

Quantum correlations in a quasi 1d BEC

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The catch: NO BEC in 1d!

The bad news: Bogoliubov's inequality:

$$\frac{1}{2} \langle \{\hat{A}, \hat{A}^\dagger\} \rangle \langle [[\hat{C}, \hat{H}], \hat{C}^\dagger] \rangle \geq k_B T |\langle [\hat{C}, \hat{A}] \rangle|^2$$

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Hohenberg-Mermin-Wagner theorem: $T > 0 \implies n_0 = 0$

$$n_{k>0} = \langle \hat{a}_k^\dagger \hat{a}_k \rangle > -\frac{1}{2} + \frac{n_0}{n_0 + (n - n_0)} \frac{T}{k^2/m}$$

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The good news: no thermodynamic limit in finite systems ¹

¹ D. Petrov *et al.*, PRL, **85**, 3745 (2000); L. Santos *et al.*, PRL, **85**, 1791 (2000)
A. Görlitz *et al.*, PRL, **87**, 130402 (2001); S. Dettmer *et al.*, PRL, **87**, 160406, (2001); F. Schreck *et al.*, PRL, **87**, 80403, (2001)

From 3d to a quasi 1d BEC

with increasing
anisotropy $\beta \gg 1$

$$V_{\text{ext}} = \frac{1}{2}x^2 + \frac{\beta}{2}r_{\perp}^2$$

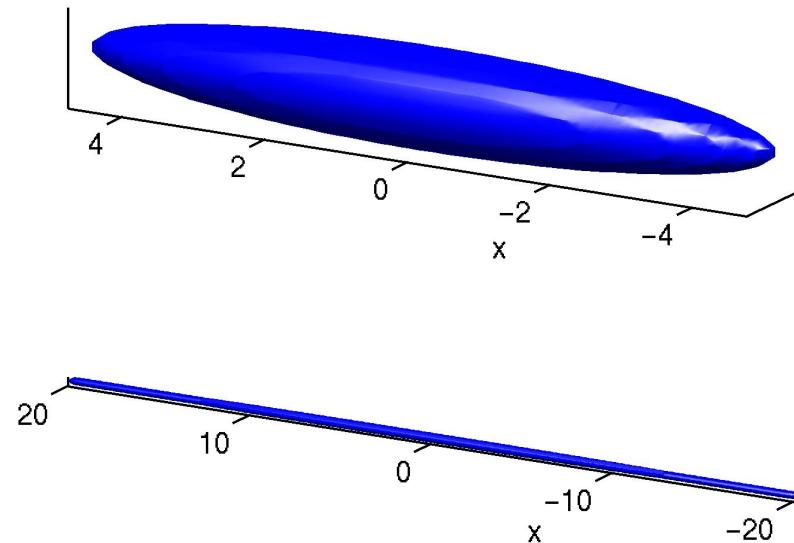
$$\beta = \frac{\omega_{\perp}}{\omega_{\parallel}}$$

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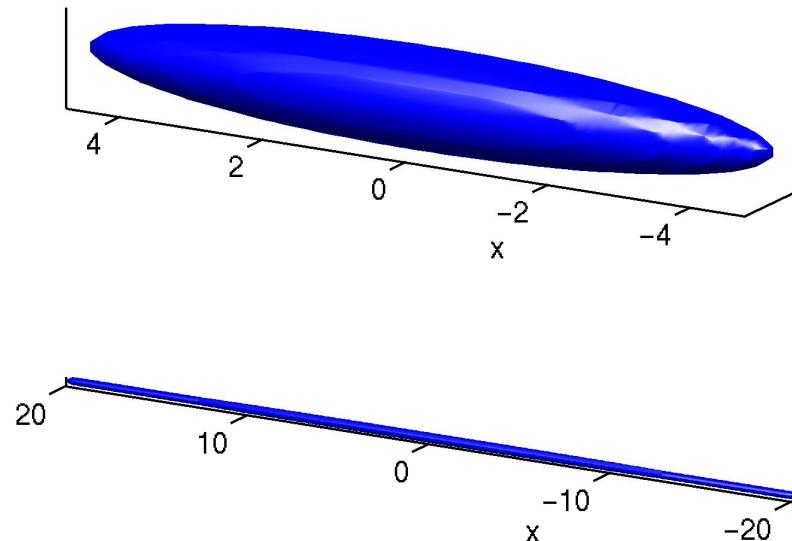


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Quasi 1d mean field limit: ² $\alpha(x, r) \approx \alpha(x)\varphi(r)$

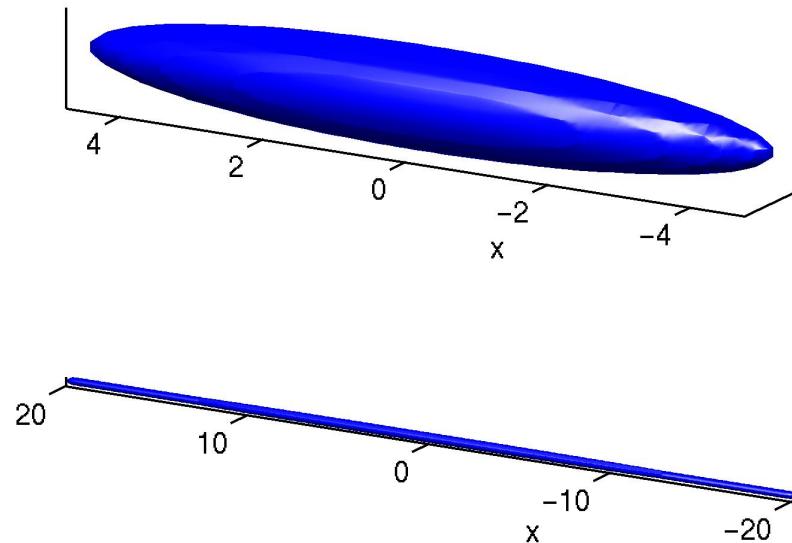
² C. Menotti and S. Stringari, PRA, **66**, 43610 (2002)

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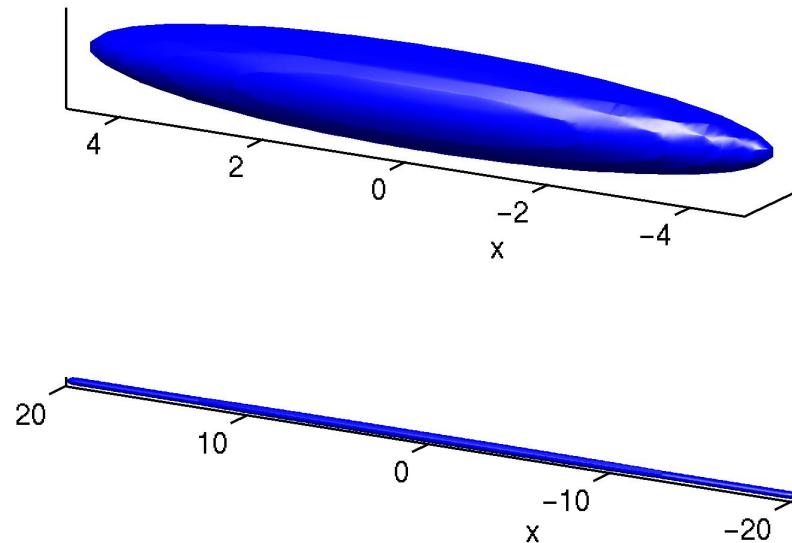
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$$a_s N / \sqrt{\beta} \ll 1 \ll n \xi$$

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Beyond mean field theories

BEC phase:

$$\alpha_x = \langle \hat{a}_x \rangle$$

Single particle density:

$$\tilde{f}(x, y) = \langle (\hat{a} - \alpha)_y^\dagger (\hat{a} - \alpha)_x \rangle$$

Two particle field:

$$\tilde{m}(x, y) = \langle (\hat{a} - \alpha)_x (\hat{a} - \alpha)_y \rangle$$

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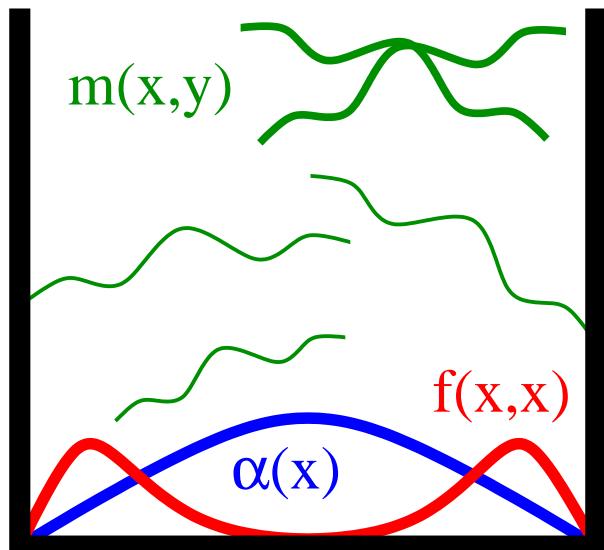
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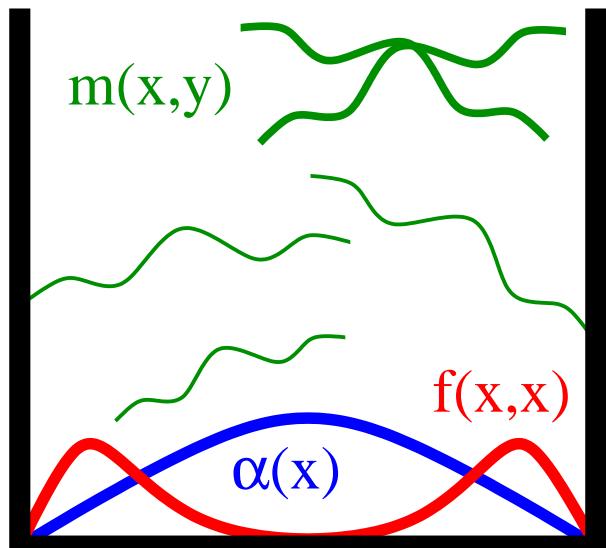
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► non-equilibrium kinetics

R. Walser *et al.*, PRA, **59**, 3878 (1999)

► atom-molecule oscillations

M. Holland *et al.*, PRL, **86**, 1915 (2001)

► resonant superfluidity of fermions

S. Kokkelmans *et al.*, PRA, **65**, 53617, (2002)

► ground state correlations in quasi 1d

R. Walser, cond-mat/0411483, Special Issue, Opt. Comm
(2004)

Coupled dynamics: $\alpha_x \Leftrightarrow \tilde{f}_{x,y} \Leftrightarrow \tilde{m}_{x,y}$

Density matrix $G^> = \begin{bmatrix} \tilde{f} & \tilde{m} \\ \tilde{m}^* & (1 + \tilde{f})^* \end{bmatrix}$, MF state $\chi = \begin{bmatrix} \alpha \\ \alpha^* \end{bmatrix}$

$$\begin{aligned} i\hbar\partial_t\chi &= (\Pi_{GP} + i(\Upsilon^> - \Upsilon^<))\chi \\ i\hbar\partial_t G^> &= (\Sigma_{HFB} + i\Gamma^>)G^> - i\Gamma^<G^< - H.c. \end{aligned}$$

Propagators Π_{GP} , Σ_{HFB} , mposCollision operators $\Upsilon^>$, $\Gamma^>$

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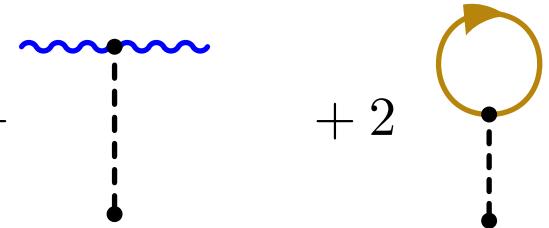
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⇒ Static, Dynamics, Relaxation

Self-energy Π_{GP} , Σ_{HFB} à la Feynman

$$\Pi_{\text{GP}11} = \text{---} + \text{---} + 2 \text{---} + \dots$$


The diagram shows a horizontal line representing a particle exchange. Above it is a wavy line representing a virtual particle. A vertical dashed line connects the two. To the right of the dashed line is a plus sign, followed by a term consisting of a yellow circle with an arrow and a vertical dashed line connecting to it. Another plus sign follows, and then ellipses.

$$\Pi_{\text{GP}12} = \text{---} + \dots$$


The diagram shows a vertical dashed line with a red circle containing an arrow, indicating a loop. A plus sign follows, and then ellipses.

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The diagrams consist of a horizontal dashed line with a vertical wavy line attached to its right end. The first row shows the self-energy $\Pi_{\text{GP}11}$ with a black horizontal line, a blue wavy line, and a yellow circle. The second row shows $\Pi_{\text{GP}12}$ with a red circle. The third row shows $\Sigma_{\text{HFB}11}$ with a black horizontal line, a blue wavy line, and a yellow circle. The fourth row shows $\Sigma_{\text{HFB}12}$ with a blue wavy line and a red circle.

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Equilibrium and coherent dynamics

No collisions, but strong self-interaction:

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Parameter for quasi 1d ^{87}Rb :

$$\omega_{\perp} = 2\pi 800 \text{ Hz}, \omega_{\parallel} = 2\pi 3 \text{ Hz}, a_{\parallel} = 6.2 \mu\text{m}, N^{(c)} = 10^4, T = 0$$

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Characteristic energy [$\hbar\omega_{\parallel}$]: $\mu = (a_S\beta N^{(c)})^{\frac{2}{3}} = 191$

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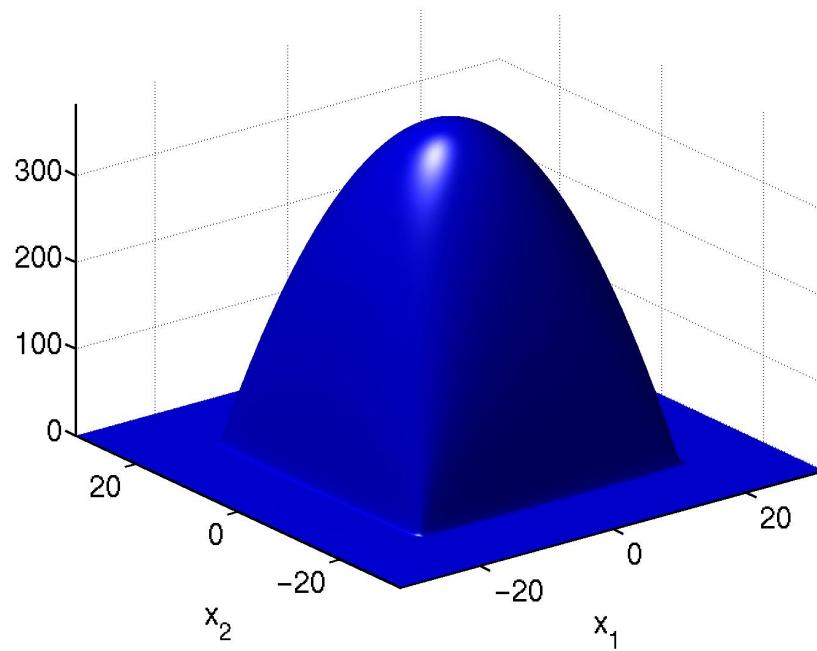
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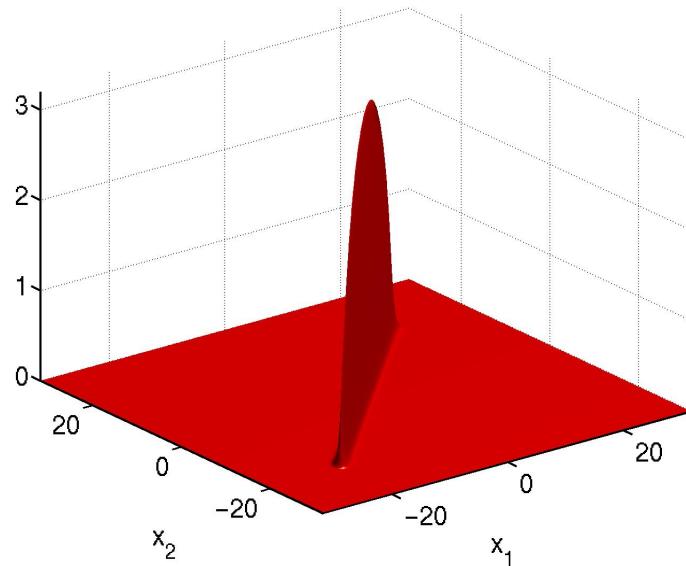
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Quasi 1d meanfield limit: $0.5 = a_S N^{(c)} / \sqrt{\beta} \ll 1 \ll n\xi = 20$

Mean field density $f^{(c)}(x_1, x_2)$



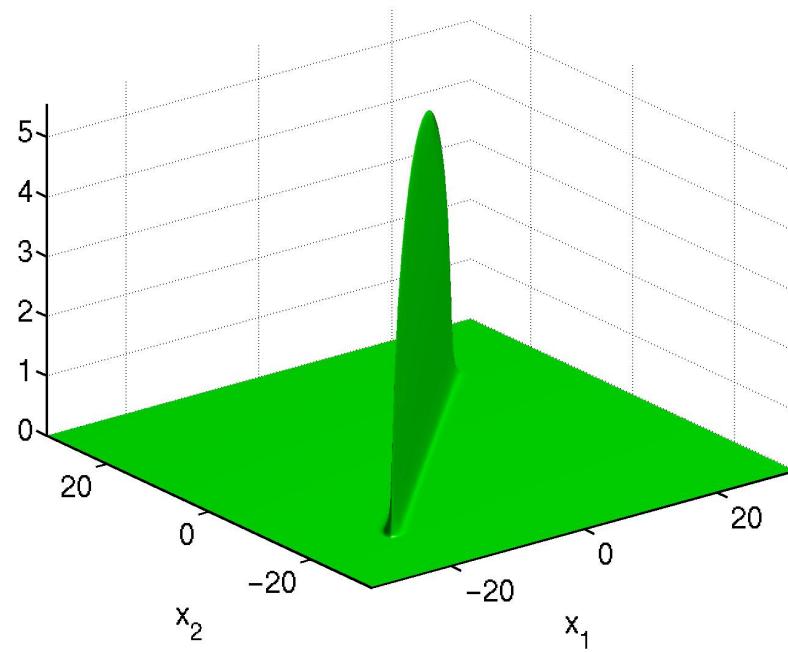
Normal density $\tilde{f}(x_1, x_2)$



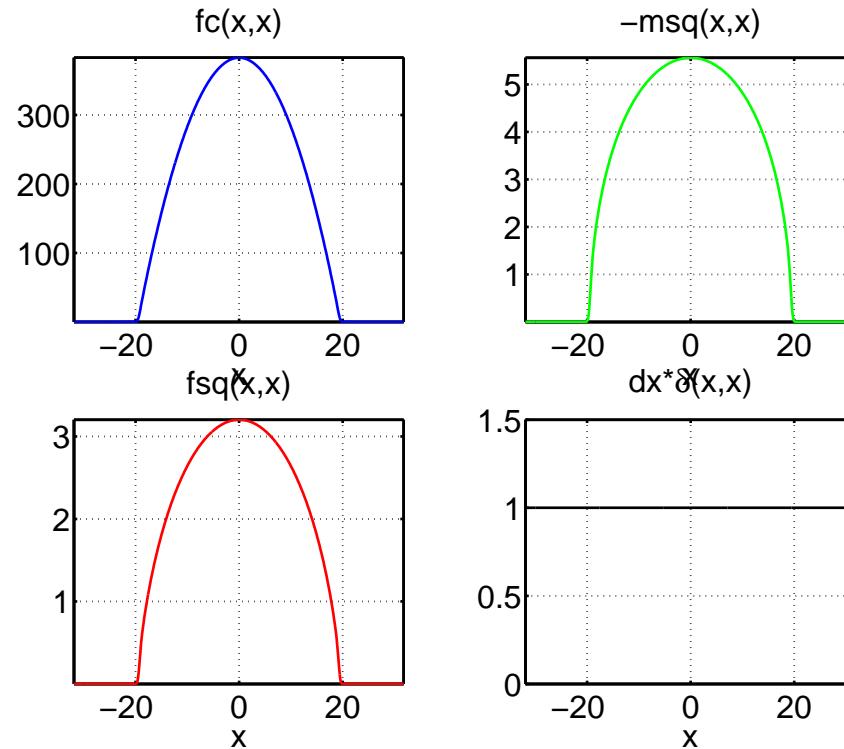
Depletion³: $0.8\% = \frac{\tilde{n}(0)}{n^{(c)}(0)+\tilde{n}(0)} \approx \frac{1}{n\xi} \left(\frac{\sqrt{2}}{4}\pi - 1 \right) = 0.6\%$

³U. Al Khawaja *et al.*, PRA, **66**, 13615 (2002)

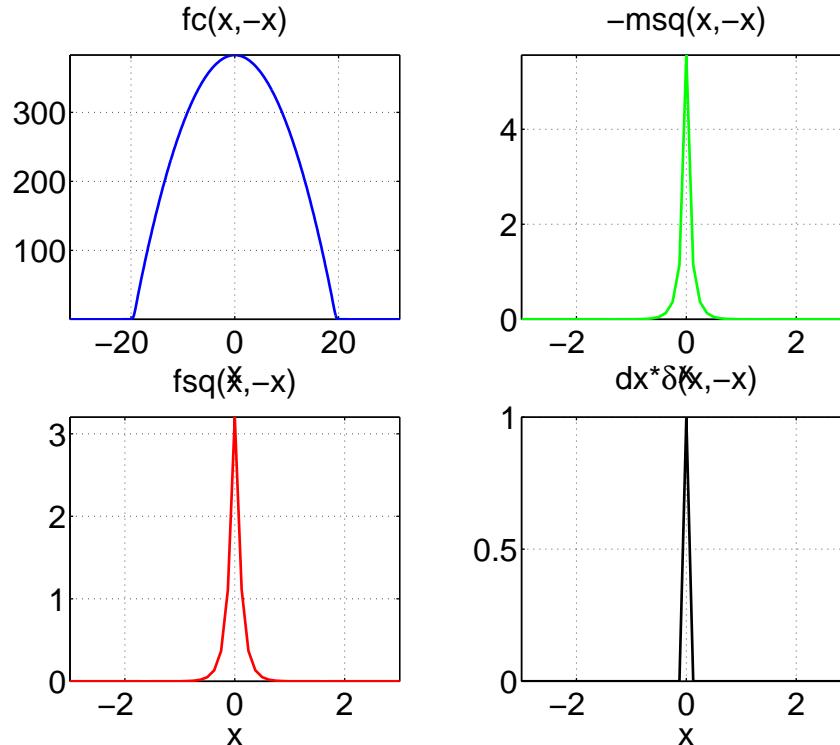
Pair correlations $-\tilde{m}(x_1, x_2)$



Diagonal order



Offdiagonal order



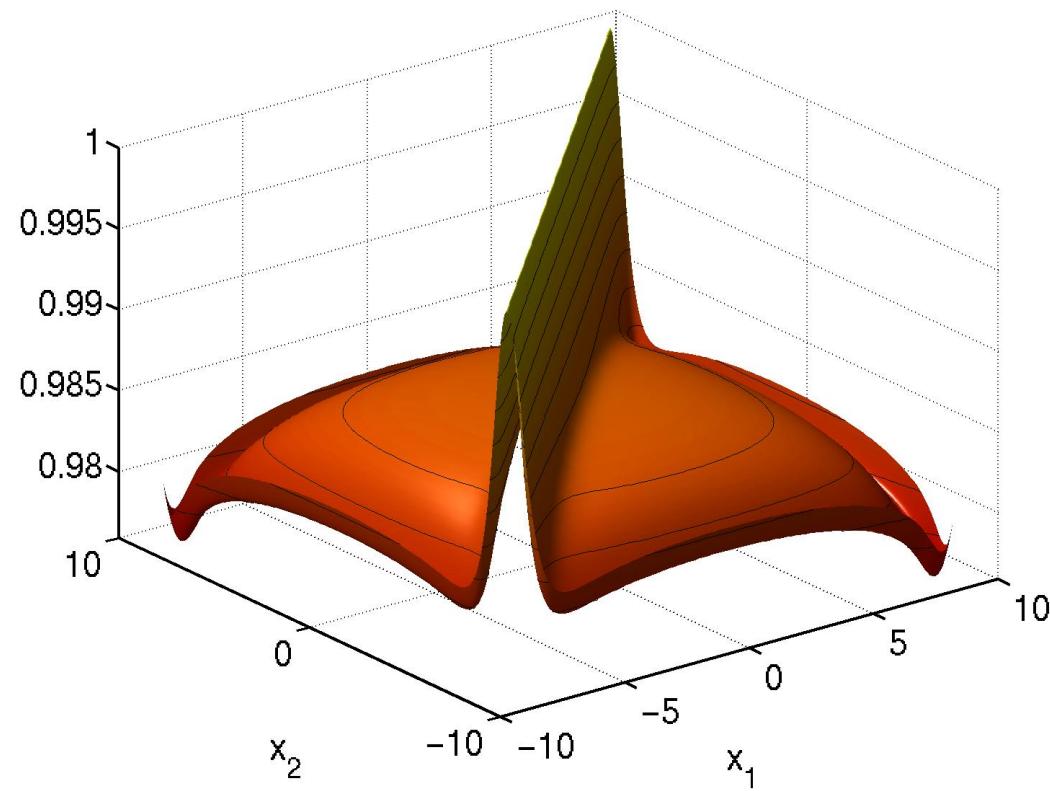
First order correlation function

Measures spatial phase coherence: $\alpha(x) = |\alpha(x)|e^{-i\phi(x)}$

$$g^{(1)}(x_1, x_2) = \frac{\langle \hat{a}_{x_2}^\dagger \hat{a}_{x_1} \rangle}{\sqrt{n(x_1) n(x_2)}} = \frac{f^{(c)}(x_1, x_2) + \tilde{f}(x_1, x_2)}{\sqrt{n(x_1) n(x_2)}}$$

total density: $n(x) = f^{(c)}(x, x) + \tilde{f}(x, x)$

$$g^{(1)}(x_1, x_2), N = 10^3$$

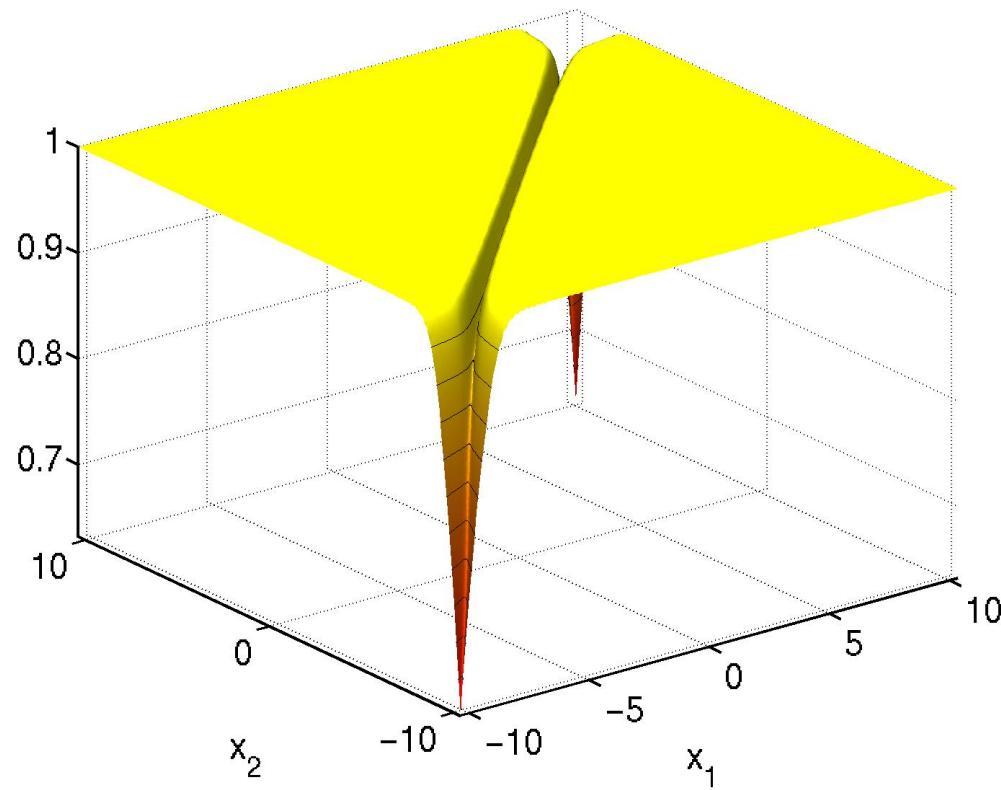


Second order correlation function

Measures density-density fluctuations:

$$\begin{aligned} g^{(2)}(x_1, x_2) &= \langle \hat{a}_{x_1}^\dagger \hat{a}_{x_2}^\dagger \hat{a}_{x_2} \hat{a}_{x_1} \rangle / n(x_1)n(x_2) \\ &= 1 + \left(2 \operatorname{Re}[\textcolor{blue}{f}^{(c)}(x_1, x_2)^* \tilde{f}(x_2, x_1) + \textcolor{blue}{m}^{(c)}(x_1, x_2)^* \tilde{m}(x_2, x_1)] + \right. \\ &\quad \left. \tilde{f}(x_1, x_2) \tilde{f}(x_2, x_1) + \tilde{m}(x_1, x_2)^* \tilde{m}(x_2, x_1) \right) / n(x_1)n(x_2) \end{aligned}$$

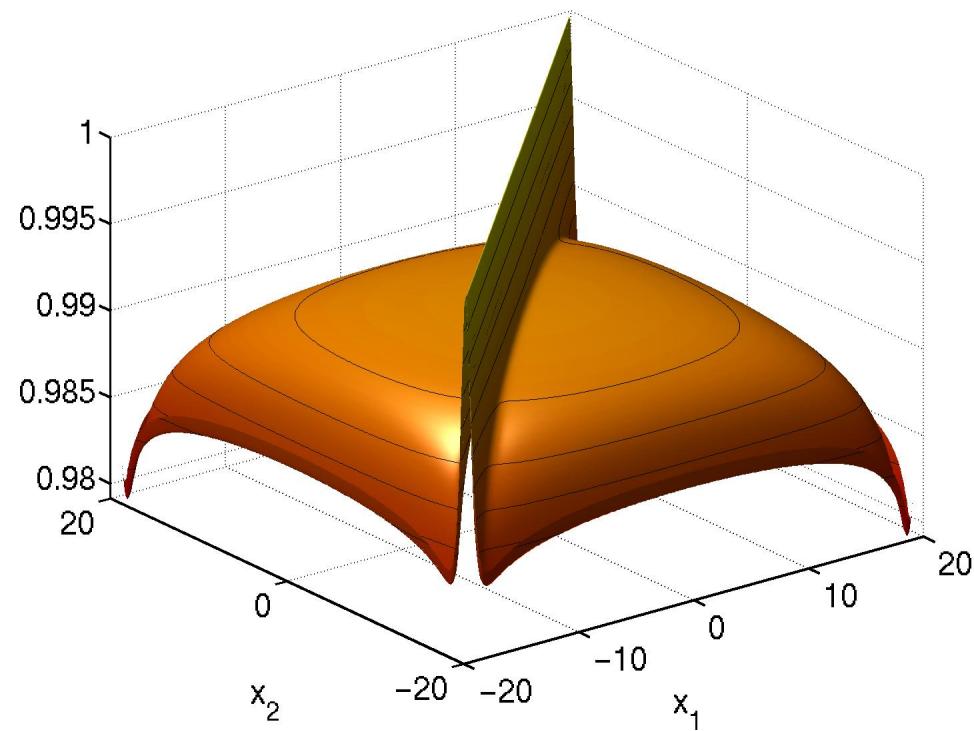
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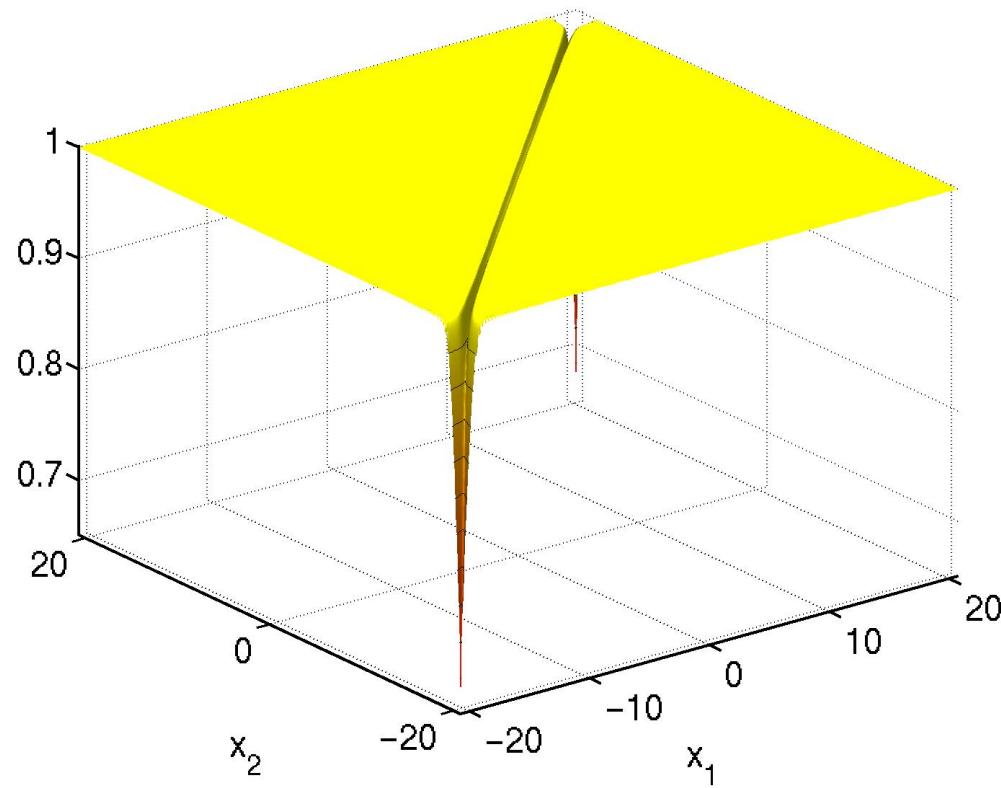
Conclusions and Outlook

- Studied quantum statistical correlations of a quasi 1d trapped gas
 \Rightarrow found non-classical suppression of density fluctuations
 (squeezing)
- extension to finite temperature
- non-equilibrium response

$$g^{(1)}(x_1, x_2), N = 10^4$$



$$g^{(2)}(x_1, x_2), N = 10^4$$



Collisions Υ , Γ à la Feynman

$$\Upsilon_{11}^> = \begin{array}{c} \text{Diagram 1: Two vertical dashed lines with yellow loops at top and bottom. Horizontal lines connect them.}\\ \text{Diagram 2: Similar to Diagram 1, but with red loops instead of yellow.} \end{array} + 2$$
$$\Upsilon_{12}^> = \begin{array}{c} \text{Diagram 3: One vertical dashed line with red loops at top and bottom.}\\ \text{Diagram 4: One vertical dashed line with yellow loops at top and bottom.} \end{array} + 2$$

$\Gamma^>$ = $\Upsilon^>$ + 3 diagrams with MF propagators