cornell and C. Wieman)



(Courtesy W. Ketterle)

Matter-wave field $\hat{\psi}(r,t)$

$$\left[\hat{\psi}(r,t),\hat{\psi}^{\dagger}(r',t)\right]_{-}=\delta(r-r')$$ bosons

$$\left[\hat{\psi}(r,t),\hat{\psi}^{\dagger}(r',t)\right]_{+}=\delta(r-r')$$ fermions

Collisions

$$H = \int d^3r\,\hat{\psi}^\dagger(r) H_0 \hat{\psi}(r) + \int d^3r_1 d^3r_2\,\hat{\psi}^\dagger(r_1)\hat{\psi}^\dagger(r_2) V(r_1 - r_2)\hat{\psi}(r_2)\hat{\psi}(r_1)$$

Single-particle

Two-body collisions

- *s*-wave approximation :

$$V\left(r_2 - r_1\right) = \frac{4\pi\hbar^2 a}{M}\delta\left(r_2 - r_1\right)$$

- Heisenberg equations of motion:

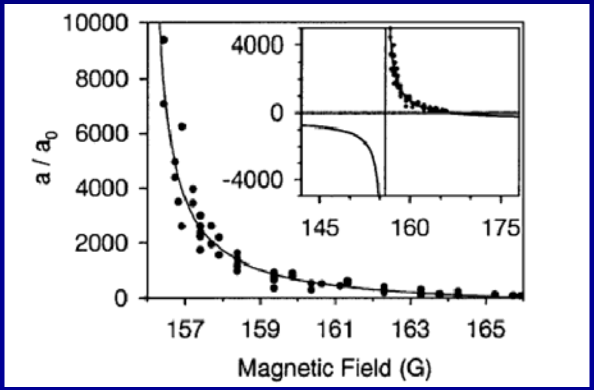
$$i\hbar\frac{d\hat{\psi}(r,t)}{dt} = H_0\hat{\psi}(r,t) + \left(\frac{4\pi\hbar^2 a}{M}\right)\hat{\psi}^\dagger(r,t)\hat{\psi}(r,t)\hat{\psi}(r,t)$$

Mean-field theory (bosons)

Bosons : $\hat{\psi}(r,t) \rightarrow \langle \hat{\psi}(r,t) \rangle \equiv \Phi(r,t)$ "condensate wave function"

Gross-Pitaevskii equation : $$i\hbar \frac{d\Phi(r,t)}{dt} = H_0 \Phi(r,t) + \underbrace{\left(\frac{4\pi\hbar^2 a}{M} \right) |\Phi(r,t)|^2 \Phi(r,t)}_{\text{Two-body collisions}}$$

a : scattering length



(From S. L. Cornish *et al*, PRL **85**, 1795 (2000))

Examples

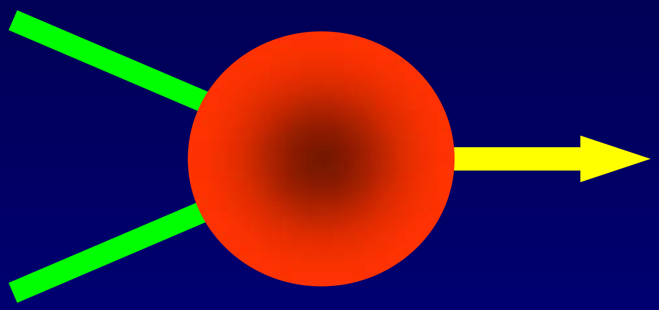
- Four-wave mixing of matter waves
- Matter-wave phase conjugation
- Atom holography
- Atomic solitons
- Second-harmonic generation
- Atom lasers
- Atom amplifiers
- Matter-wave superradiance
- Mixing of optical and matter waves
- Nonclassical and entangled states
- Coherence control
- Quantum information
- Sensors

- Fermionic matter waves
- Fermi-Bose wave mixing
- Coherent molecular fields
- ...

Recent trends

- **Beyond mean field**
  - Fluctuations
  - Strongly correlated systems (Mott insulator transition, ...)
  - Quantum atom optics (number states, entanglement, ...)
- **Fermions**
  - BCS pairing
  - BEC-BCS cross-over
  - Fermionic atom optics
- **Molecules**
  - Feshbach resonances
  - Photoassociation
  - Fermions vs. bosons
  - Molecular condensates
  - Molecular optics
- **Cavity atom optics**
  - Quantum control of matter-wave field

Three-wave mixing

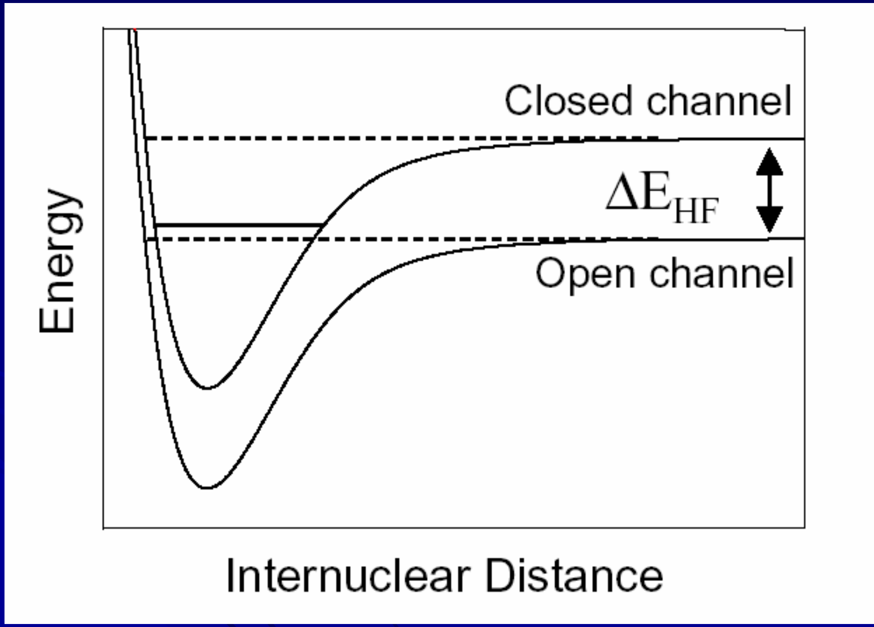


- Particle number conservation ?
- **Molecules !**

Atoms	Molecules	Property
identical bosons	bosonic	short-lived
identical fermions (different internal state)	bosonic	long-lived
different bosonic or fermionic atoms	bosonic	heteronuclear, polar
one boson and one fermion	fermionic	heteronuclear, polar

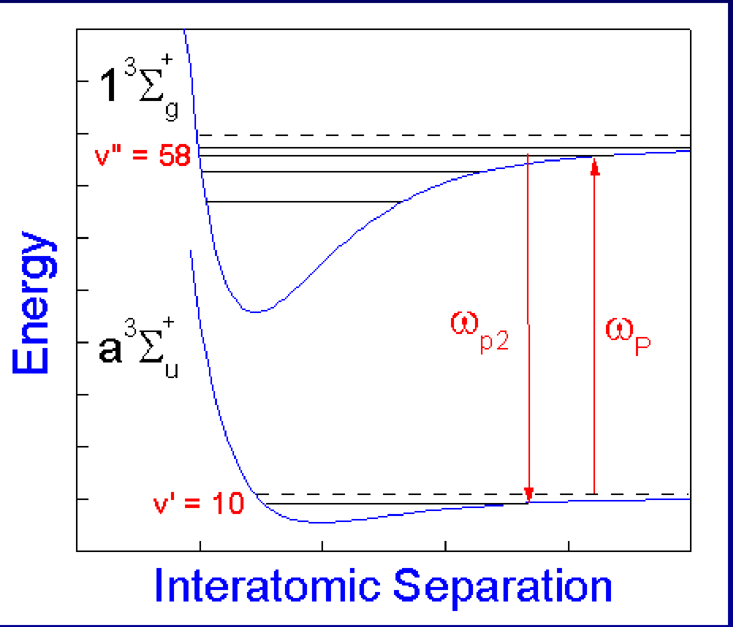
Coherent molecule formation

- Feshbach resonances



(Picture from N. R.Claussen, PhD Thesis, U. Colorado http://jilawww.colorado.edu/www/sro/thesis/claussen/)

- Photoassociation

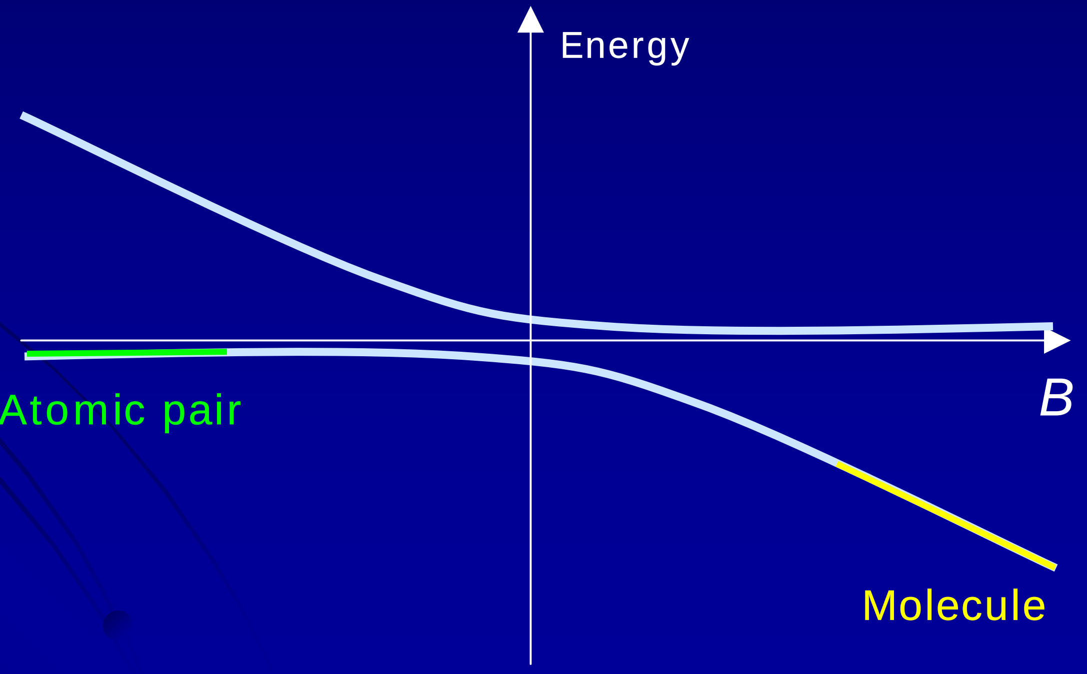


(Picture from R. Hulet, http://atomcool.rice.edu)

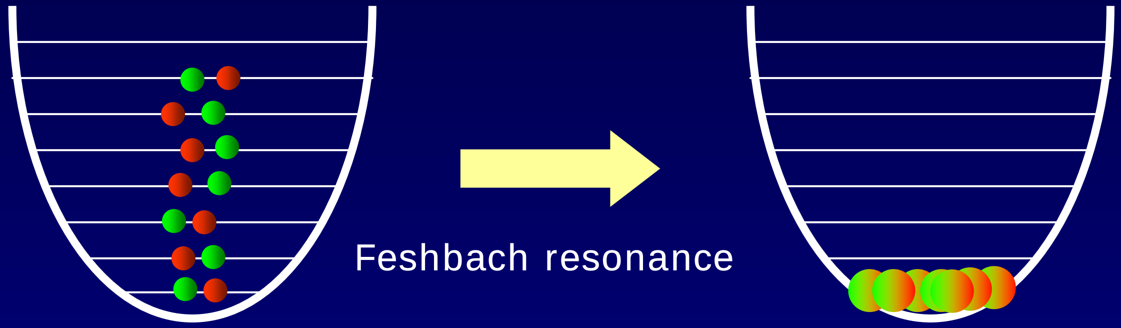
Adiabatic sweep

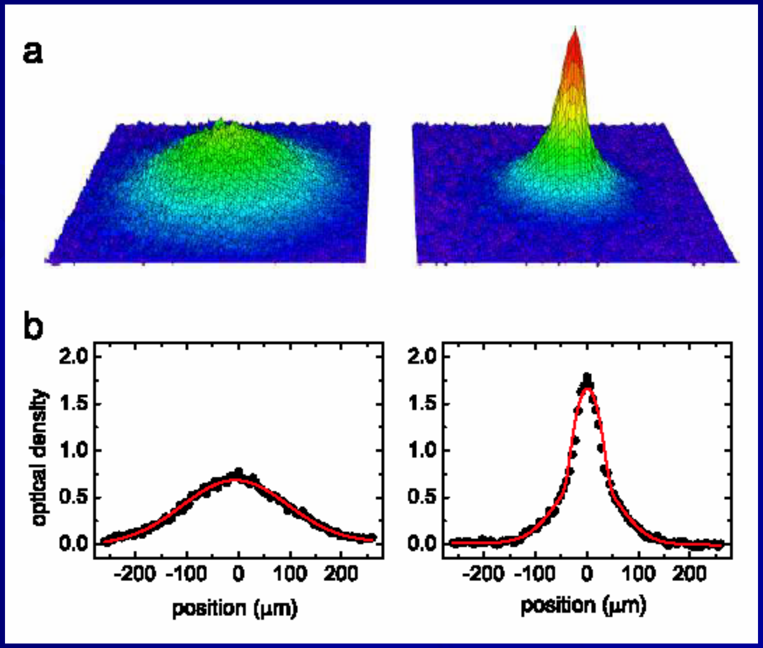
$$H=\sum_{\sigma=\uparrow,\downarrow}\int d^3r\left[\hat{\psi}_\sigma^\dagger(r)\left(-\frac{\hbar^2\nabla^2}{2m}+V_\sigma(r)\right)\hat{\psi}_\sigma(r)+\hat{\varphi}^\dagger(r)\left(-\frac{\hbar^2\nabla^2}{2m}+V_\sigma(r)+\varepsilon(B)\right)\hat{\varphi}(r)\right]$$

$$+\tfrac{1}{2}\sum_{\sigma,\sigma'}\int d^3r\left(\chi_{\sigma,\sigma'}\hat{\psi}_\sigma^\dagger(r)\hat{\psi}_{\sigma'}^\dagger(r)\hat{\varphi}(r)+H.c.\right)$$



Molecular BEC





"A molecular condensate emerges from the Fermi sea",

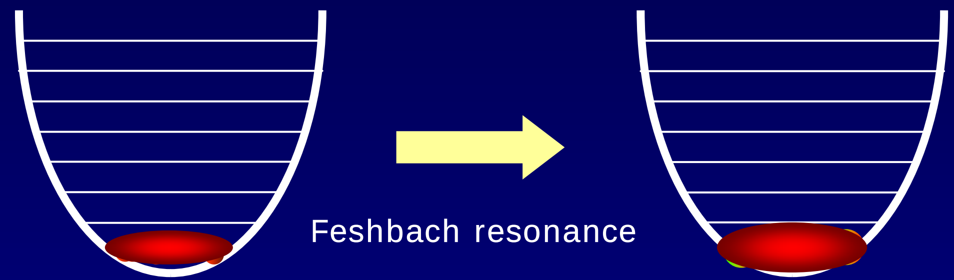
M. Greiner, C. Regal, & D. Jin
Nature **426**, 537 (2003)

Also:

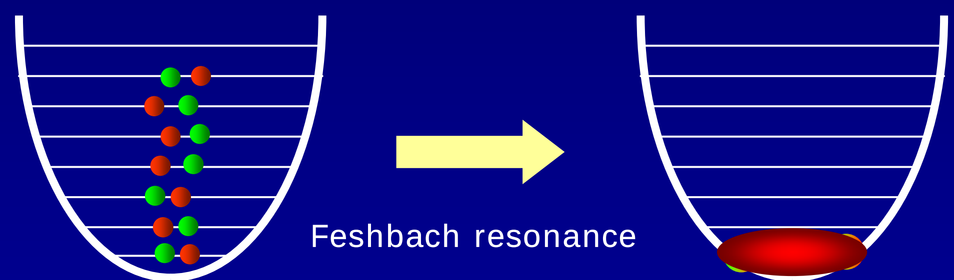
- R. Grimm *et al.*, Science Express, Nov. 13, 2003
- W. Ketterle *et al.*, PRL **91**, 250401 (2003)

Molecule statistics

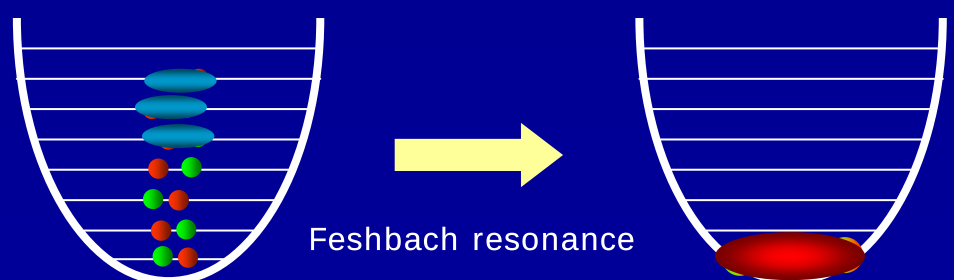
- Atomic BEC



- Normal Fermi system

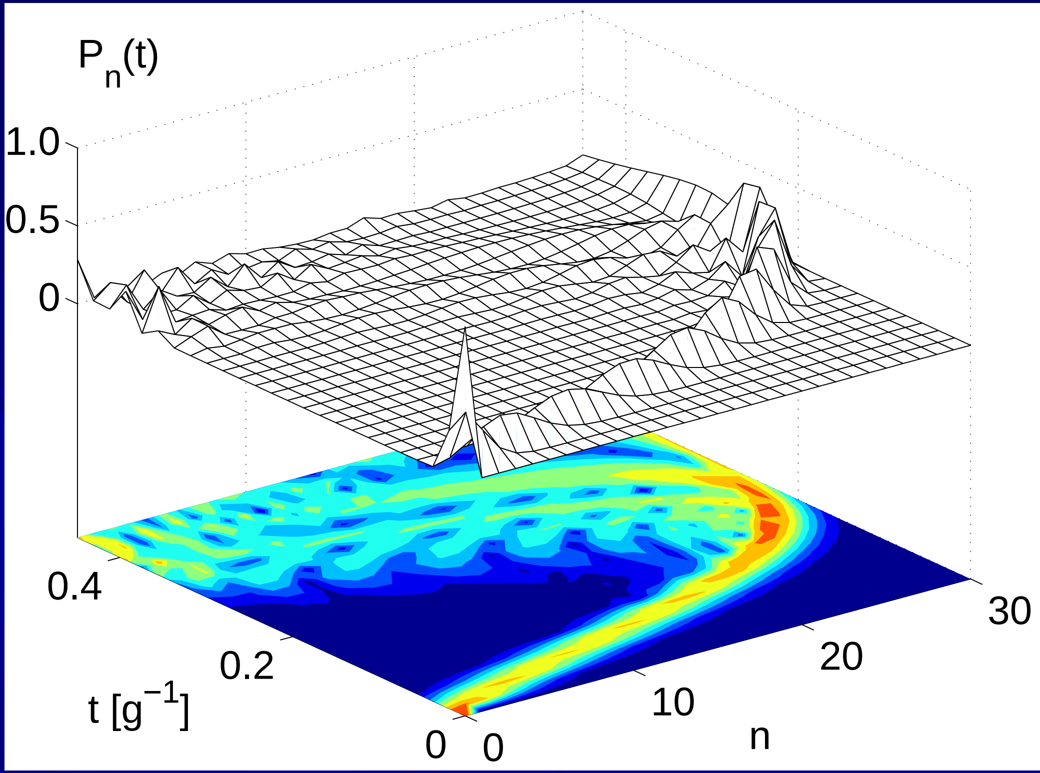


- BCS state



Atomic BEC

Two-mode model: $H_{\mathrm{BEC}} = \delta \hat{a}^{\dagger}\hat{a} + g\left(\hat{a}^{\dagger}\hat{c}^{2} + H.c.\right)$



Short times:

$$n(t) = \langle \hat{a}^{\dagger}\hat{a} \rangle = (gt)^{2}\, 2N_a (2N_a - 1)$$

$$g^{(2)}(t_1, t_2) = 1 - \frac{2}{N_a} + \dots$$

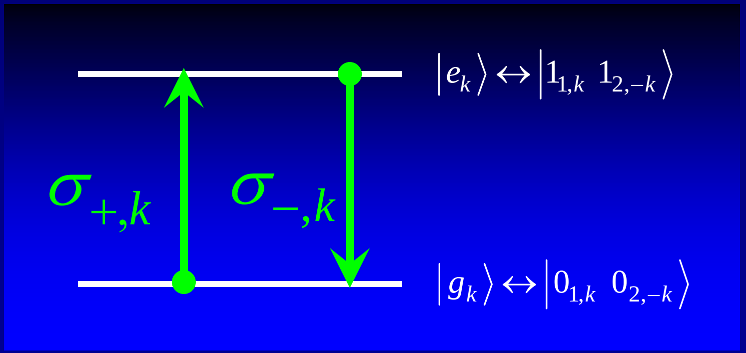
Coherent state !

Normal Fermi Gas

$$H_{\mathrm{NFG}} = \sum_k E_k \left(\hat{c}^\dagger_{1,k}\hat{c}_{1,k} + \hat{c}^\dagger_{2,-k}\hat{c}_{2,-k} \right) + \delta \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \sum_k \hat{c}_{1,k}\hat{c}_{2,-k} + H.c. \right)$$

- Anderson mapping

$$\sigma_{-,k} = \hat{c}_{1,k}\hat{c}_{2,-k}$$
$$\sigma_{+,k} = \hat{c}^\dagger_{2,-k}\hat{c}^\dagger_{1,k}$$
$$\sigma_{z,k} = \hat{c}^\dagger_{1,k}\hat{c}_{1,k} + \hat{c}^\dagger_{2,-k}\hat{c}_{2,-k} - 1$$

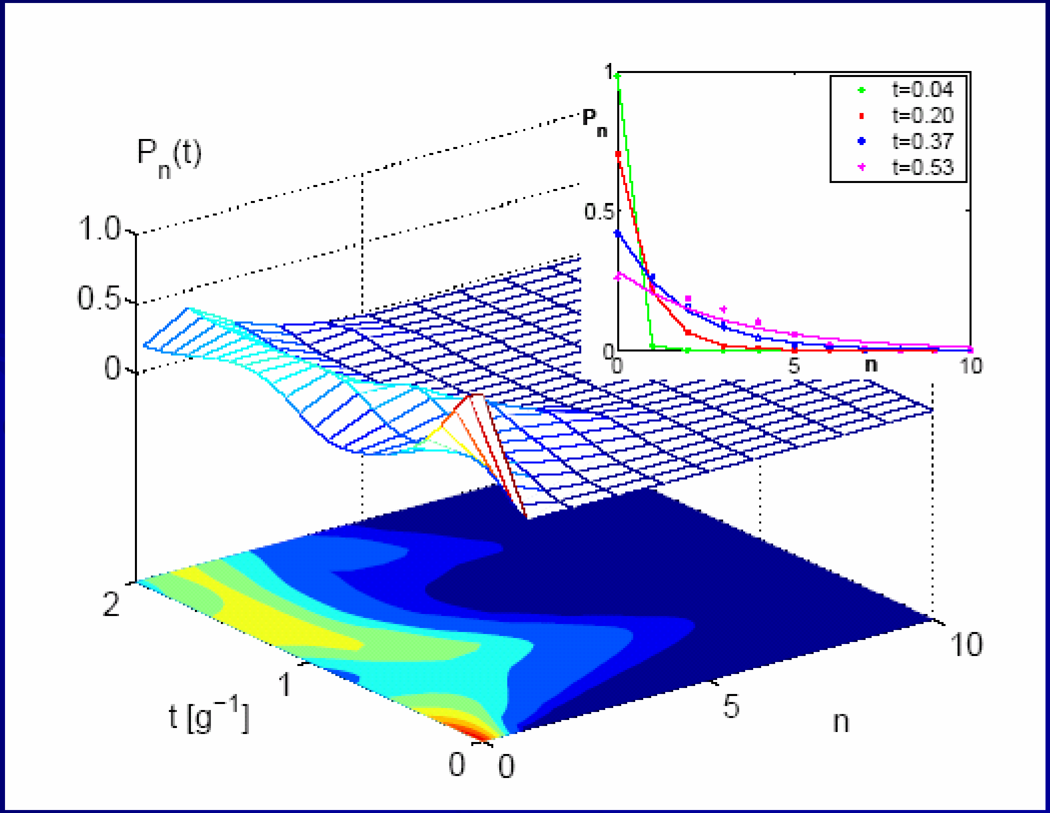


$$\Rightarrow \quad H_{\mathrm{NFG}} = \sum_k E_k \hat{\sigma}_{k,z} + \delta \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \sum_k \hat{\sigma}_{k,-} + H.c. \right)$$

Normal Fermi Gas (2)

$$H_{\mathrm{NFG}} = \sum_k E_k \hat{\sigma}_{k,z} + \delta \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \sum_k \hat{\sigma}_{k,-} + H.c. \right)$$

$$\begin{cases} \hat{\sigma}_{k,z} = \hat{c}_k^\dagger \hat{c}_k + \hat{c}_{-k}^\dagger \hat{c}_{-k} - 1 \\ \hat{\sigma}_{k,-} = \hat{c}_{-k} \hat{c}_k \end{cases}$$



Short times:

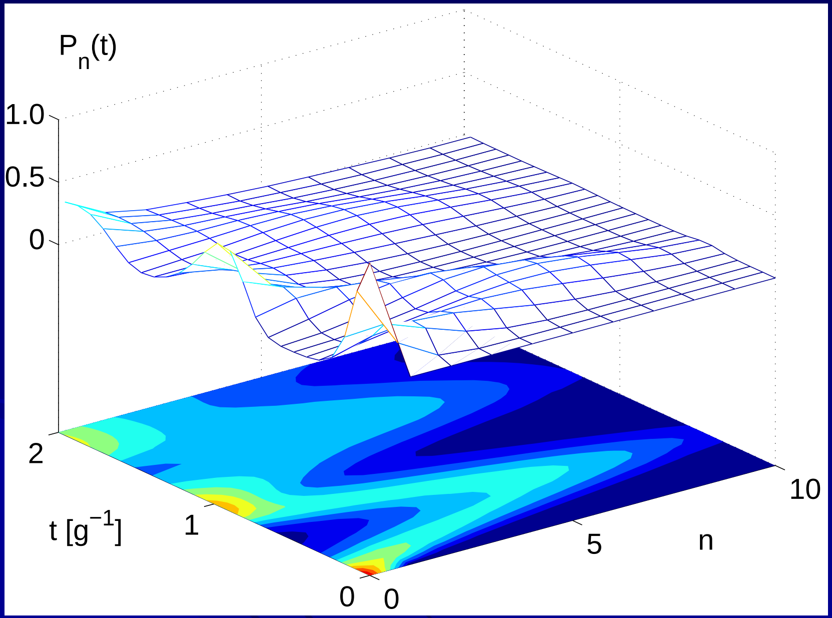
$$n(t) = \langle \hat{a}^\dagger \hat{a} \rangle = (gt)^2 \, 2N_a$$

$$g^{(2)}(t_1, t_2) = 2\left(1 - \frac{1}{2N_a}\right) + \ldots$$

Chaotic state !

BCS state

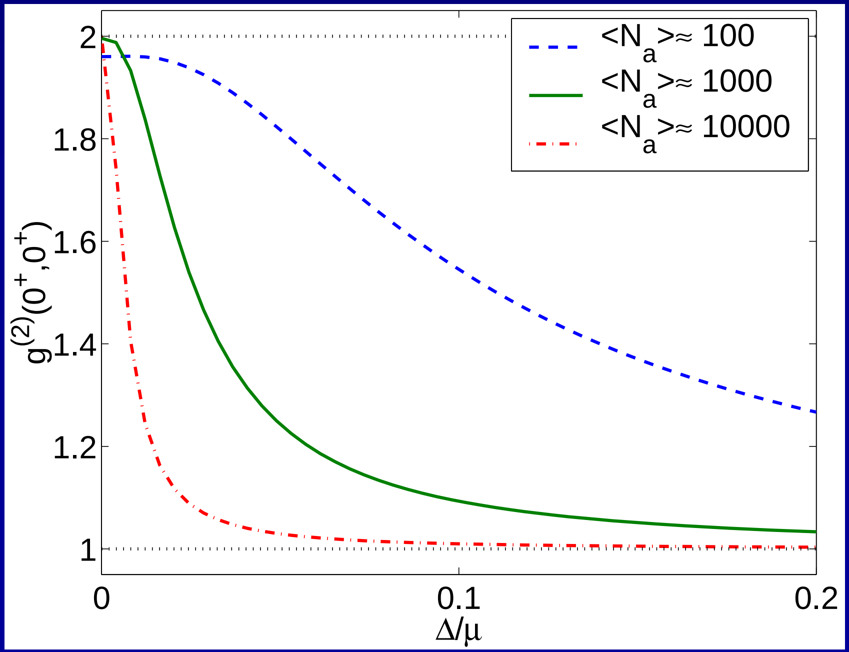
$$H_{BCS} = H_{NFG} - V\hat{\sigma}_{k,+}\hat{\sigma}_{k',-}$$



Short times:

$$n(t) = \langle \hat{a}^{\dagger}\hat{a} \rangle = (gt)^2 \left[\left(\frac{\Delta}{V} \right)^2 + N_a \right]$$

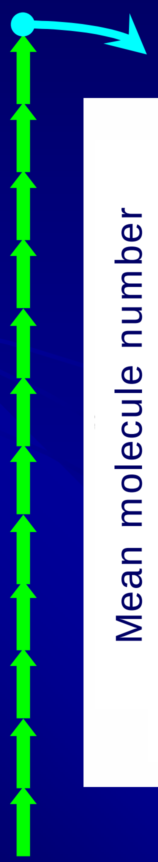
(Number of Cooper pairs)²

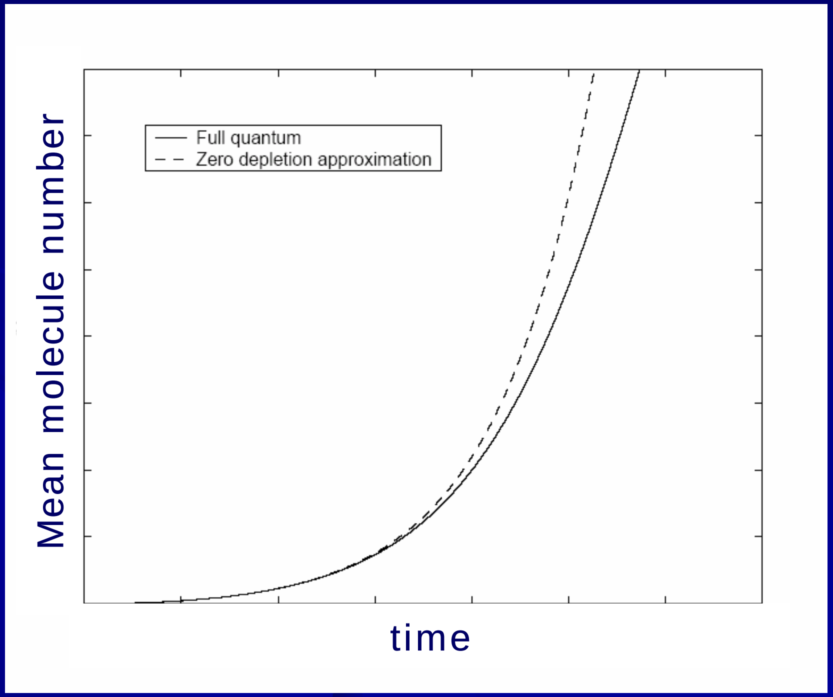


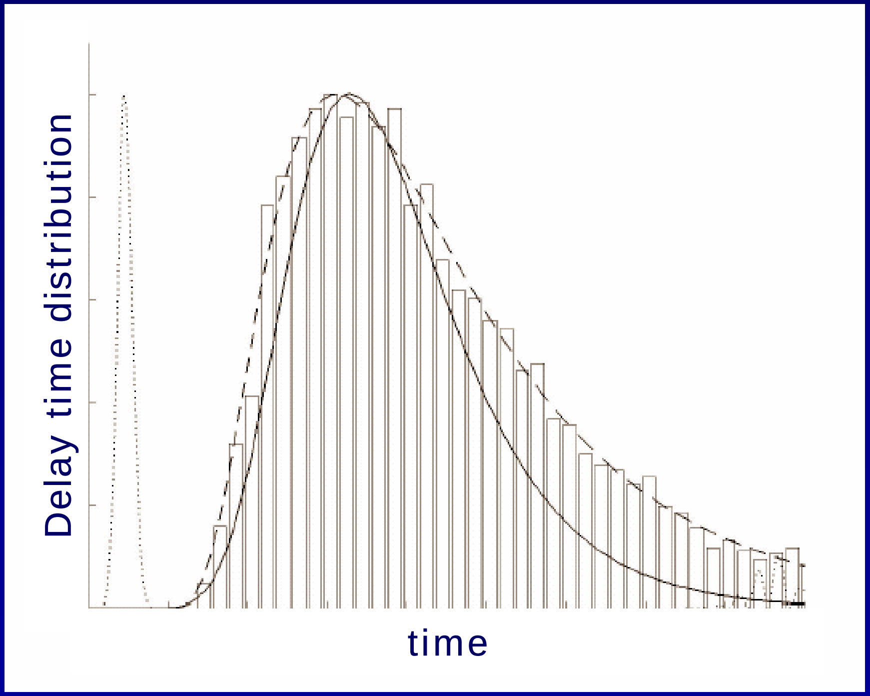
Delay time statistics (NFG)

Normal Fermi gas: $$H_{\text{NFG}} = \sum_k E_k \hat{\sigma}_{k,z} + \delta \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \sum_k \hat{\sigma}_{k,-} + H.c. \right)$$

analogous to superradiance problem







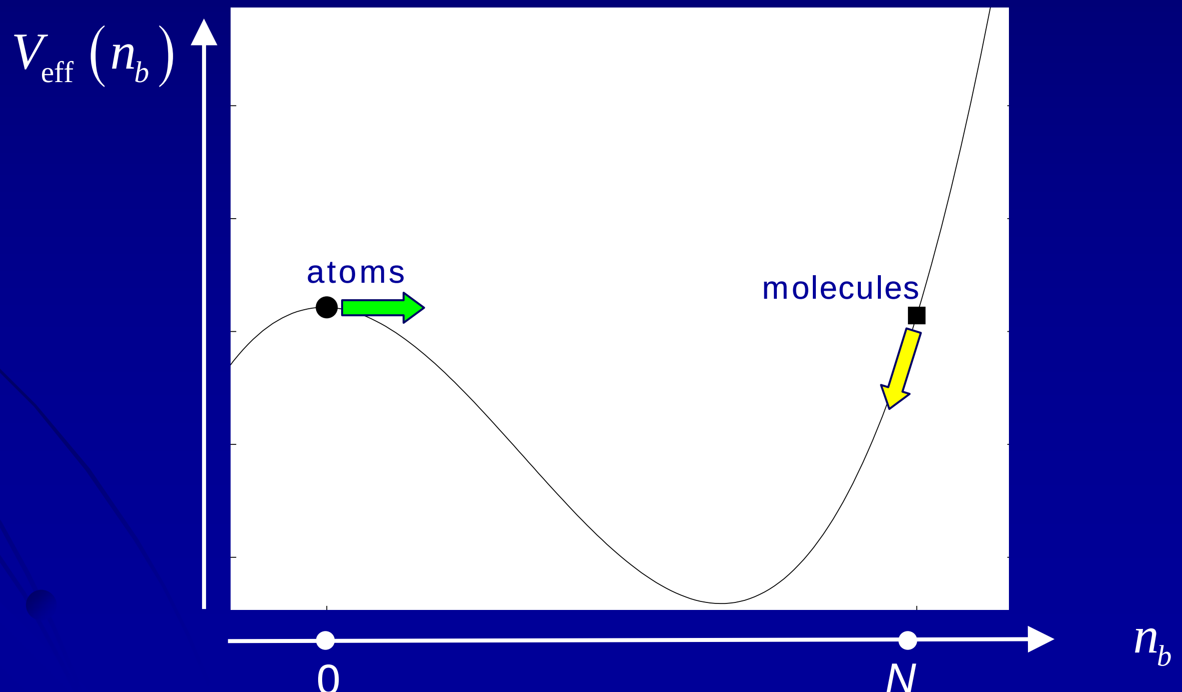
Effective potential

- Heisenberg equations of motion (Exact resonance)

$$\frac{d\hat{n}_b}{dt} = -2\chi \hat{J}_y \qquad \frac{d\hat{J}_y}{dt} = -\chi\left(2\hat{S}_z\hat{J}_y + \hat{S}_+\hat{S}_-\right)$$

$$\hat{J}_y \equiv \left(\hat{b}\hat{S}_+ - \hat{S}_-\hat{b}^\dagger\right)/2i$$

- “Semiclassical” description



Outlook

- Quantum control of molecular field
- Heteronuclear molecules

  Ketterle *et al.*, PRL **93**, 160406 (2004);
  D. Jin *et al.*, PRL **93**, 112002 (2004)
- Detection
- Molecular matter-wave amplifier

  C. P. Search & PM, PRL **93**, 140405 (2004)
- Molecular micromasers

  C. P. Search, W. Zhang, & PM, PRL **91**, 190401 (2003)

For more details:

- D. Meiser & PM, cond-mat/0410349
- H. Uys, T. Miyakawa, D. Meiser, & PM, cond-mat/0412105
- D. Meiser, PM, & C. P. Search, in preparation