Quantum correlations in a quasi 1d BEC

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The catch: NO BEC in 1d!

The bad news: Bogoliubov's inequality:

$$\frac{1}{2}\langle\{\hat{A},\hat{A}^\dagger\}\rangle\,\langle[[\hat{C},\hat{H}],\hat{C}^\dagger]\rangle \geq k_{\mathrm{B}}T\,|\langle[\hat{C},\hat{A}]\rangle|^2$$

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Hohenberg-Mermin-Wagner theorem: $T > 0 \Longrightarrow n_0 = 0$

$$n_{k>0} = \langle\hat{a}_k^{\dagger}\hat{a}_k\rangle > -\frac{1}{2} + \frac{n_0}{n_0 + (n - n_0)}\,\frac{T}{k^2/m}$$

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The good news: no thermodynamic limit in finite systems [1]

[1] D. Petrov *et al.*, PRL, **85**, 3745 (2000); L. Santos *et al.*, PRL, **85**, 1791 (2000)
A. Görlitz *et al.*, PRL., **87**, 130402 (2001); S. Dettmer *et al.*, PRL. **87**, 160406, (2001); F. Schreck *et al.*, PRL, **87**, 80403, (2001)

From 3d to a quasi 1d BEC

with increasing
anisotropy $\beta \gg 1$

$$
\begin{aligned}
V_{\text{ext}} &= \frac{1}{2}x^2 + \frac{\beta}{2}r_\perp^2 \\
\beta &= \frac{\omega_\perp}{\omega_\parallel}
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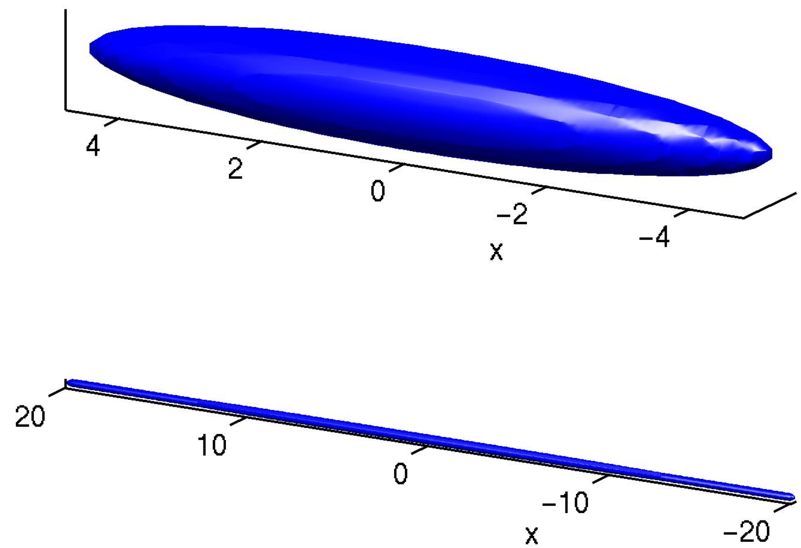

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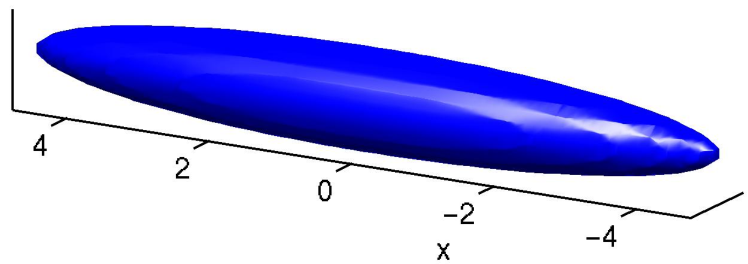

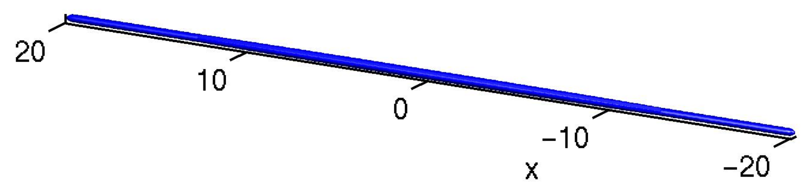

Quasi 1d mean field limit: [2] $\alpha(x,r) \approx \alpha(x)\varphi(r)$

[2] C. Menotti and S. Stringari, PRA, **66**, 43610 (2002)

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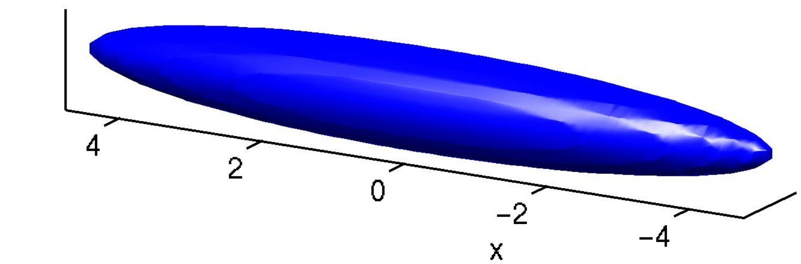

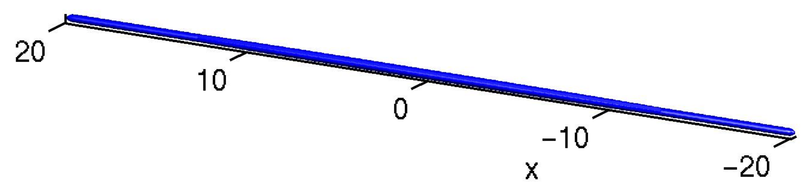

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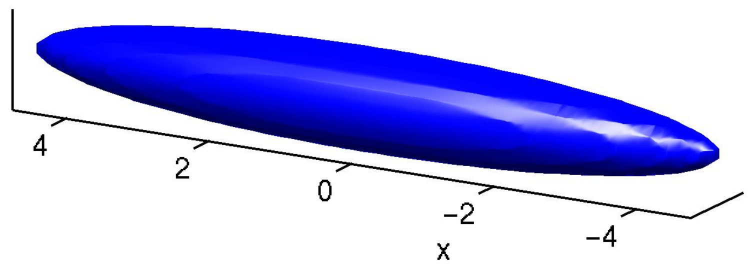

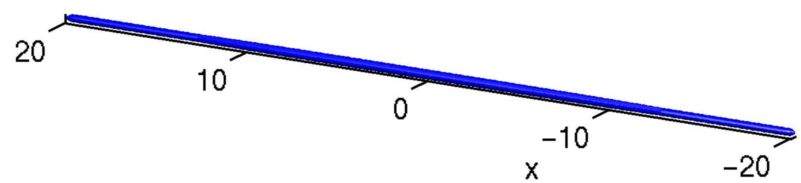

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Beyond mean field theories

BEC phase: $\alpha_{\mathbf{x}} = \langle \hat{a}_{\mathbf{x}} \rangle$
Single particle density: $\tilde{f}(\mathbf{x}, \mathbf{y}) = \langle (\hat{a} - \alpha)^{\dagger}_{\mathbf{y}} (\hat{a} - \alpha)_{\mathbf{x}} \rangle$
Two particle field: $\tilde{m}(\mathbf{x}, \mathbf{y}) = \langle (\hat{a} - \alpha)_{\mathbf{x}} (\hat{a} - \alpha)_{\mathbf{y}} \rangle$

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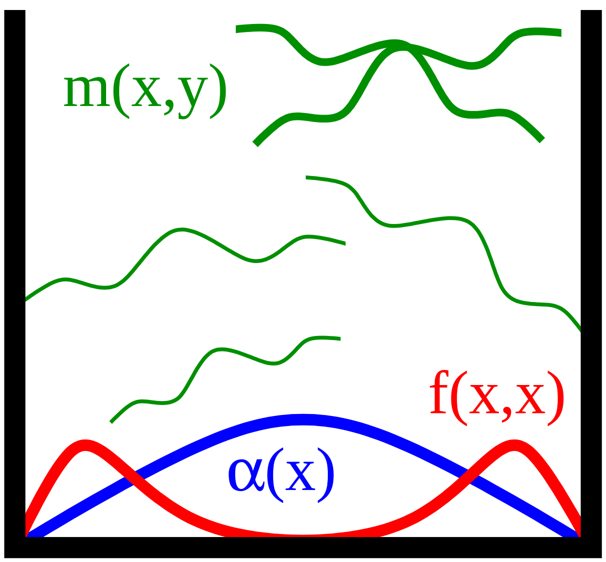

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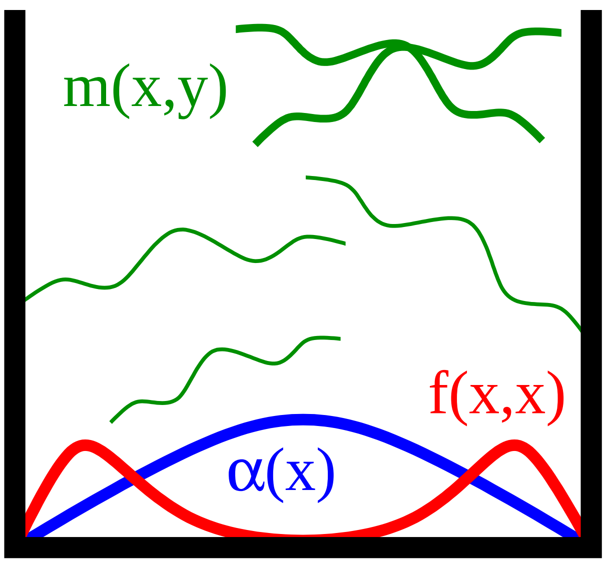

- non-equilibrium kinetics

R. Walser *et al.*, PRA, **59**, 3878 (1999)

- atom-molecule oscillations

M. Holland *et al.*, PRL, **86**, 1915 (2001)

- resonant superfluidity of fermions

S. Kokkelmans *et al.*, PRA, **65**, 53617, (2002)

- ground state correlations in quasi 1d

R. Walser, cond-mat/0411483, Special Issue, Opt. Comm (2004)

Coupled dynamics: $\alpha_{\mathbf{x}} \Leftrightarrow \tilde{\mathbf{f}}_{\mathbf{x},\mathbf{y}} \Leftrightarrow \tilde{\mathbf{m}}_{\mathbf{x},\mathbf{y}}$

Density matrix $G^{>} = \begin{bmatrix} \tilde{f} & \tilde{m} \\ \tilde{m}^{*} & (1+\tilde{f})^{*} \end{bmatrix}$, MF state $\chi = \begin{bmatrix} \alpha \\ \alpha^{*} \end{bmatrix}$

$$\begin{aligned} \mathbf{i}\,\hbar\,\partial_{\mathbf{t}}\chi &= \left(\mathbf{\Pi}_{\mathsf{GP}} + \mathbf{i}(\mathbf{\Upsilon}^{>} - \mathbf{\Upsilon}^{<})\right)\chi \\ \mathbf{i}\,\hbar\,\partial_{\mathbf{t}}\mathbf{G}^{>} &= \left(\mathbf{\Sigma}_{\mathsf{HFB}} + \mathbf{i}\,\mathbf{\Gamma}^{>}\right)\mathbf{G}^{>} - \mathbf{i}\,\mathbf{\Gamma}^{<}\,\mathbf{G}^{<} - \text{H.c.} \end{aligned}$$

Propagators $\mathbf{\Pi}_{\mathsf{GP}}$, $\mathbf{\Sigma}_{\mathsf{HFB}}$, mposCollision operators $\mathbf{\Upsilon}^{\gtrless}$, $\mathbf{\Gamma}^{\gtrless}$

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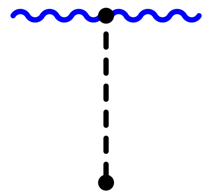
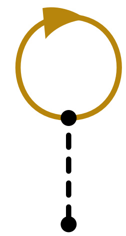
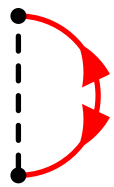
$\Longrightarrow$ Static, Dynamics, Relaxation

Self-energy Π_{GP}, Σ_{HFB} à la Feynman

$\Pi_{\mathsf{GP}11} \quad =$ $+$  $+\,2$  $+ \ldots$

$\Pi_{\mathsf{GP}12} \quad =$  $+ \ldots$

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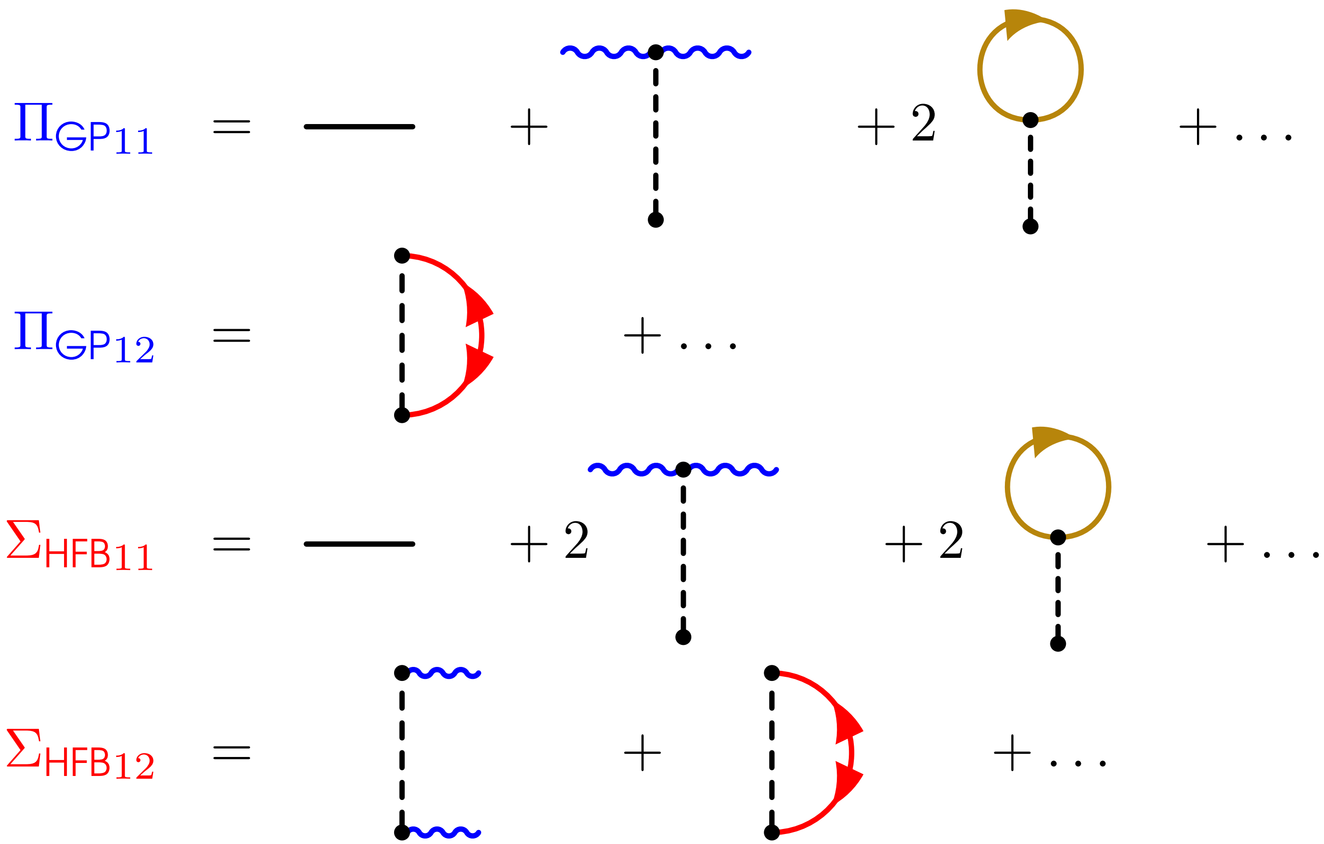

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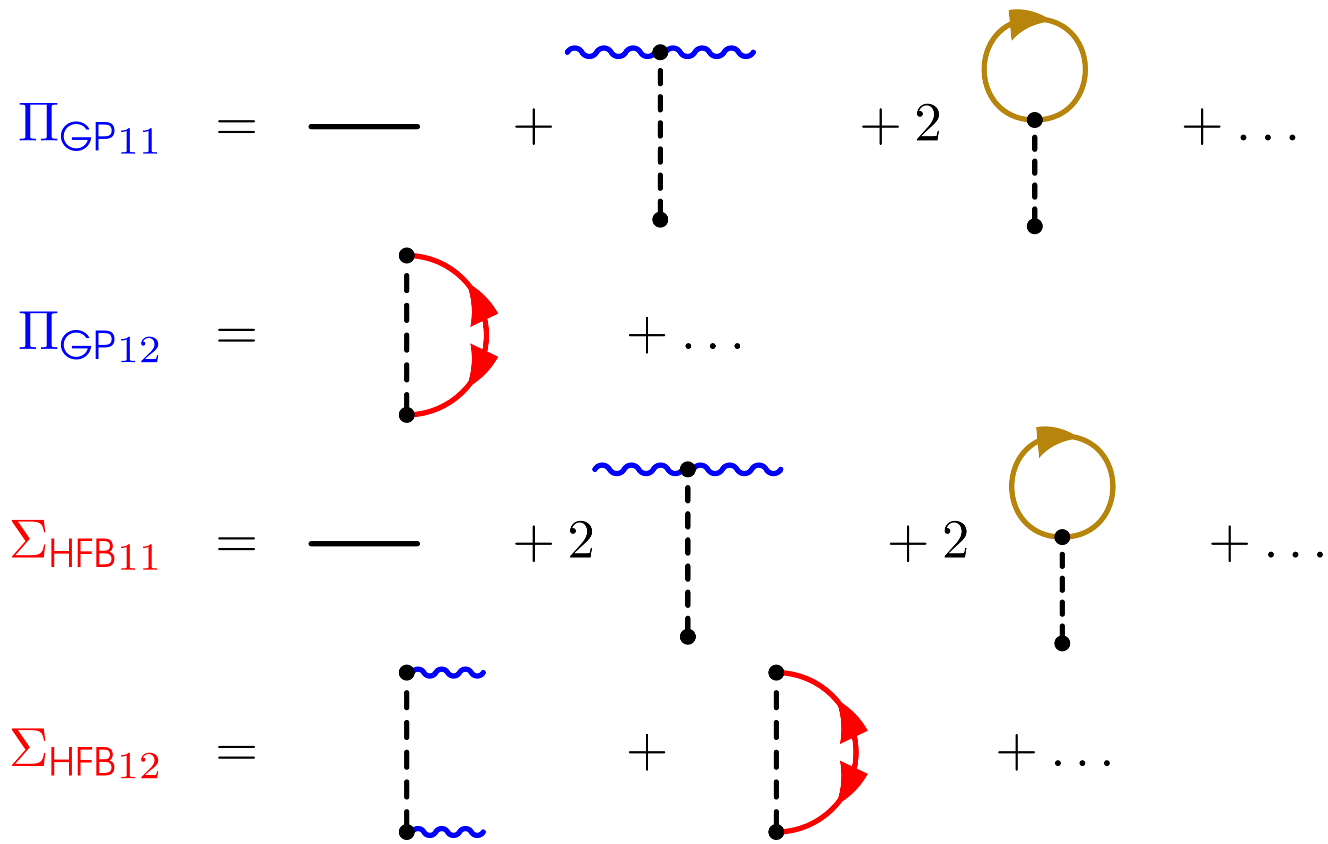

Equilibrium and coherent dynamics

No collisions, but strong self-interaction:

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Parameter for quasi 1d ^{87}Rb:

$\omega_\perp = 2\pi\,800$ Hz, $\omega_{||} = 2\pi\,3$ Hz, $a_{||} = 6.2\mu m$, $N^{(c)} = 10^4$, $T = 0$

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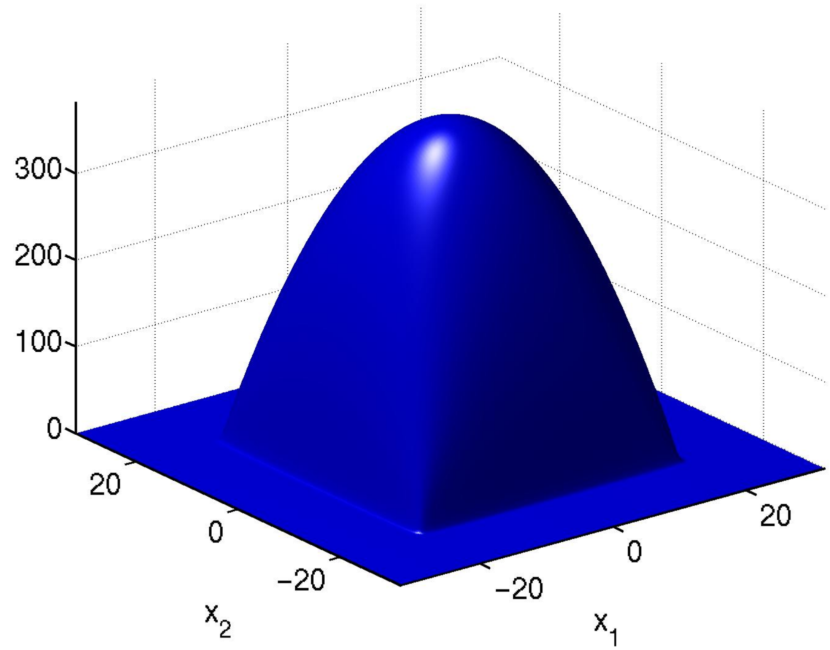
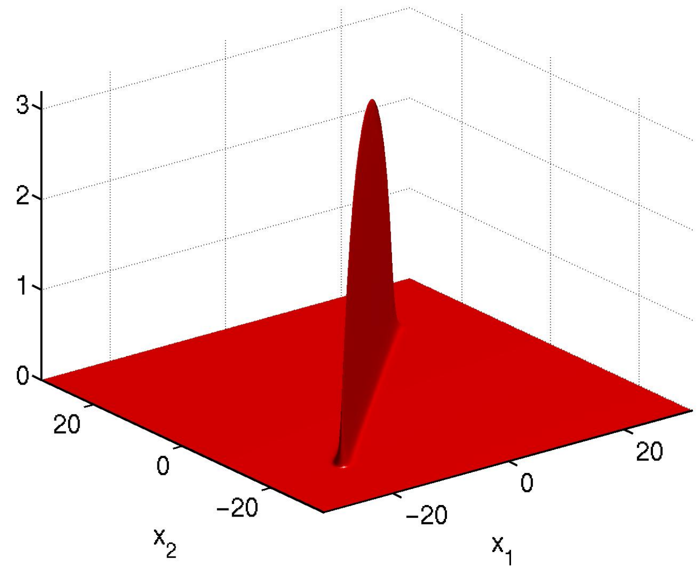
Quasi 1d meanfield limit: $0.5 = a_{\text{S}} N^{(c)}/\sqrt{\beta} \ll 1 \ll n\xi = 20$

Mean field density $f^{(c)}(x_1, x_2)$



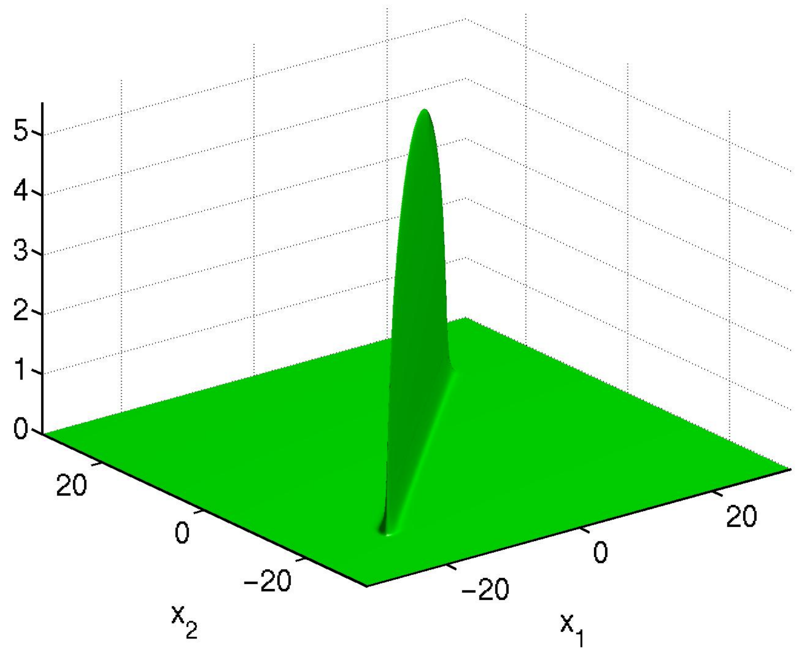
Normal density $\tilde{f}(x_1, x_2)$



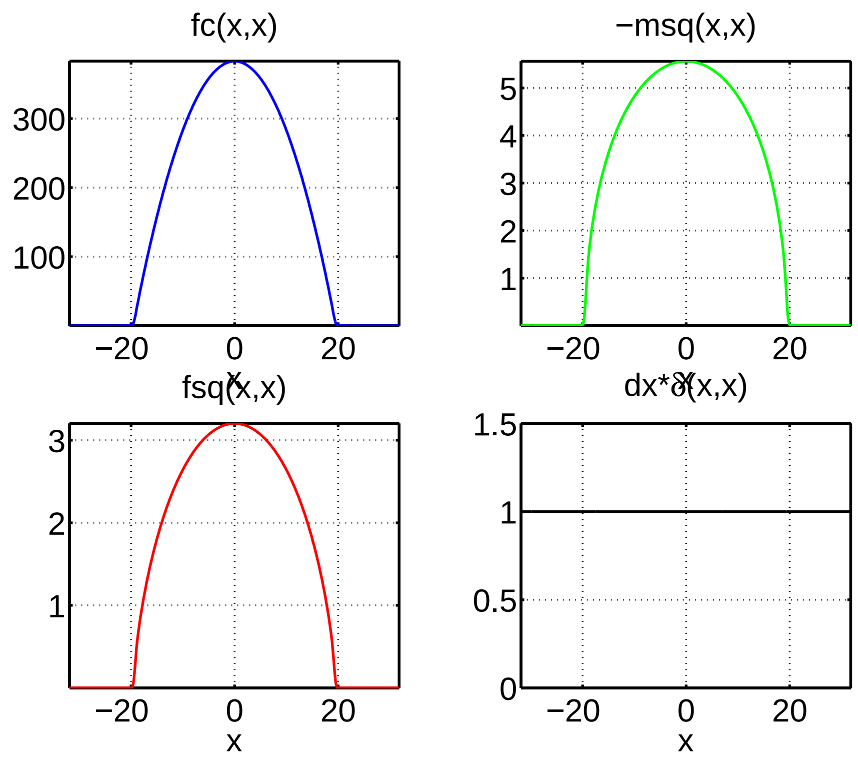
Depletion[3]: $0.8\% = \frac{\tilde{n}(0)}{n^{(c)}(0)+\tilde{n}(0)} \approx \frac{1}{n\xi}(\frac{\sqrt{2}}{4}\pi - 1) = 0.6\%$

[3] U. Al Khawaja *et al.*, PRA, **66**, 13615 (2002)

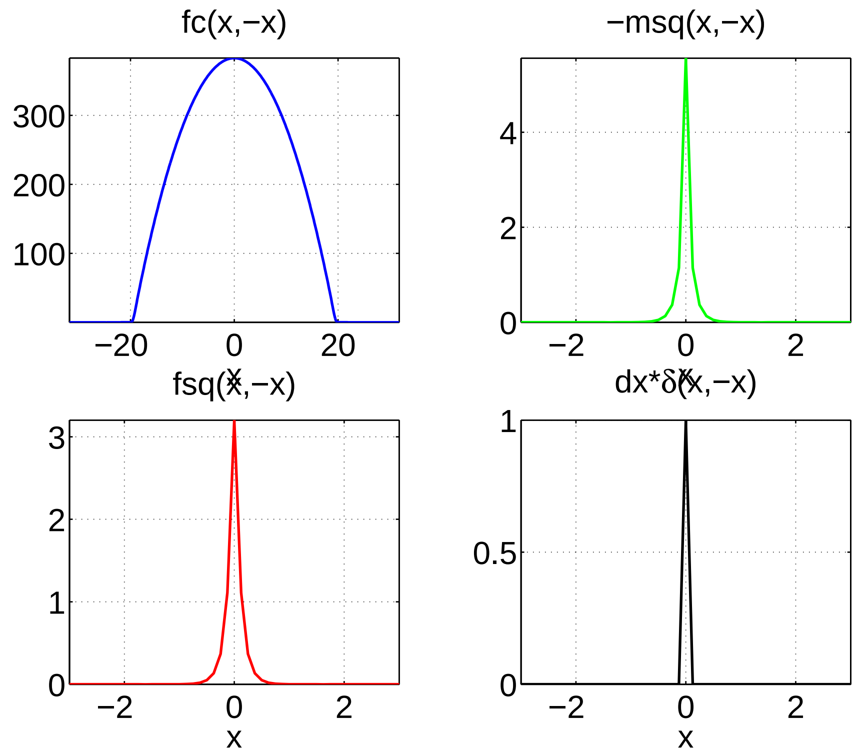
Pair correlations $-\tilde{m}(x_1, x_2)$



Diagonal order



Offdiagonal order



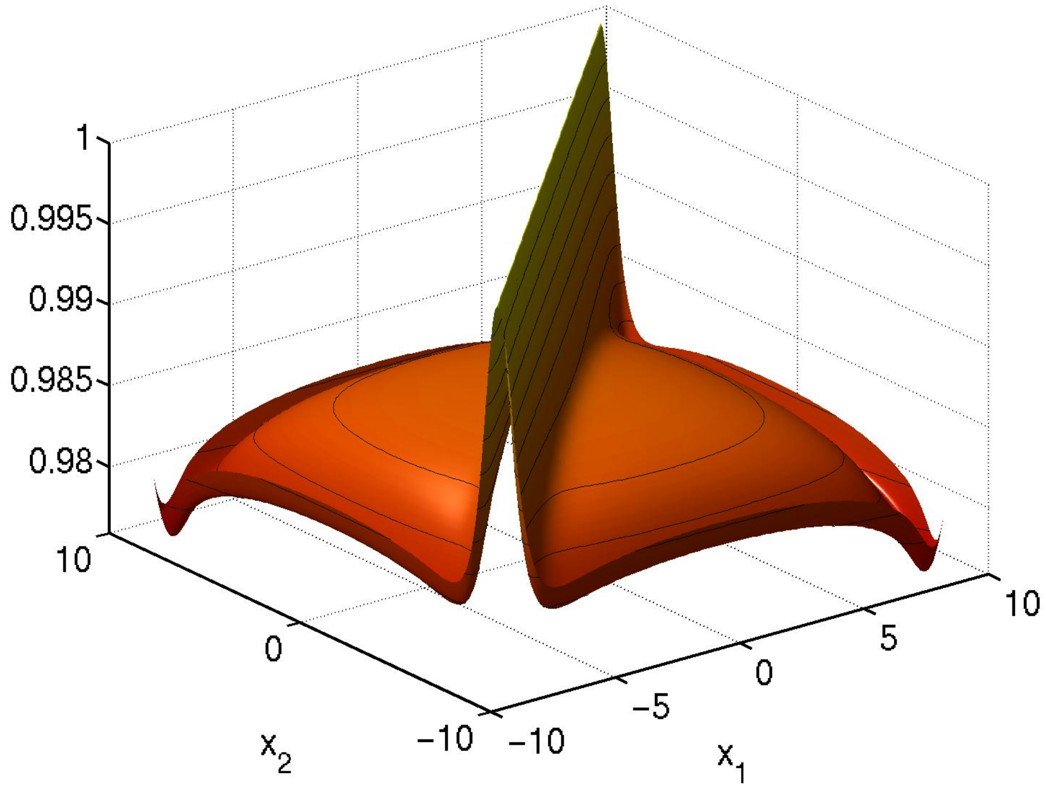
First order correlation function

Measures spatial phase coherence: $\alpha(x) = |\alpha(x)|e^{-i\phi(x)}$

$$g^{(1)}(x_1, x_2) = \frac{\langle \hat{a}^\dagger_{x_2} \hat{a}_{x_1} \rangle}{\sqrt{n(x_1)\, n(x_2)}} = \frac{f^{(c)}(x_1, x_2) + \tilde{f}(x_1, x_2)}{\sqrt{n(x_1)\, n(x_2)}}$$

total density: $n(x) = f^{(c)}(x, x) + \tilde{f}(x, x)$

$$g^{(1)}(x_1, x_2), N = 10^3$$



Second order correlation function

Measures density-density fluctuations:

$$g^{(2)}(x_1, x_2) = \langle \hat{a}^\dagger_{x_1} \hat{a}^\dagger_{x_2} \hat{a}_{x_2} \hat{a}_{x_1} \rangle / n(x_1) n(x_2)$$

$$= 1 + \Big(2\,\mathrm{Re}[f^{(c)}(x_1, x_2)^* \tilde{f}(x_2, x_1) + m^{(c)}(x_1, x_2)^* \tilde{m}(x_2, x_1)] +$$

$$\tilde{f}(x_1, x_2) \tilde{f}(x_2, x_1) + \tilde{m}(x_1, x_2)^* \tilde{m}(x_2, x_1) \Big) / n(x_1) n(x_2)$$

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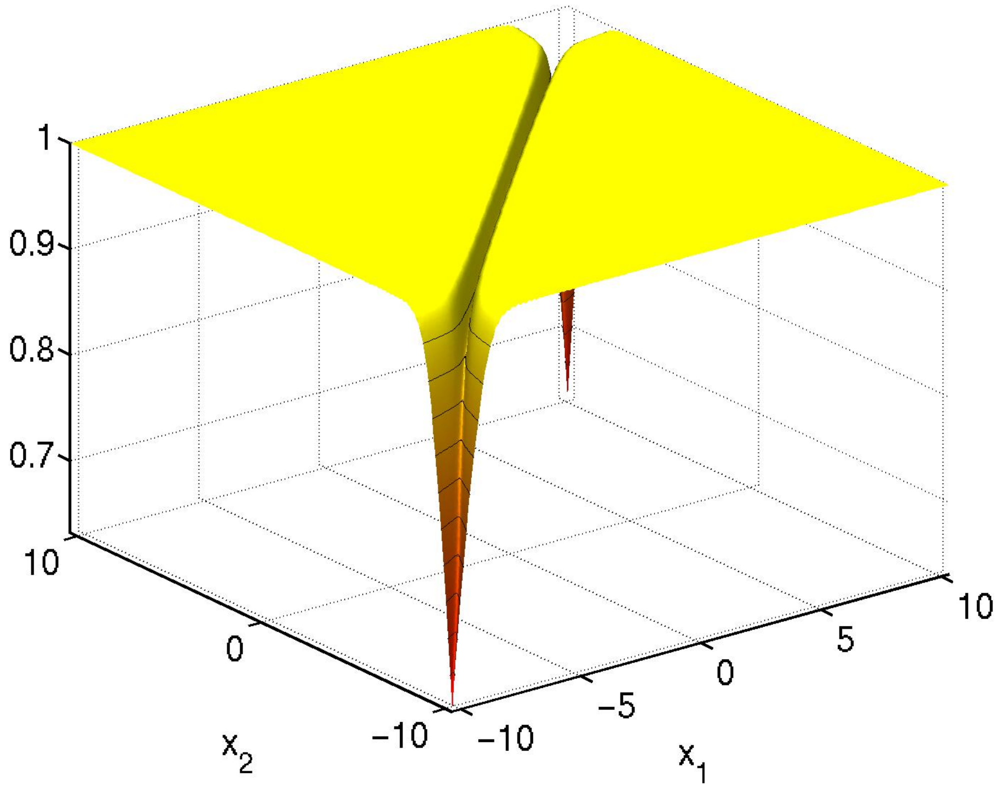
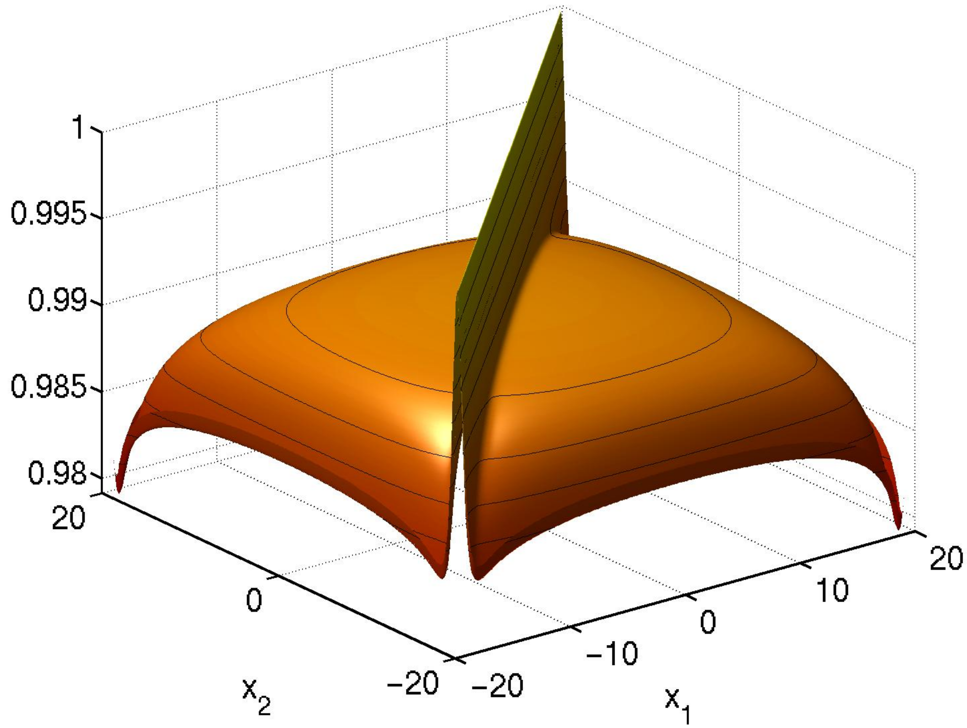


Conclusions and Outlook

- Studied quantum statistcial correlations of a quasi 1d trapped gas
  $\Longrightarrow$ found non-classical suppression of density fluctuations (squeezing)
- extension to finite temperature
- non-equilibrium response

$$g^{(1)}(x_1, x_2), N = 10^4$$



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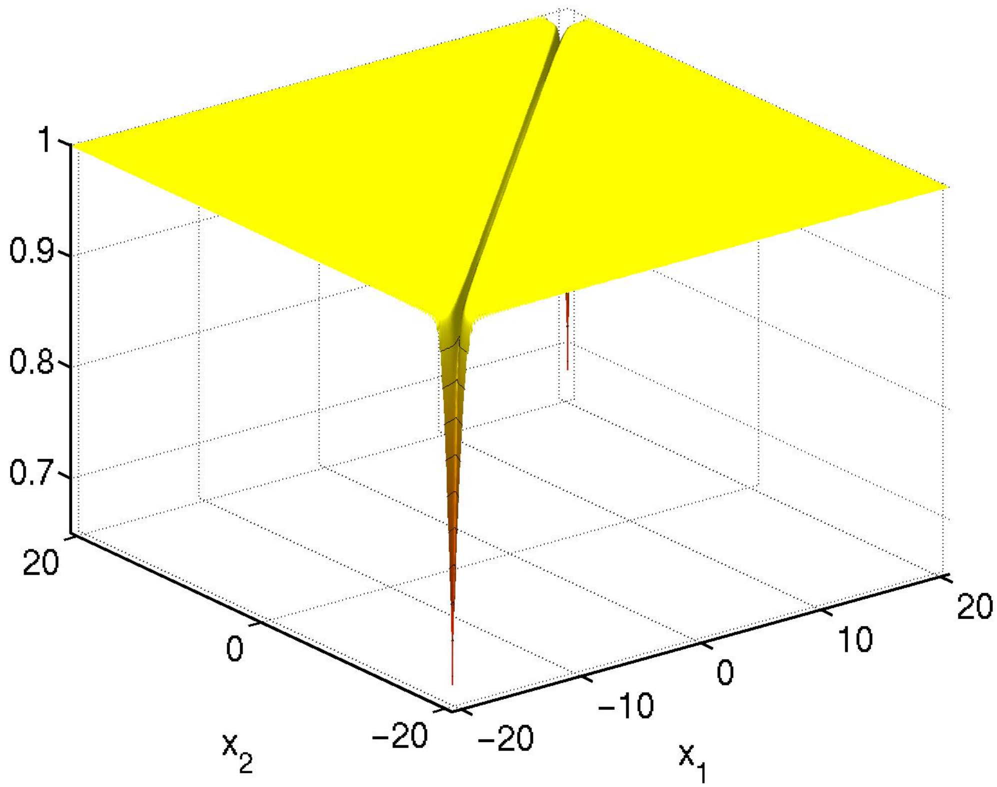
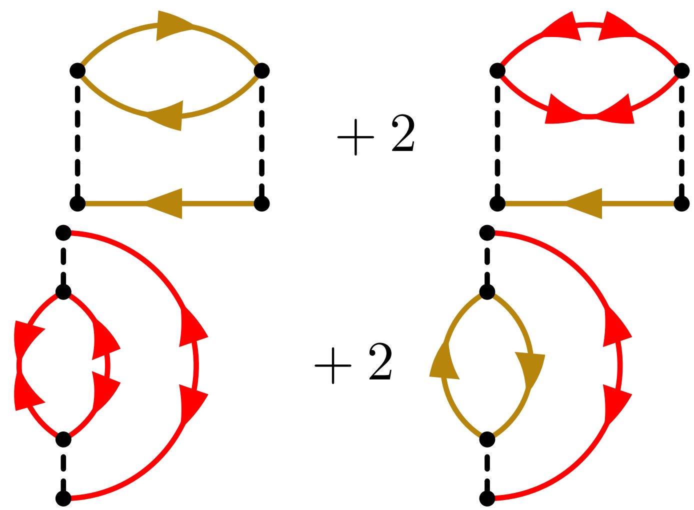


Collisions Υ, Γ à la Feynman



$\Upsilon^{>}_{11} \quad = \quad \ldots + 2 \ldots$

$\Upsilon^{>}_{12} \quad = \quad \ldots + 2 \ldots$

$\Gamma^{>} \quad = \Upsilon^{>}$+ 3 diagrams with MF propagators