

# Atom Optics and Ultra-Cold Atoms

Dr J Rogel-Salazar

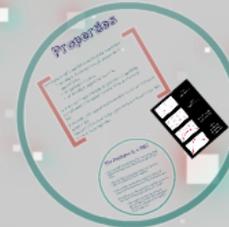
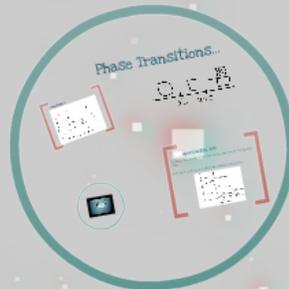
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London



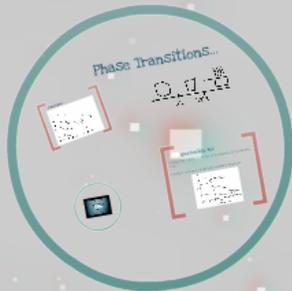
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# Light waves

Can the same happen with matter??

Because light behaves as a wave it can be reflected, refracted, diffracted, or interfere with each other.

## Reflection

Light waves are waves, so when they hit a surface they can bounce off it.



## Refraction

When light waves pass from one medium to another, they change direction.



## Diffraction

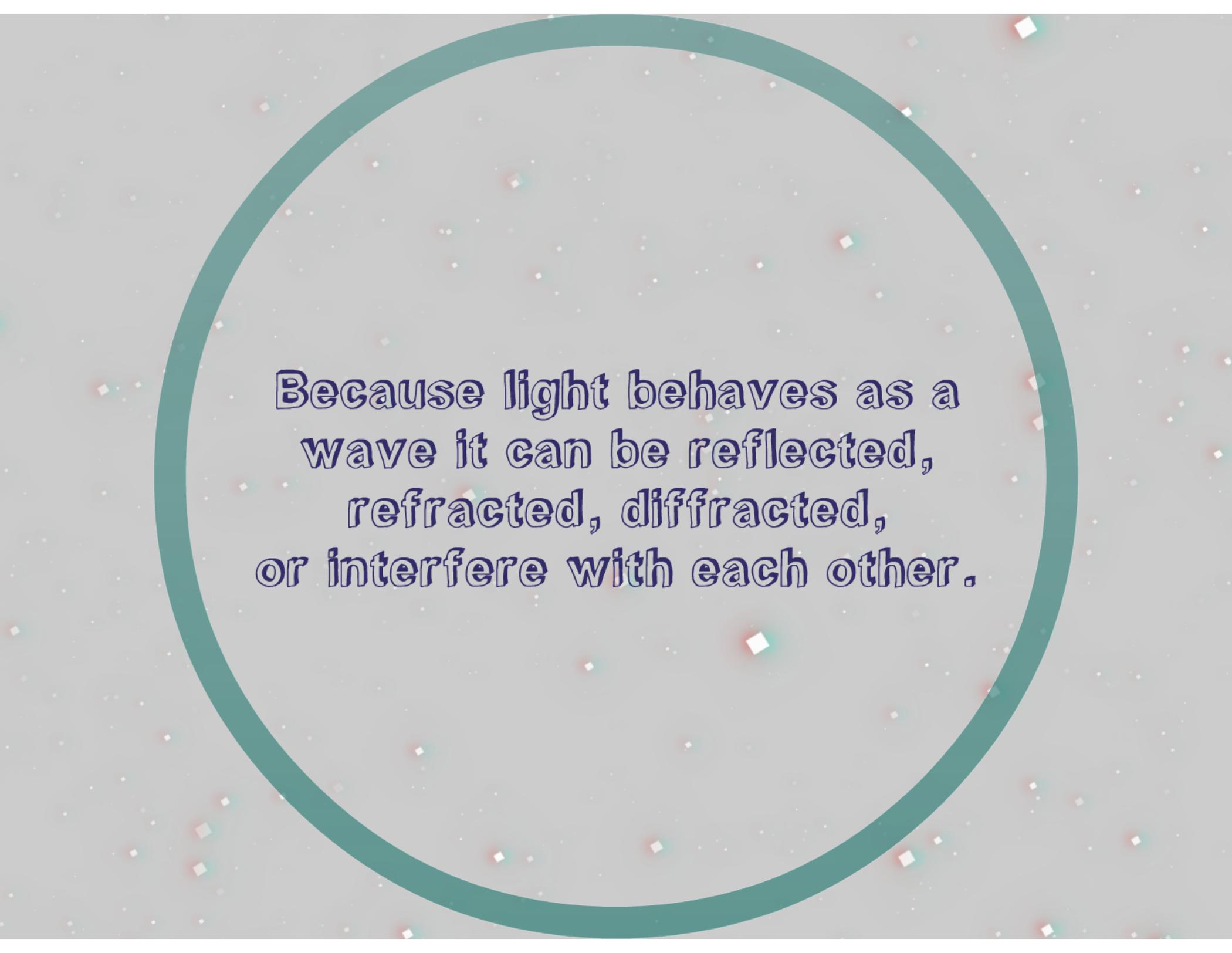
Light waves can pass through small openings and bend around corners.



## Interference

When two light waves meet, they can combine to form a larger wave or cancel each other out.



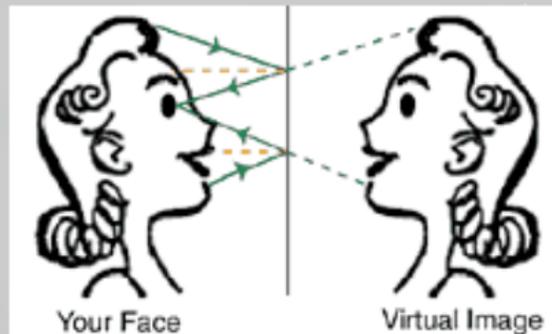


**Because light behaves as a  
wave it can be reflected,  
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or interfere with each other.**

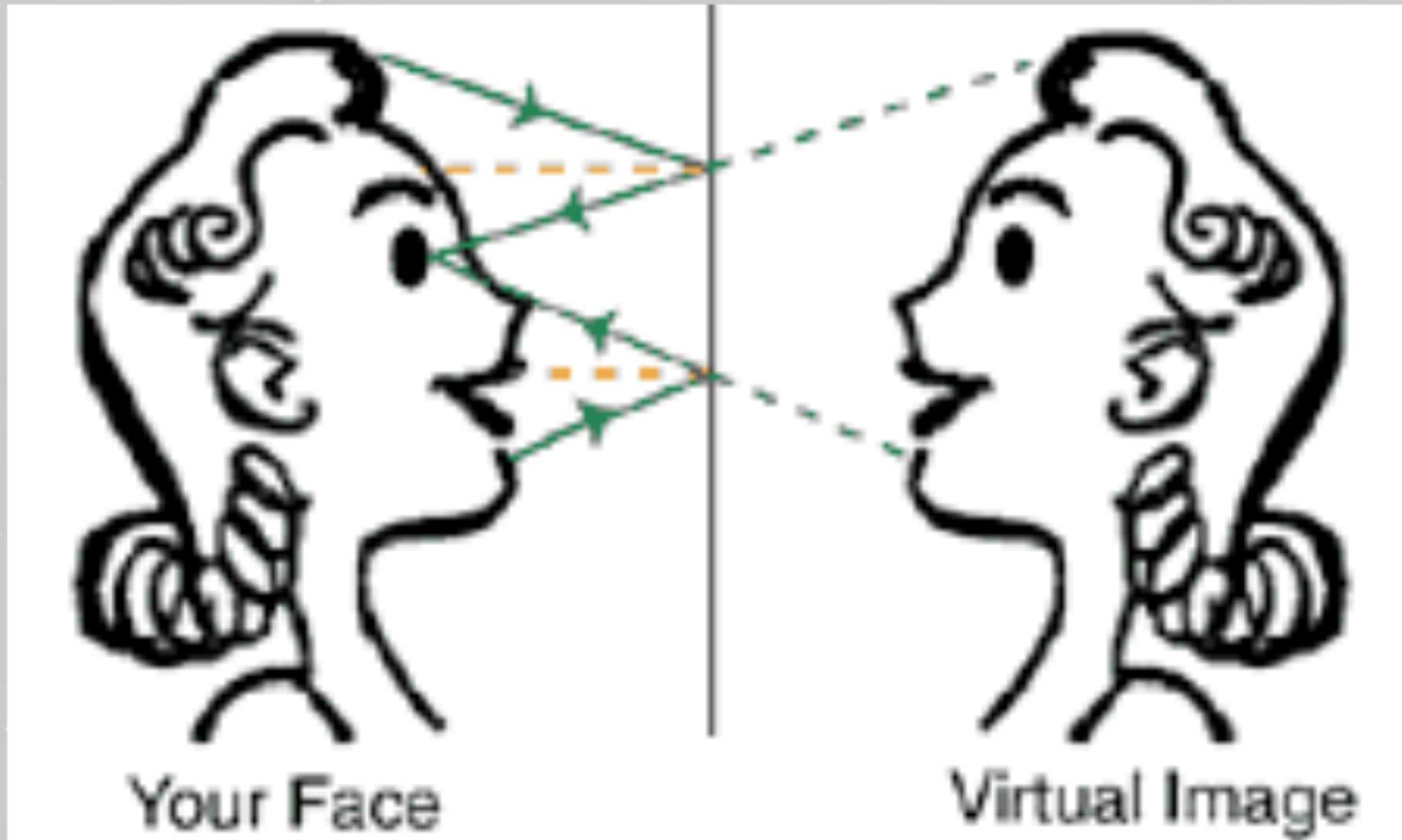
# Reflection

Reflection occurs when incoming light strikes an object and then bounces off.

Light waves obey the Law of Reflection when reflected off of any surface.



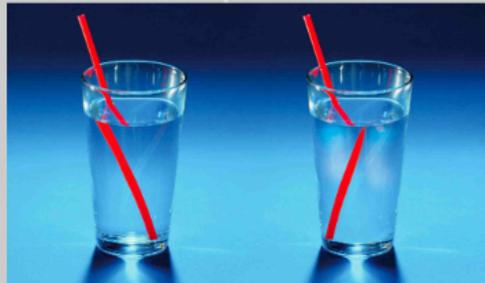
Light rays being reflected off the mirror's surface and into your eyes.



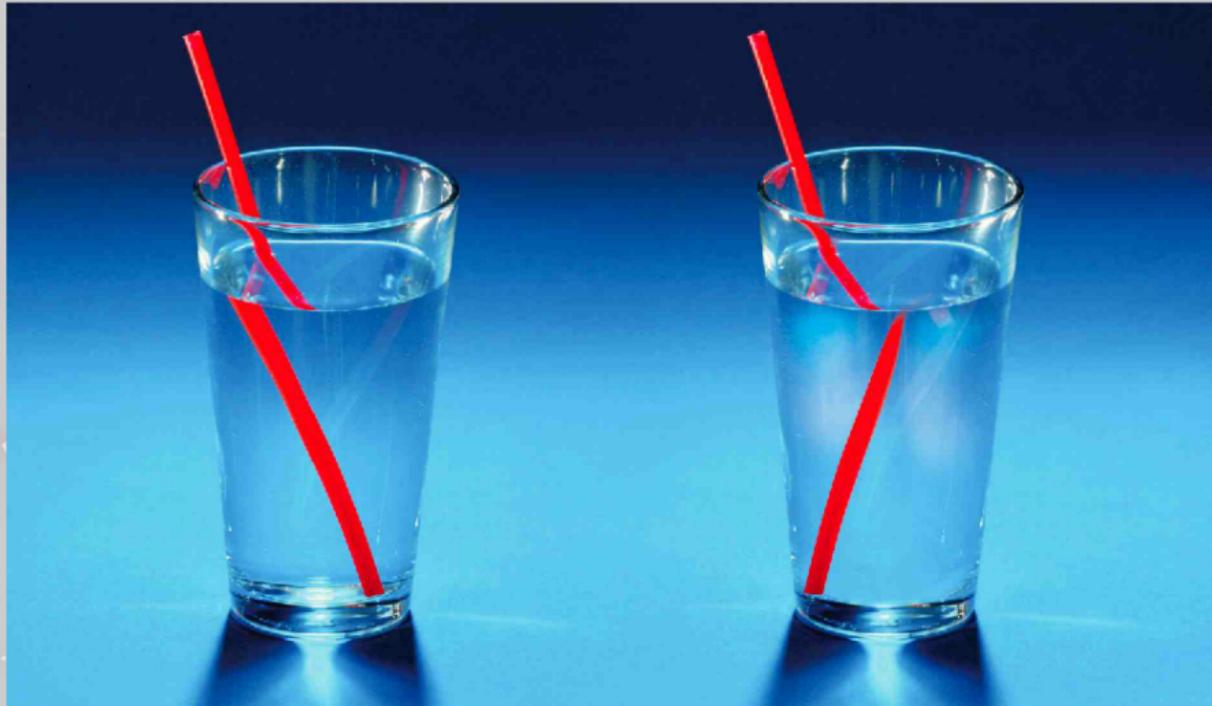
Light rays being reflected off the mirror's surface and into your eyes.

# Refraction

Refraction occurs when a light wave changes its speed while it is passes from one material to another.



The illusion of a straw in a glass of water created by the refraction of light in water.



The illusion of a straw in a glass of water created by the refraction of light in water.

# Diffraction

Diffraction occurs when light waves bend around an object and change direction.

Diffraction can also occur when waves pass through narrow openings.



Light diffraction through clouds.

Diffraction can also occur when waves pass through narrow openings.

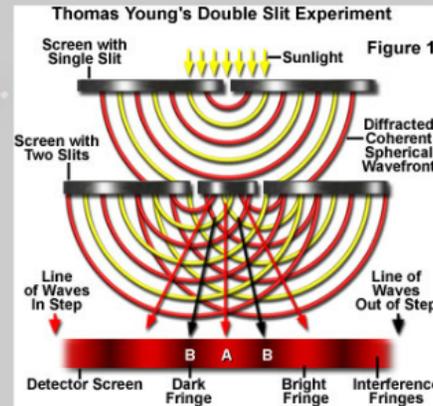


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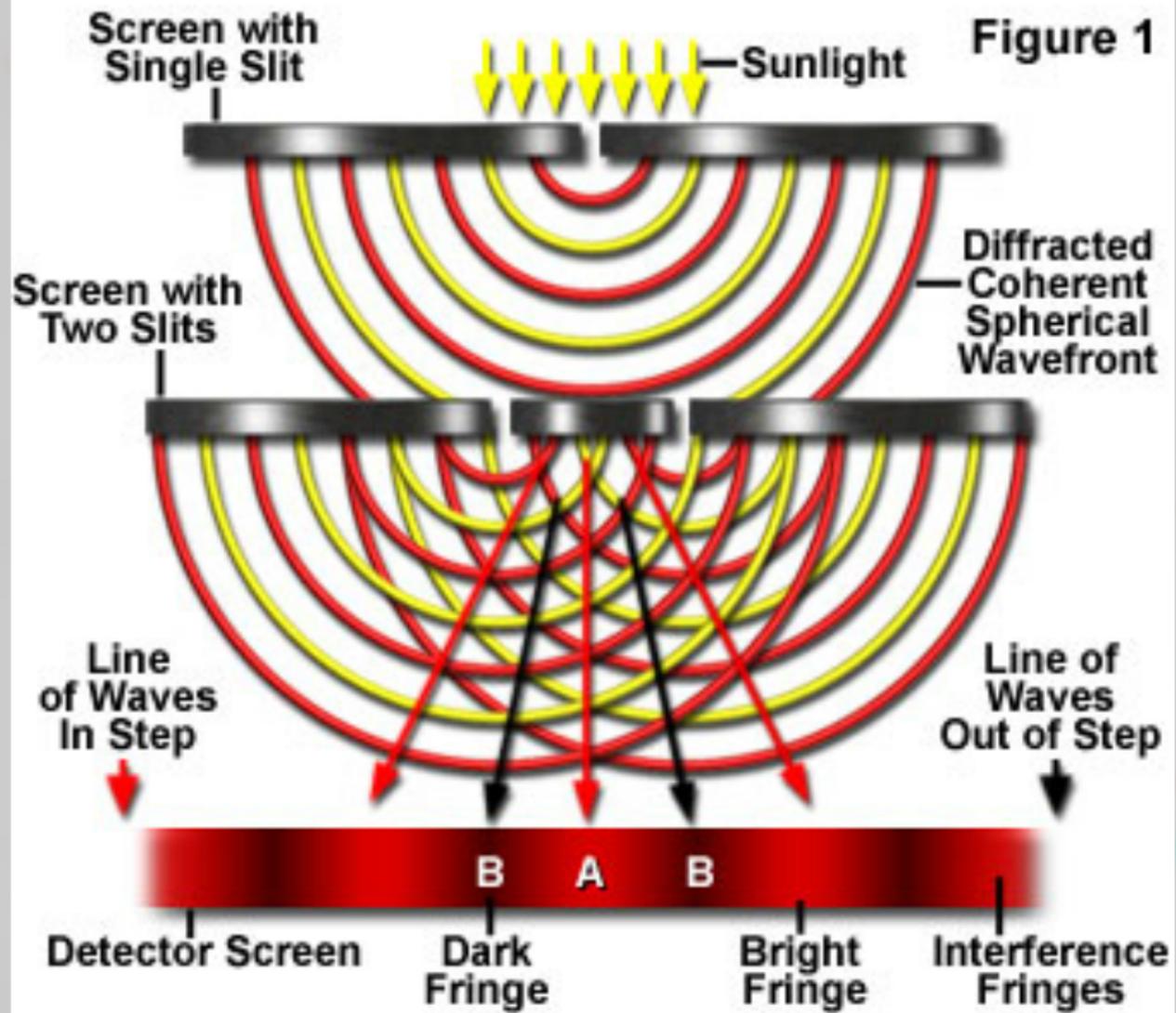
# Interference

Interference occurs when two or more waves overlap and combine to create a new wave.

Waves can combine in either constructive interference or destructive interference.



# Thomas Young's Double Slit Experiment



# Light waves

Can the same happen with matter??

Because light behaves as a wave it can be reflected, refracted, diffracted, or interfere with each other.

## Reflection

Light waves are waves, so when they hit a boundary they can be reflected back.



## Refraction

When light waves pass from one medium to another, they change direction.



## Diffraction

Light waves can pass through small openings and bend around corners.



## Interference

When two waves meet, they can combine to form a larger wave or cancel each other out.



# VECS

Can the same happen with matter??

# A bit of a detour...

Back in 1924...



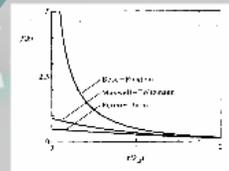
Satyendranath Bose

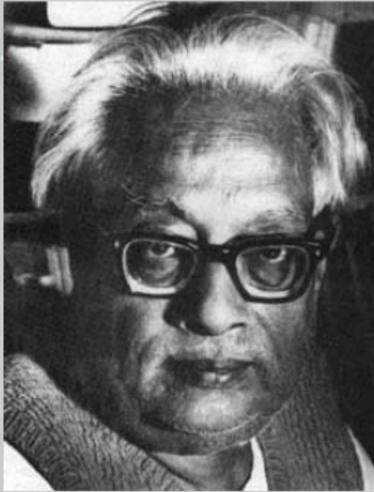


Albert Einstein

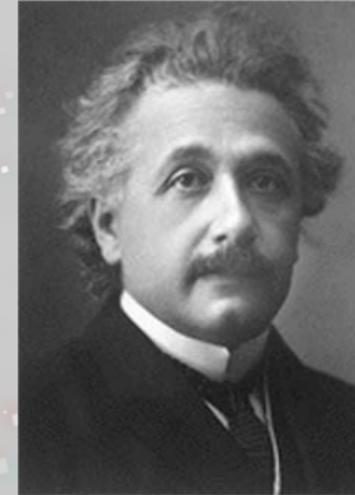
Planck's Law and the Hypothesis of Light Quanta (1924)

Respected Sir, I have ventured to send you the accompanying article for your personal and opinion. You will see that I have tried to deduce the coefficient  $a$  in Planck's law independent of classical electrodynamics...





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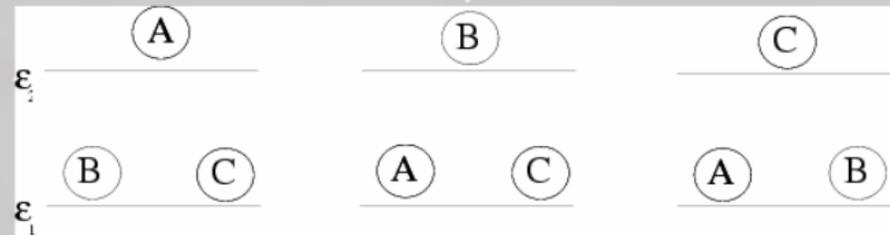
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Let us consider three identical particles which can be distributed in two energy levels.

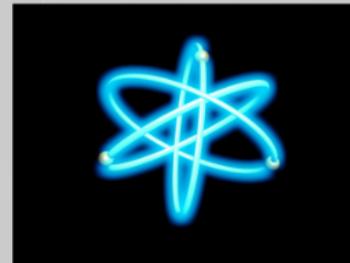
### Classical statistics

$$\left[ \begin{array}{l} \text{Maxwell-Boltzmann} \\ f(E) = \frac{1}{2^N} \end{array} \right]$$



### Quantum statistics

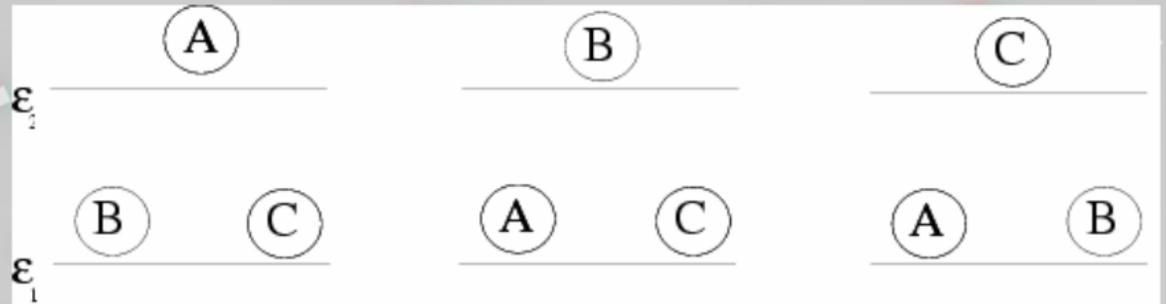
$$\left[ \begin{array}{l} \text{Fermi-Dirac} \\ f(E) = \frac{1}{2^{2N+1}-1} \end{array} \right]$$



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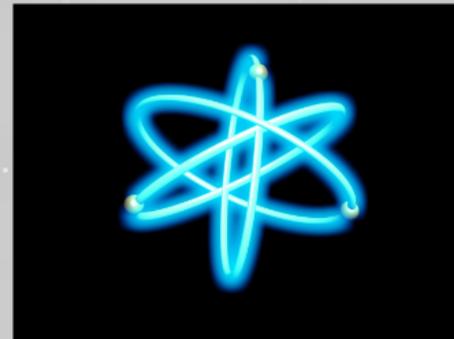
## Classical statistics

$$\left[ \begin{array}{l} \text{Maxwell-Boltzmann} \\ f(E) = \frac{1}{e^{E/kT}} \end{array} \right]$$



## Quantum statistics

$$\left[ \begin{array}{l} \text{Fermi-Dirac} \\ f(E) = \frac{1}{e^{E/kT} + 1} \\ \text{Bose-Einstein} \\ f(E) = \frac{1}{e^{E/kT} - 1} \end{array} \right]$$

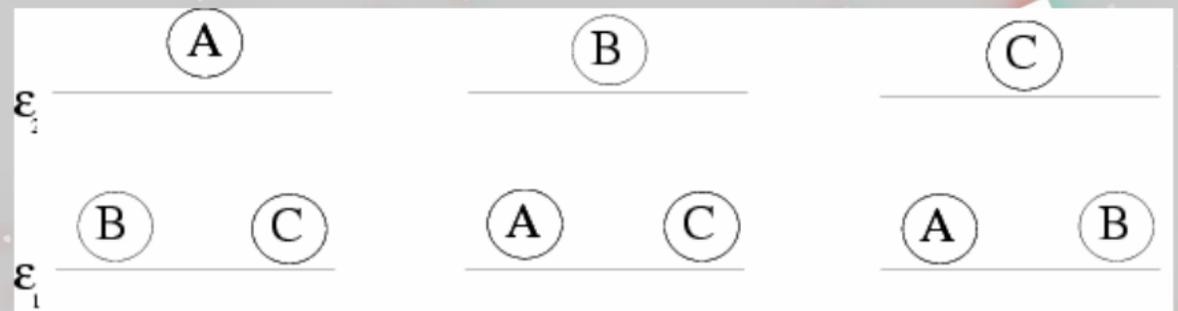


Maxwell-Boltzmann

$$f(E) = \frac{1}{e^{E/k_B T}}$$

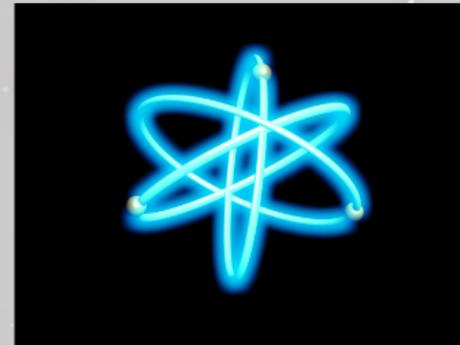
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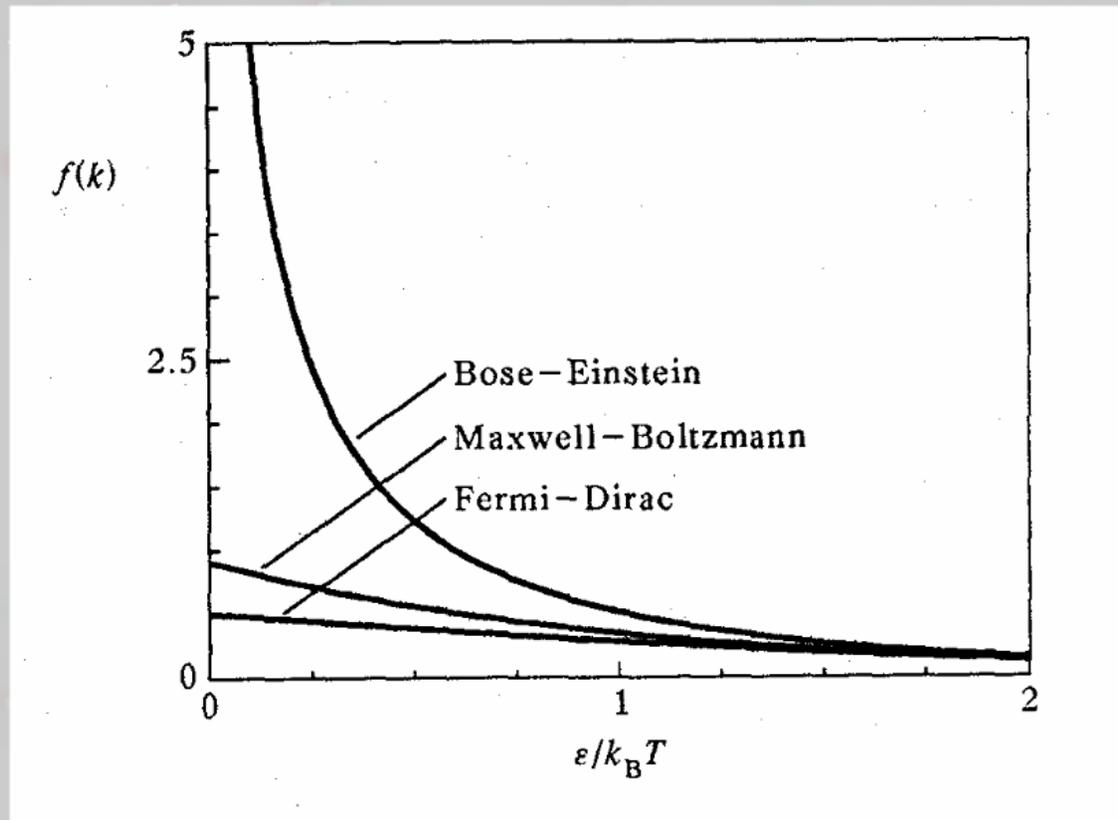


Fermi-Dirac

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

Bose-Einstein

$$f(E) = \frac{1}{e^{(E-\mu)/k_B T} - 1}$$



# Bose-Einstein Condensation

Bose-Einstein condensate (BEC) is a state of matter made up of atoms that have been cooled down to their absolute lowest points, close to absolute zero.

A BEC is a couple of billionths of a degree warmer than absolute zero, where atoms meld together to form one "super-atom".

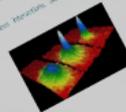
- Bosons do not have a restriction in the number of particles that can occupy a particular quantum state.
- When temperature  $T$  goes to zero, the particles will occupy the lower energy states.
- BEC is characterized by the macroscopic occupation of a particular quantum state, the ground state.
- De Broglie's wavelength is comparable to the interparticle distance:

$$\rho \lambda_{dB}^3 = 2.612...$$

$$\lambda_{dB} = \left( \frac{2\pi\hbar^2}{m k_B T} \right)^{1/2}$$

## Why do atoms interact?

- Atoms interact due to the potential energy that arises from a macroscopic occupation of an energy level at finite temperature.
- The first condensate was observed in a dilute vapor of  $^{87}\text{Rb}$  atoms in 1995. Nobel Prize in BEC shows additional support for BEC. Condensed Matter Physics.
- BEC atoms, gases show long interactions. Along with to form the system in a single way.



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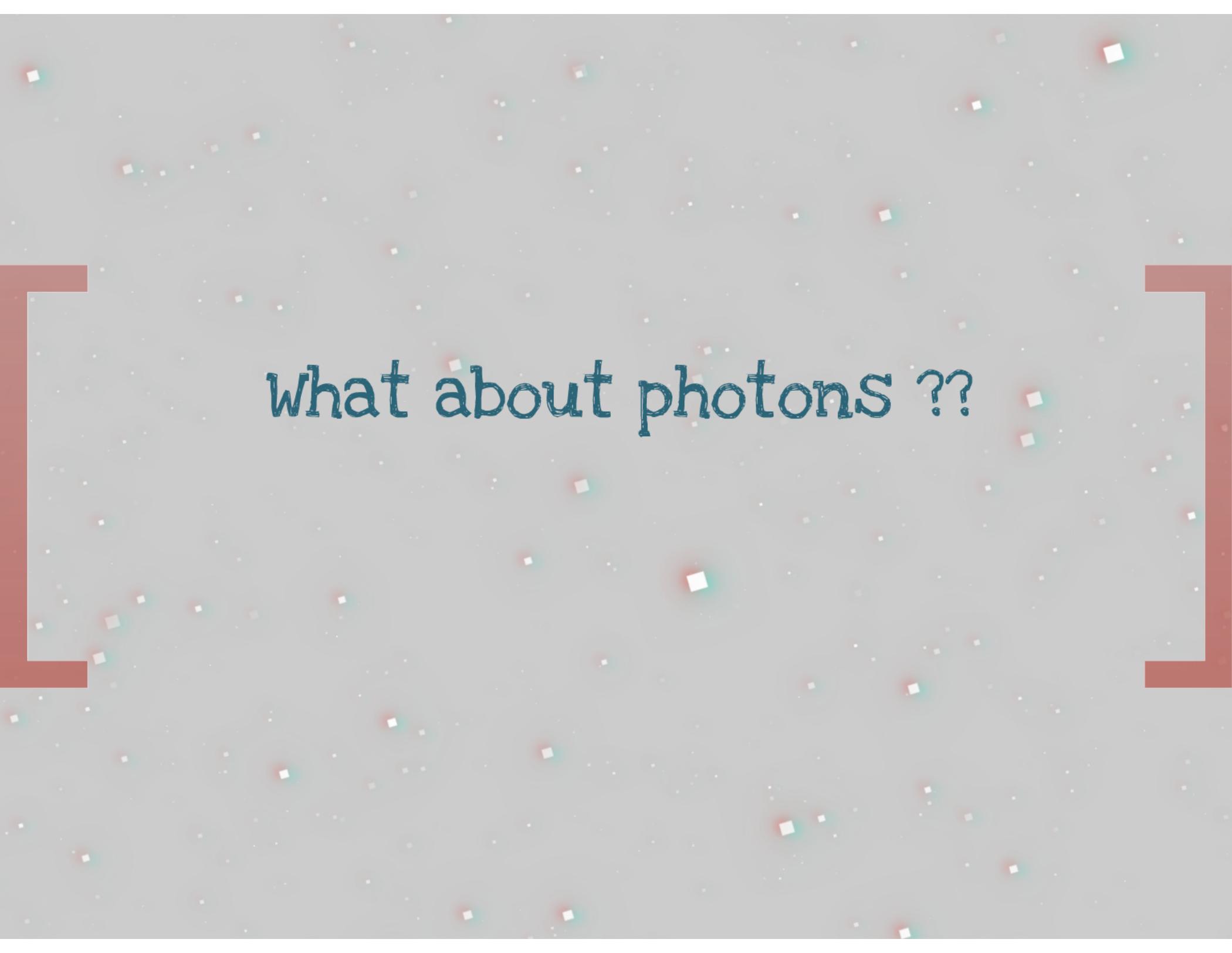
... energy levels allows a macroscopic occupation  
... atoms in 1995 (2001 Nobel Prize)

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[ What about photons ? ]



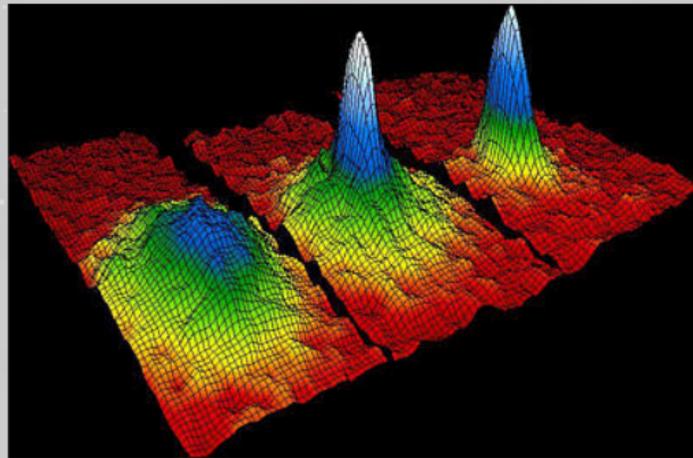
what about photons ??

## Why so much interest?

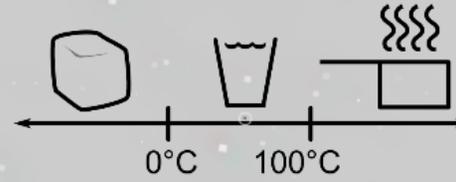
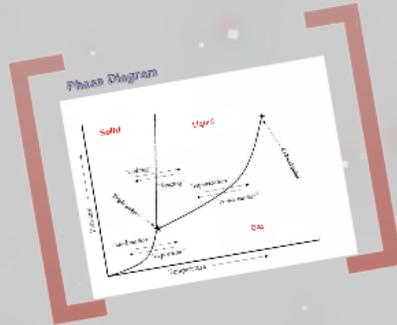
Bose statistics along with the quantised energy levels allows a macroscopic occupation of an energy level at finite temperature.

The first condensate was achieved in a diluted vapour of alkali atoms in 1995 (2001 Nobel Prize). A BEC shows well-known superfluid effects (2003 Nobel Prize)

Alkali atomic gases show less interactions, allowing us to treat the system in a simpler way.



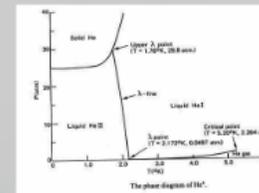
# Phase Transitions...



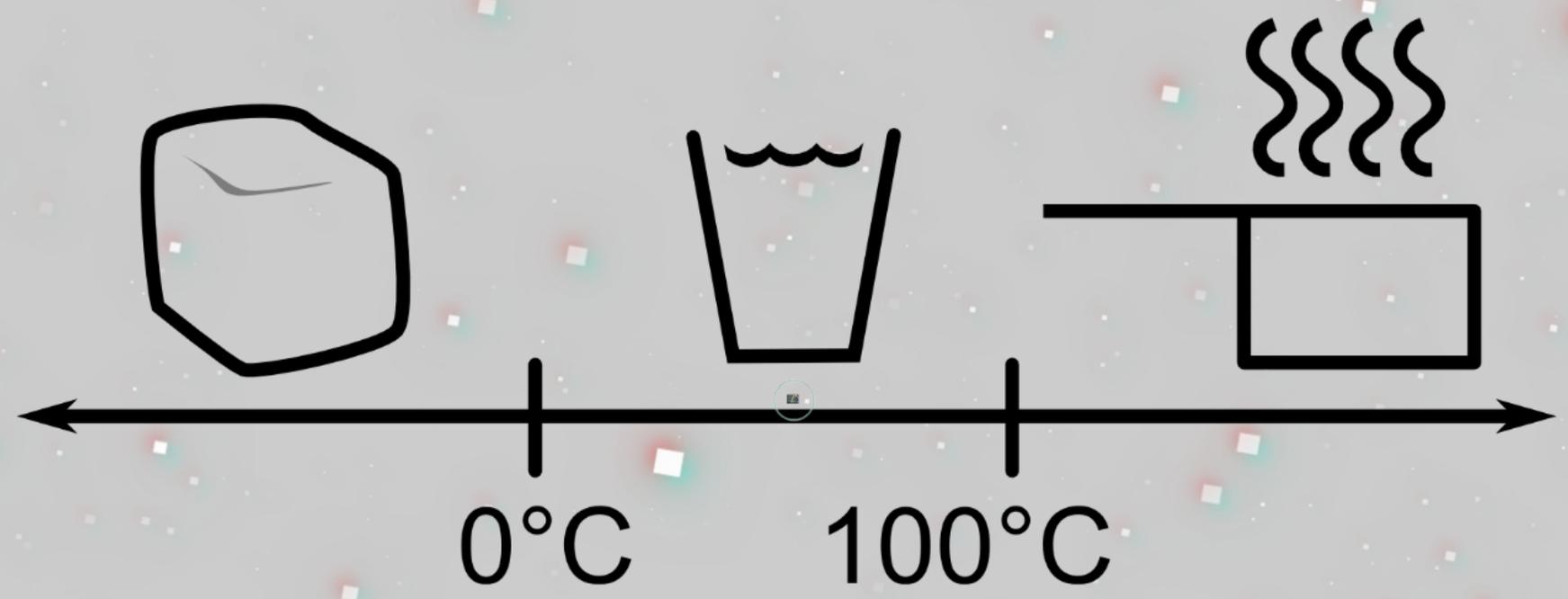
## Superfluidity: He4

The study of the transition in liquid helium can be considered as the first observation of BEC.

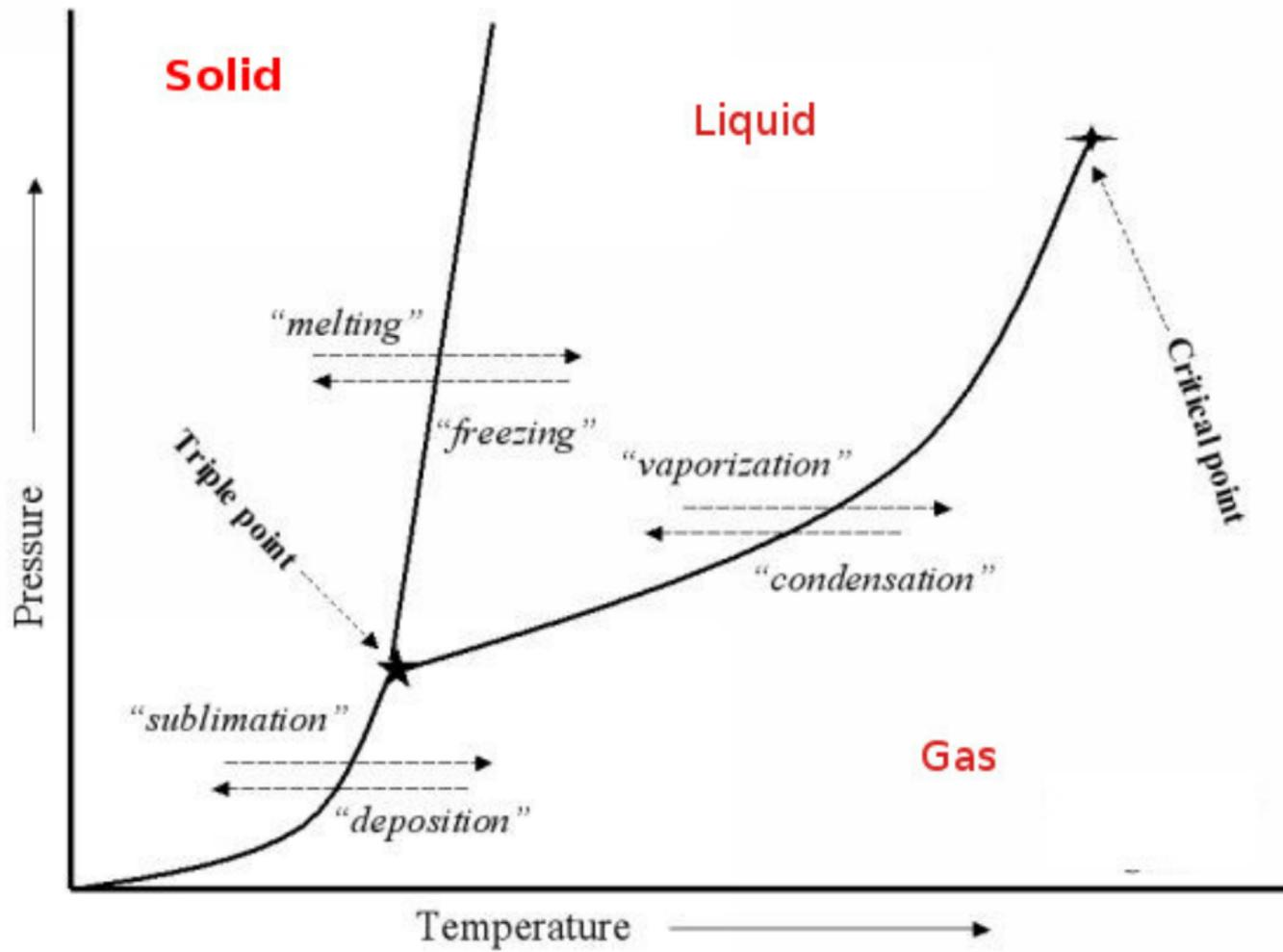
In the diagram we can see how the He II phase is extended to absolute zero.



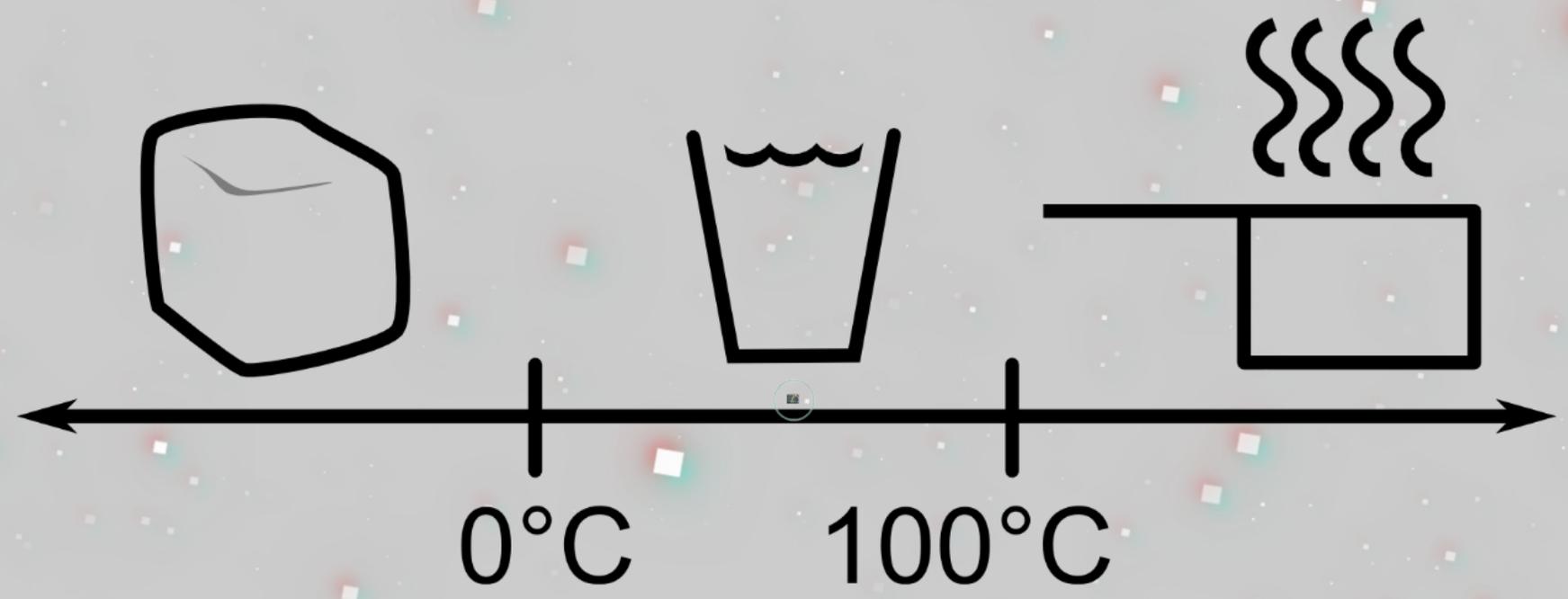
# Transitions...



# Phase Diagram



# Transitions...





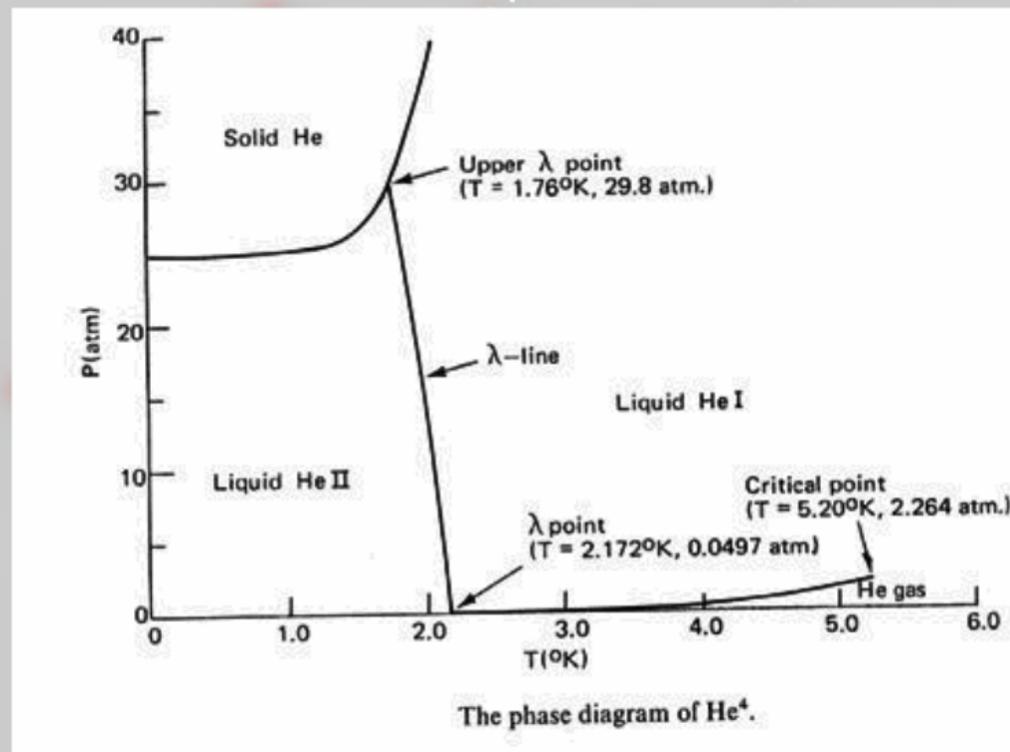


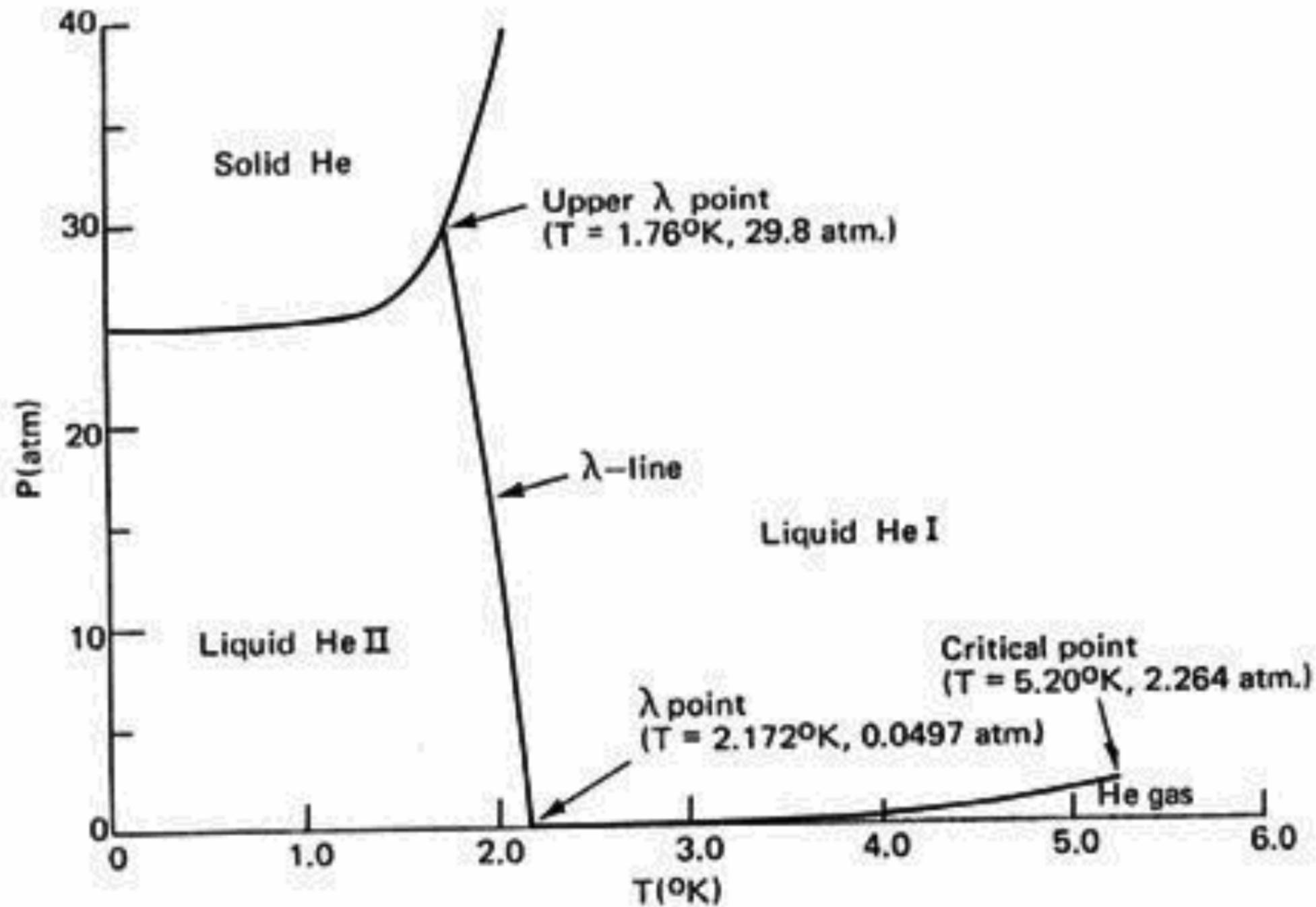


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The phase diagram of  $\text{He}^4$ .





# Properties

The He II phase is called a superfluid because its peculiar characteristics:

- the ability to flow through microscopic passages with no apparent friction;
- the quantisation of vortices;
- and the ability to support four wave modes.

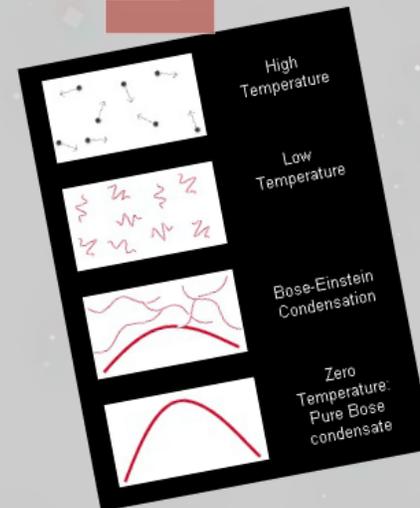
As in the case of superconductivity, the phenomenon of superfluidity is a manifestation of the quantum mechanical effects at a macroscopic level.

In 1938, Fritz London suggested that the transition from He I and He II was an example of BEC.

Tisza used London's idea to develop a phenomenological model based on two fluids in order to describe liquid helium.

## The Particles in a BEC

- The more heat particles have, the more energy they have. As particles cool, they lose energy and slow down.
- When particles lose energy, they move less and start to collect and clump together.
- Bose-Einstein condensation begins to happen at super cold temperatures, just shy of absolute zero.
- Absolute zero is the coldest temperature known to man, when particles give off almost no heat.
- When a gas is cooled near absolute zero, it begins to act like a superfluid. The atoms in the gas come together into a single blob.



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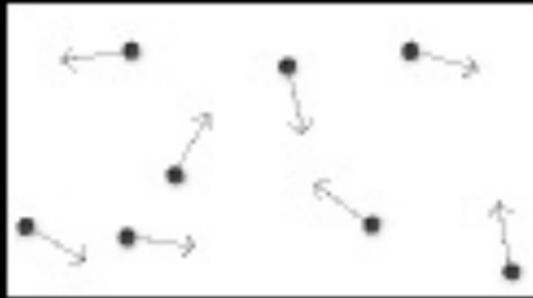
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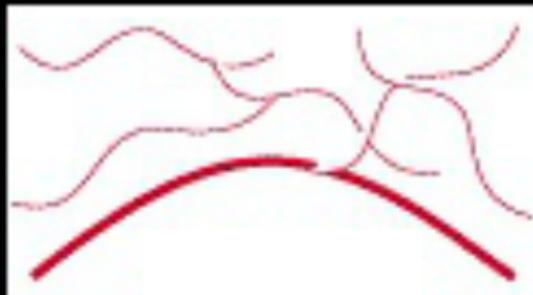
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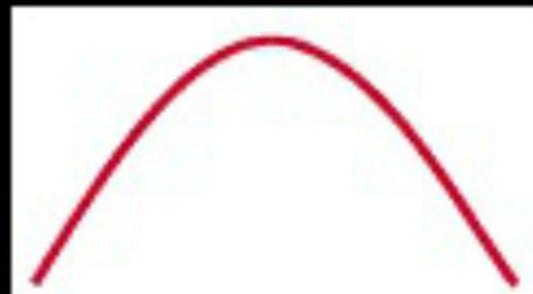
High  
Temperature



Low  
Temperature



Bose-Einstein  
Condensation



Zero  
Temperature:  
Pure Bose  
condensate

# How do we describe a BEC?

## Quantum Field Theory

(Many-body)

$$i\partial_\tau \langle \Psi(\xi, \tau) \rangle = \langle [\Psi(\xi, \tau), \hat{H}] \rangle$$

In the second quantisation formalism, the many-body Hamiltonian can be written as:

$$\hat{H} = \int d^3\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}, t) \left[ -\frac{\hbar^2}{2m} \nabla_r^2 + V_{trap}(\mathbf{r}) \right] \hat{\Psi}(\mathbf{r}, t) + \frac{1}{2} \int d^3\mathbf{r} d^3\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}^\dagger(\mathbf{r}', t) \hat{V}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}', t)$$

Mean-field approximation (Bogoliubov)

$$\hat{\Psi}(\mathbf{r}, t) = \psi(\mathbf{r}, t) + \hat{\delta}(\mathbf{r}, t)$$

## Gross-Pitaevskii Equation (GPE)

$$i\hbar\partial_t \psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + V_{trap}(\mathbf{r}) + C_0 |\psi|^2 \right) \psi$$

where  $C_0 = N \bar{V}_0 = \frac{4\pi\hbar^2 a}{m}$

Optics link:

GPE  $\rightarrow$  Beam in a medium with third order susceptibility

Hydrodynamics link:

The GPE can be written as the equations that describe the hydrodynamic theory of superfluids.

MATTER WAVES!!!

Atom Optics

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Statistical Mechanics  
It is possible to derive the GPE using arguments from statistical mechanics.  
A description can be obtained from:  
Rogel-Gabster, Rev. Z. Phys. 36, pp. 247-257 (1933)

Optics link:

GPE → Beam in a medium with third order susceptibility

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Attribution: Maxhonor  
It is possible to obtain the GPE, only parameters from  
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Bogoliubov theory (1946, 1947, 1948, 1949, 1950)

## Optics link:

GPE → Beam in a medium with third order susceptibility

## Hydrodynamics link:

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**MATTER  
WAVES!!!!**



Atom Optics

# Atom Optics

## Creation of the condensate

- Take a collection of bosons at room temperature and put them in a magneto-optical trap.
- Laser cool them to approx. 20 mK and compress.
- Transfer them to a time orbiting potential trap (TOP).
- Evaporative cool to  $\sim 170$ nK
- Result - Condensate + Excited states
- BEC has been achieved with several bosons: Rb, Na, Li

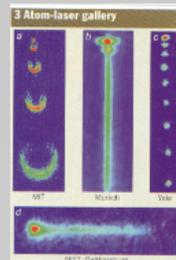
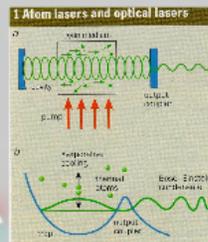


## Atom Laser?!

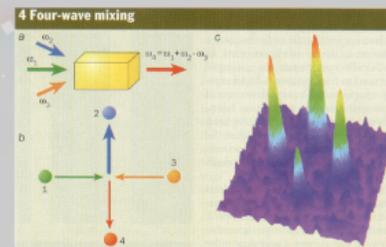


The exotic quantum phenomenon of Bose-Einstein condensation is the main ingredient for a new kind of "laser" based on atoms rather than photons.

In a conventional laser, photons share the same wave function (bosons). Laser light has the property of being monochromatic. In a BEC all the atoms have the same energy and therefore the same de Broglie wavelength. If we keep this property when output coupling the atoms from the condensate, we would basically have highly monochromatic matter waves.



## Non-linear processes



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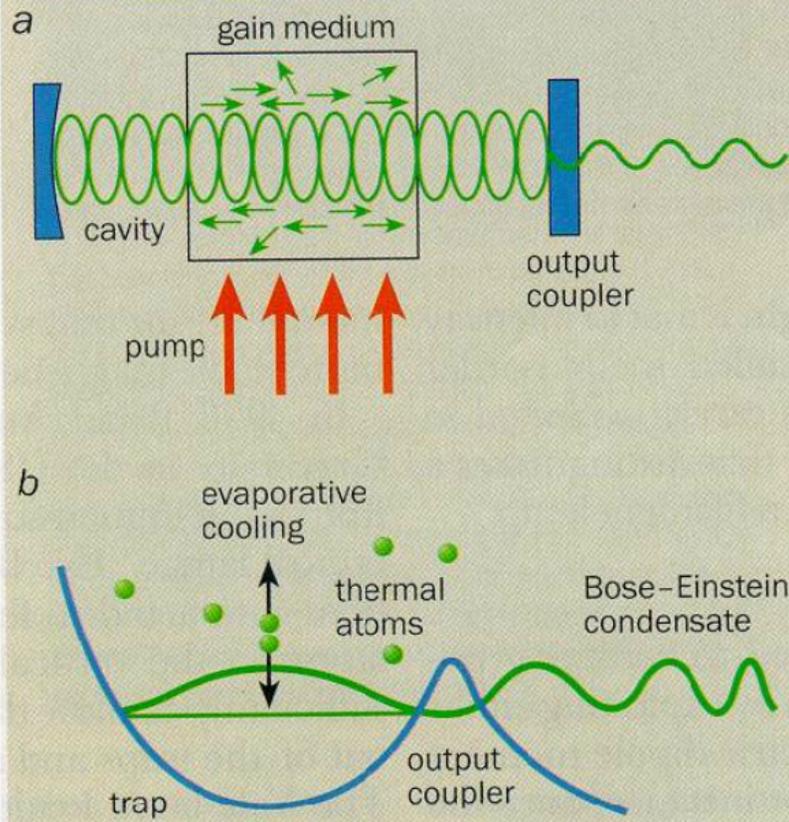
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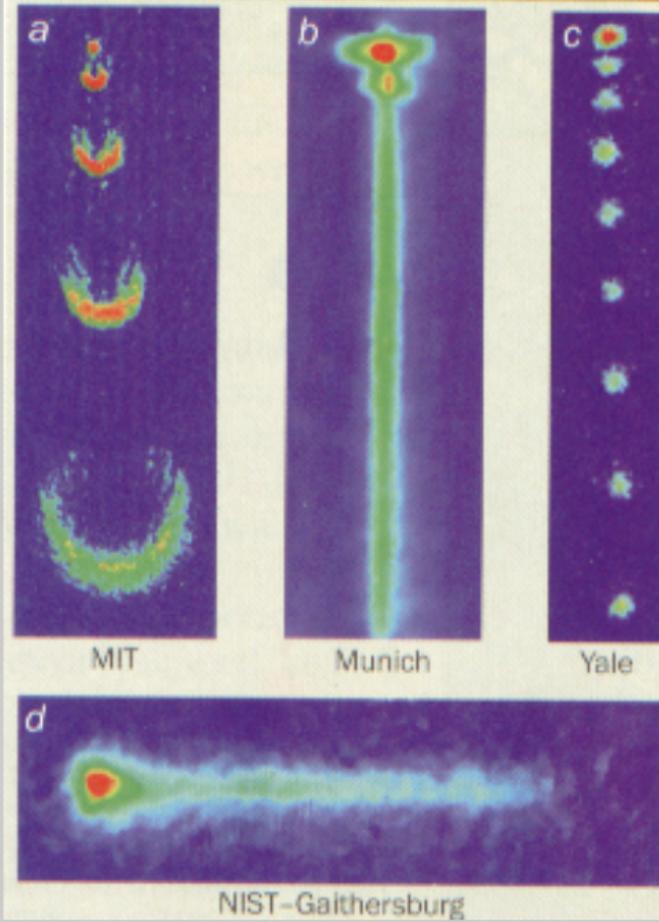
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# 1 Atom lasers and optical lasers

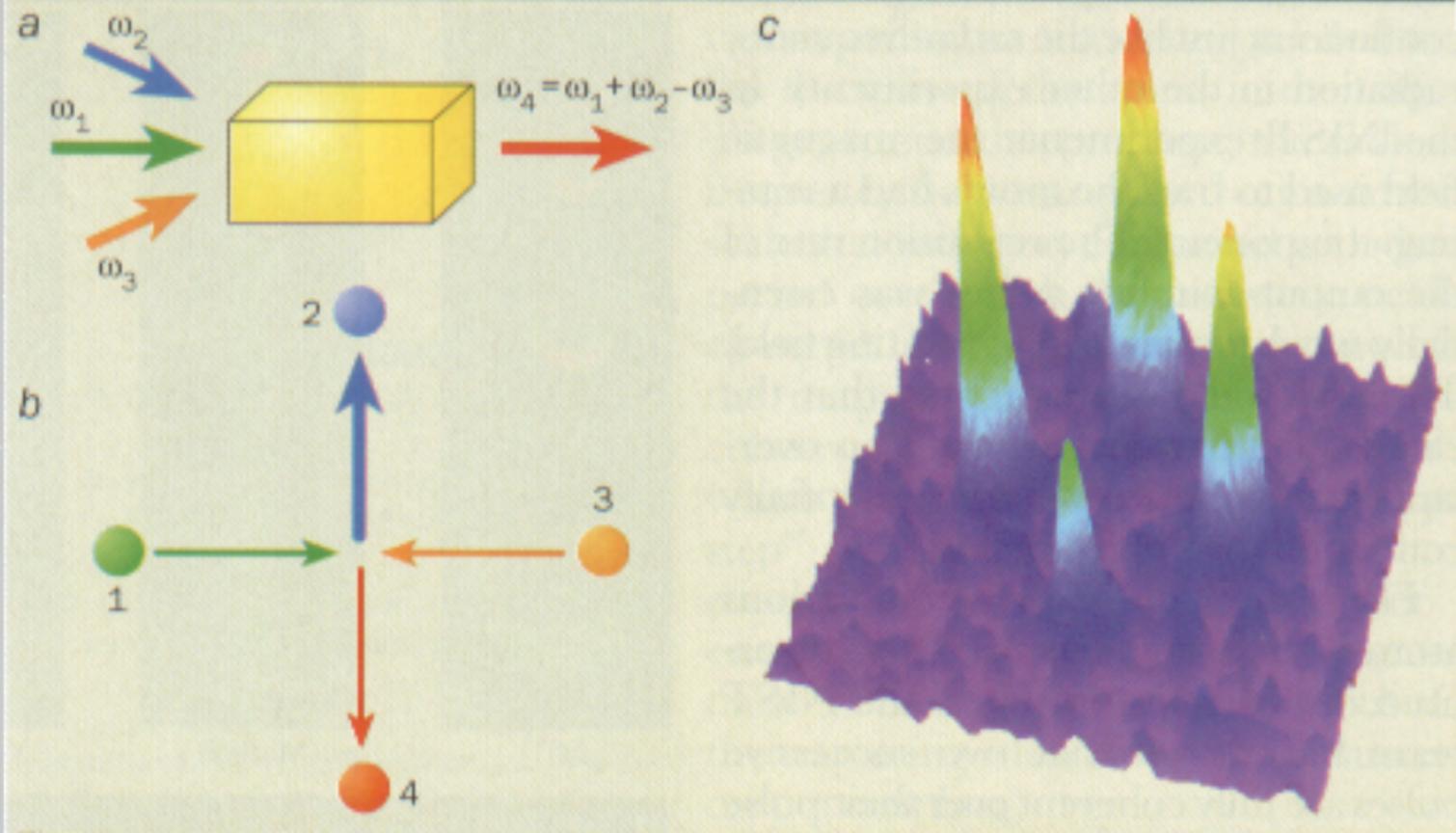


# 3 Atom-laser gallery



# Non-linear processes

## 4 Four-wave mixing



## Final remarks

There is a clear analogy between the general behaviour of optical lasers and atomic ones.

This is due to the possibility of having a macroscopic occupation of the single particle states by the atoms.

In the mean-field approximation, the dynamics of the condensate are described by the Gross-Pitaevskii equation.

Studies related to the quantum effects in the condensate are being carried out, e.g. quantum state engineering, quantum phase transitions, low-dimensionality effects, etc.

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# Atom Optics and Ultra-Cold Atoms

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