In-fiber acousto-optics

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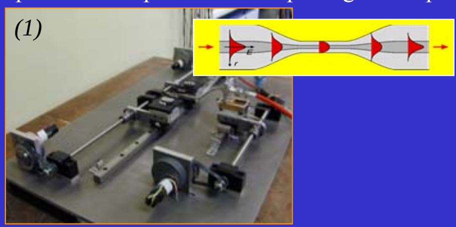
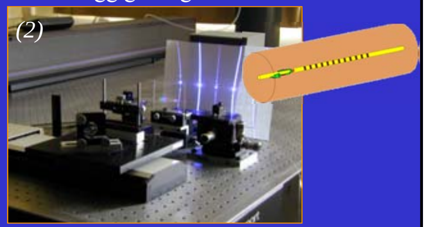
Fiber-optics components technologies

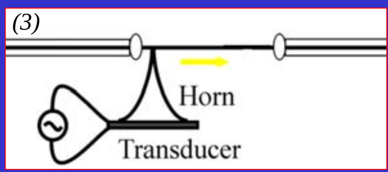
- **Fabrication of fiber-optics components**
  - Optical fiber tapers: fusion & pulling technique



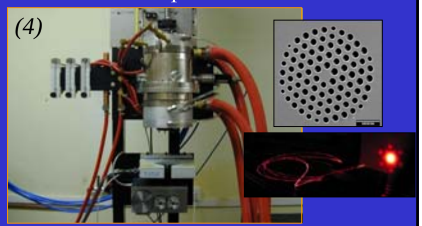
  - Fiber Bragg gratings and LPG



  - In-Fiber acousto-optic devices



  - Microstructured optical fiber



- **Applications:**
  - All-fiber light sources
  - Optical communications and microwave photonics
  - Optical fiber sensors

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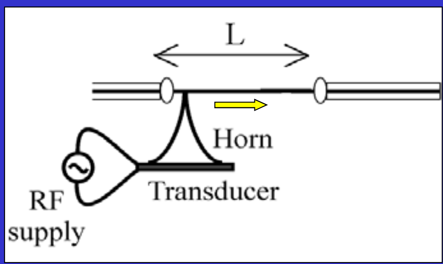
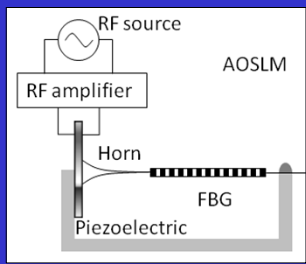
Objective

• All-fiber dynamic devices

- Silica cylinders have a good performance as acoustic waveguides
- The simultaneous propagation of guided acoustic waves and light permits long interaction lengths, enabling full transfer of power between optical modes of the fiber
- The interaction is controlled by the RF voltage that generates the acoustic wave
- Being all-fiber devices, they can handle high power optical signals



Excitation of the fundamental flexural acoustic mode



Excitation of the fundamental longitudinal acoustic mode

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In-fiber acousto-optics

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- I.3. Acousto-optic interaction: coupled modes theory
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- I.5. Longitudinal waves: applications

II. Recent developments
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- II.4. Group delay measurements
- II.5. Time-domain distributed acousto-optic interaction

III. Conclusions

- The wave equation for acoustic waves in isotropic homogeneous media

$$(\lambda+2\mu)\vec{\nabla}(\vec{\nabla}\cdot\vec{\mathbf{u}})-\mu\,\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\mathbf{u}}\right)-\rho\frac{\partial^2\vec{\mathbf{u}}}{\partial t^2}=0$$

  - λ and μ are the Lamé coefficients:
    $\lambda = c_{12}$ and $\mu = c_{44}$ (where c_{ij} is the stiffness tensor)
  - ρ is the density
  - $\vec{\mathbf{u}}$ is the vector displacement

The velocities of propagation of compressional, c_D, and shear waves, c_t:

$$c_D^{\;2}=\frac{\lambda+2\mu}{\rho}\qquad c_t^{\;2}=\frac{\mu}{\rho}$$

The Young's modulus, E, and the Poisson's coefficient, σ:

$$E=\mu\cdot\frac{3\lambda+2\mu}{\lambda+\mu}\qquad \sigma=\frac{\lambda}{2(\lambda+\mu)}$$

Fused silica:

$\lambda = 1.61\times10^{10}$ kg/m·s^2

$\mu = 3.12\times10^{10}$ kg/m·s^2

$\rho = 2.2\times10^{3}$ kg/m^3

$c_D = 5975$ m/s

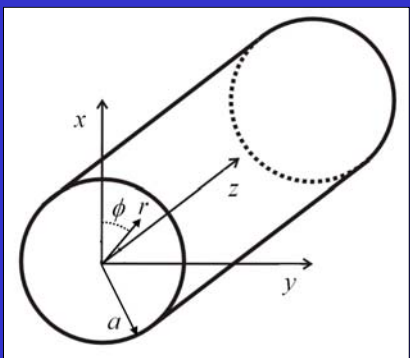
$c_t = 3764$ m/s

$E = 7.3\times10^{10}$ kg/m·s^2

$\sigma = 0.17$

I. Fundamentals and applications
I.1. The fundamental acoustic modes

- The solution for a circular solid cylinder



$$u_r = U(r)\begin{Bmatrix}\sin(n\phi)\\ \cos(n\phi)\end{Bmatrix}e^{i(\Omega t-\kappa z)}$$

$$u_\phi = V(r)\begin{Bmatrix}\cos(n\phi)\\ -\sin(n\phi)\end{Bmatrix}e^{i(\Omega t-\kappa z)}$$

$$u_z = W(r)\begin{Bmatrix}\sin(n\phi)\\ \cos(n\phi)\end{Bmatrix}e^{i(\Omega t-\kappa z)}$$

$$U(r)=A\kappa_d\,J'_n(\kappa_d r)+B\kappa\,J'_n(\kappa_t r)+C\frac{n}{r}J_n(\kappa_t r)$$

$$V(r)=A\frac{n}{r}J_n(\kappa_d r)+B\frac{\kappa\,n}{\kappa_t r}J_n(\kappa_t r)+C\kappa_t\,J'_n(\kappa_t r)$$

$$W(r)=-i\left[A\kappa\,J_n(\kappa_d r)-B\kappa_t\,J_n(\kappa_t r)\right]$$

$$\kappa_d^{\;2}=\frac{\Omega^2}{c_d^{\;2}}-\kappa^2$$

$$\kappa_t^{\;2}=\frac{\Omega^2}{c_t^{\;2}}-\kappa^2$$

I. **Fundamentals and applications**

I.1. The fundamental acoustic modes

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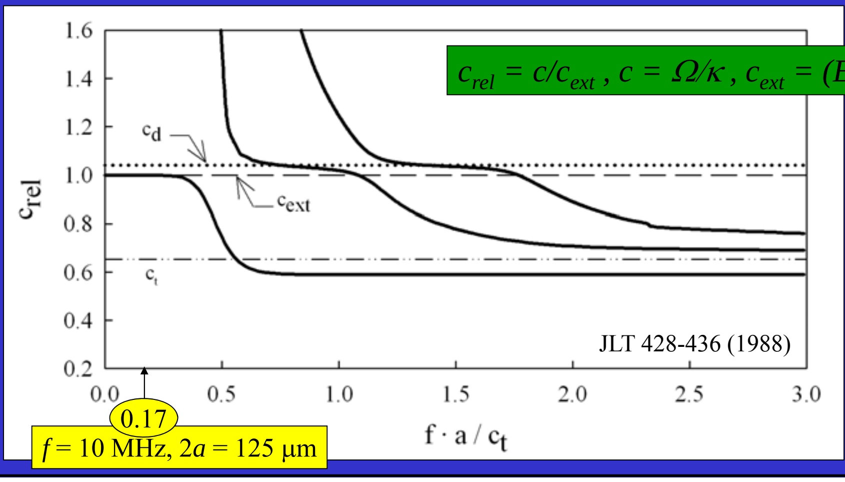
- The longitudinal acoustic modes

  – Boundary conditions (the outer surface at $r = a$ free of tractions): $T_{rr} = T_{rz} = T_{r\phi} = 0$

  – Longitudinal modes correspond to $n = 0$ and $u_\phi = 0$

$c_{rel} = c/c_{ext}$, $c = \Omega/\kappa$, $c_{ext} = (E/\rho)^{1/2}$



$f = 10$ MHz, $2a = 125$ µm

I. **Fundamentals and applications**

I.1. The fundamental acoustic modes

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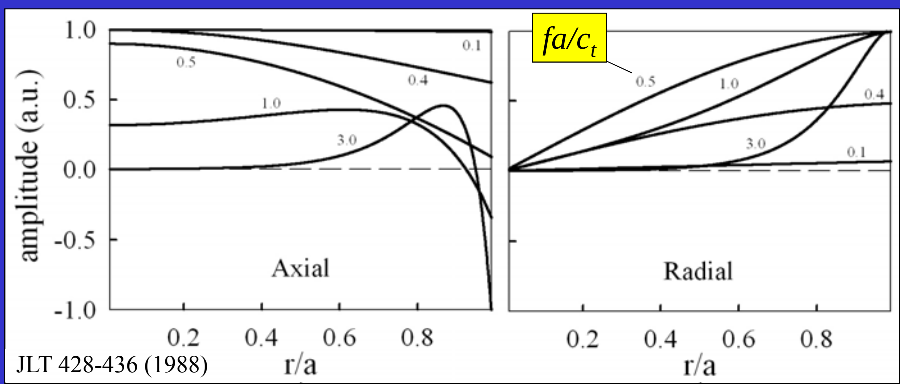
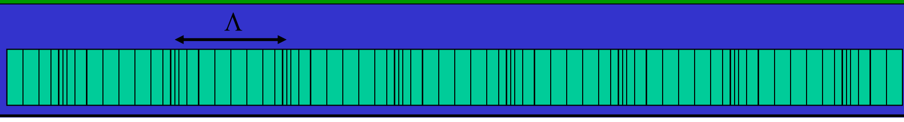
- The fundamental longitudinal acoustic mode

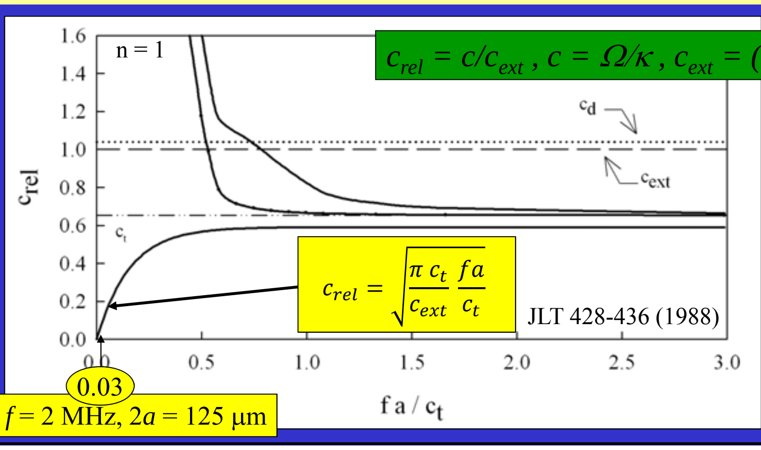


Conclusion: For an optical fiber, $2a \sim 125$ µm, and at f < 10 MHz, the fundamental longitudinal mode will have constant velocity $c = (E/\rho)^{1/2} = 5760$ m/s, and the strain will be approximatly axial ($u_z \neq 0$ and $u_r \cong 0$)



• The flexural acoustic modes

- Boundary conditions (the outer surface at $r = a$ free of tractions): $T_{rr} = T_{rz} = T_{r\phi} = 0$
- Flexural modes correspond to $n > 0$
- Only for n = 1 there is one mode with no cutoff



$c_{rel} = c/c_{ext}$, $c = \Omega/\kappa$, $c_{ext} = (E/\rho)^{1/2}$

$$c_{rel} = \sqrt{\frac{\pi\, c_t}{c_{ext}} \frac{fa}{c_t}}$$

$f = 2$ MHz, $2a = 125$ µm

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I. Fundamentals and applications
I.1. The fundamental acoustic modes
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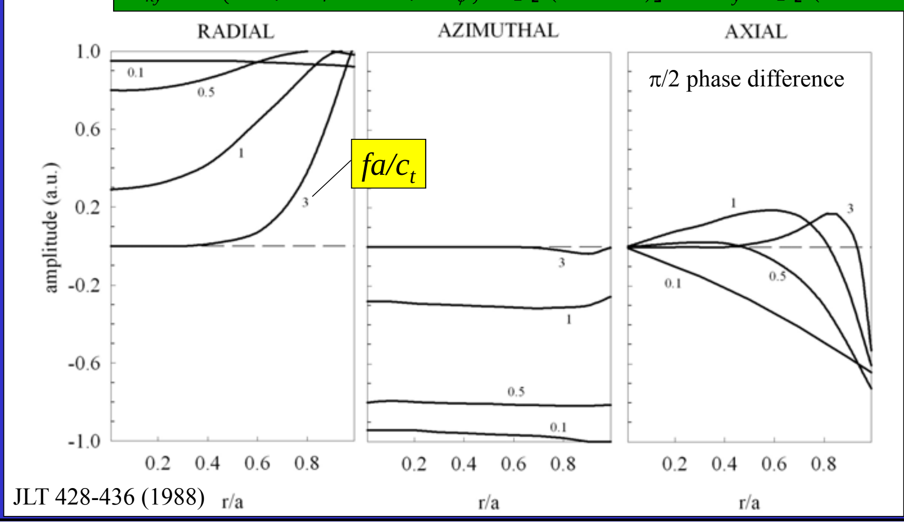
• The fundamental flexural acoustic mode

Si $fa/c_t << 1$:

$$\vec{\mathbf{u}}_{xy} \cong A\{\sin\phi\ \vec{\mathbf{u}}_r - \cos\phi\ \vec{\mathbf{u}}_\phi\}\exp[i(\Omega t - \kappa z)] = A\vec{\mathbf{u}}_y \exp[i(\Omega t - \kappa z)]$$



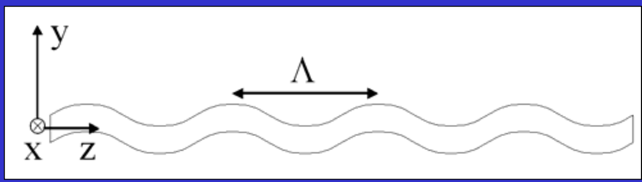
I. Fundamentals and applications
I.1. The fundamental acoustic modes

- The fundamental flexural acoustic mode

Conclusion: For an optical fiber, $2a \sim 125\ \mu$m, and at f < 2 MHz, the fundamental flexural mode will have a velocity:

$$c_{rel} = \sqrt{\frac{\pi \cdot c_{ext}}{c_t} \cdot \frac{f \cdot a}{c_t}}$$

and the displacement will be approximatly transversal: $A\bar{\mathbf{u}}_y \exp[i(\Omega t - \kappa z)]$



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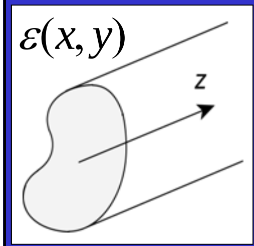
In-fiber acousto-optics

I. Fundamentals and applications

II.1. Core and cladding optical modes

- Maxwell equations far from the sources



$$\vec{\nabla}\vec{\mathbf{D}}=0 \qquad \vec{\nabla}\times\vec{\mathbf{E}}=-\frac{\partial\vec{\mathbf{B}}}{\partial t} \qquad \vec{\nabla}\vec{\mathbf{B}}=0 \qquad \vec{\nabla}\times\vec{\mathbf{H}}=-\frac{\partial\vec{\mathbf{D}}}{\partial t}$$

$$\Delta\vec{\mathbf{H}}-\varepsilon\mu\frac{\partial^2\vec{\mathbf{H}}}{\partial t^2}=0 \qquad \Delta\vec{\mathbf{E}}-\varepsilon\mu\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2}=0 \Rightarrow \Delta\vec{\mathbf{H}}+k^2\,\vec{\mathbf{H}}=0 \qquad \Delta\vec{\mathbf{E}}+k^2\,\vec{\mathbf{E}}=0$$

$$\vec{\mathbf{H}}(x,y,z,t)=\vec{\mathbf{h}}(x,y,z)\,\mathrm{e}^{-j\omega t} \qquad k=\omega\sqrt{\varepsilon\mu}$$

- Separation of variables

$$H_x(x,y,z,t)=h_x(x,y)\,f(z)\,\mathrm{e}^{-j\omega t} \Rightarrow \underbrace{\frac{1}{h_x}\nabla_t^2 h_x+k^2}_{+\beta^2}+\underbrace{\frac{1}{f}\frac{\partial^2 f}{\partial^2 z^2}}_{-\beta^2}=0$$

$$\vec{\nabla}=\vec{\nabla}_t+\vec{\mathbf{u}}_z\frac{\partial}{\partial z}$$

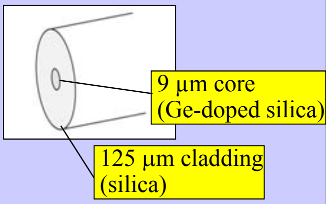
$$\Rightarrow f(z)=e^{j\beta z} \quad \text{y} \quad \left[\nabla_t^2+k^2\right]h_x=\beta^2\,h_x \qquad H_x(x,y,z,t)=h_x(x,y)\,e^{j\beta z}\,e^{-j\omega t}$$

I. Fundamentals and applications

II.1. Core and cladding optical modes

- The fiber mode spectrum

Conventional optical fibre



$$\mathbf{H}(x,y,z,t)=\mathbf{h}(x,y)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}$$

$$\mathbf{h}=(\mathbf{h}_x,\mathbf{h}_y,\mathbf{h}_z)$$

$$\left(\frac{1}{k_0^2}\nabla_t^2+\varepsilon_r\right)\mathbf{h}_z=n_m^2\,\mathbf{h}_z\,, \quad n_m=\frac{\beta}{k_0}=\frac{c}{v_f}$$

$$\left.\begin{aligned}\vec{\mathbf{E}}_{z1}(\rho,\phi,z,t)&=a_1\,J_n(k_{t1}\rho)\cos(n\phi)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}\,\vec{\mathbf{u}}_z\\ \vec{\mathbf{H}}_{z1}(\rho,\phi,z,t)&=b_1\,J_n(k_{t1}\rho)\,\mathrm{sen}(n\phi)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}\,\vec{\mathbf{u}}_z\end{aligned}\right\}\rho<a$$

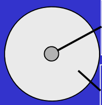
$$\left.\begin{aligned}\vec{\mathbf{E}}_{z2}(\rho,\phi,z,t)&=\left[a_2J_n(k_{t2}\rho)+a_3Y_n(k_{t2}\rho)\right]\cos(n\phi)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}\,\vec{\mathbf{u}}_z\\ \vec{\mathbf{H}}_{z2}(\rho,\phi,z,t)&=\left[b_2J_n(k_{t2}\rho)+b_3Y_n(k_{t2}\rho)\right]\mathrm{sen}(n\phi)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}\,\vec{\mathbf{u}}_z\end{aligned}\right\}\begin{aligned}\rho&>a\\ \rho&<b\end{aligned}$$

$$\left.\begin{aligned}\vec{\mathbf{E}}_{z3}(\rho,\phi,z,t)&=a_4\,K_n(h\rho)\cos(n\phi)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}\,\vec{\mathbf{u}}_z\\ \vec{\mathbf{H}}_{z3}(\rho,\phi,z,t)&=b_4\,K_n(h\rho)\,\mathrm{sen}(n\phi)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}\,\vec{\mathbf{u}}_z\end{aligned}\right\}\rho>b$$

$$k_{t1}=\sqrt{k_1^2-\beta^2} \qquad k_{t2}=\sqrt{k_2^2-\beta^2} \qquad h=\sqrt{\beta^2-k_3^2}$$

– Boundary conditions → characteristic equation

- Step index optical fiber: core modes



Medium 1: core radius a, ε_1 and μ_1

Medium 2: cladding radius $b >> a$, ε_2 and μ_2

$$\left.\begin{array}{l} K_n(x)a_2 - J_n(y)a_1 = 0 \\ \dfrac{\omega\varepsilon_2 a}{x}K_n'(x)a_2 + \dfrac{\omega\varepsilon_1 a}{y}J_n'(y)a_1 + \dfrac{n\beta a}{x^2}K_n(x)b_2 + \dfrac{n\beta a}{y^2}J_n(y)b_1 = 0 \\ \dfrac{n\beta a}{x^2}K_n(x)a_2 + \dfrac{n\beta a}{y^2}J_n(y)a_1 + \dfrac{\omega\mu_2 a}{x}K_n'(x)b_2 + \dfrac{\omega\mu_1 a}{y}J_n'(y)b_1 = 0 \\ K_n(x)b_2 - J_n(y)b_1 = 0 \end{array}\right\}$$

- Characteristic equation: determinant of the system equals to zero

$$\begin{vmatrix} K_n(x) & -J_n(y) & 0 & 0 \\ \dfrac{\omega\varepsilon_2 a}{x}K_n'(x) & \dfrac{\omega\varepsilon_1 a}{y}J_n'(y) & \dfrac{n\beta a}{x^2}K_n(x) & \dfrac{n\beta a}{y^2}J_n(y) \\ \dfrac{n\beta a}{x^2}K_n(x) & \dfrac{n\beta a}{y^2}J_n(y) & \dfrac{\omega\mu_2 a}{x}K_n'(x) & \dfrac{\omega\mu_1 a}{y}J_n'(y) \\ 0 & 0 & K_n(x) & -J_n(y) \end{vmatrix} = 0$$

- Characteristic equation:

$$[\varepsilon_{r1} G_n + \varepsilon_{r2} F_n][\mu_{r1} G_n + \mu_{r2} F_n] - \left(\frac{n\beta}{k_0}\right)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)^2 = 0$$

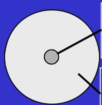
$$x^2 + y^2 = V^2$$

$$G_n = \frac{J_n'(y)}{y\,J_n(y)} \qquad F_n = \frac{K_n'(x)}{x\,K_n(x)} \qquad x = ha \qquad y = ka$$

$$V = \omega\, a\sqrt{\varepsilon_1\mu_1 - \varepsilon_2\mu_2} \qquad \beta = \sqrt{h^2 + \omega^2\varepsilon_2\mu_2}$$

– Modes TE_{0m}, TM_{0m} and hybrid (HE_{nm} and EH_{nm}, with $n > 0$)

- Step index optical fiber: LP approximation



Medium 1: core radius a, ε_1 and μ_1

Medium 2: cladding radius $b >> a$, ε_2 and μ_2

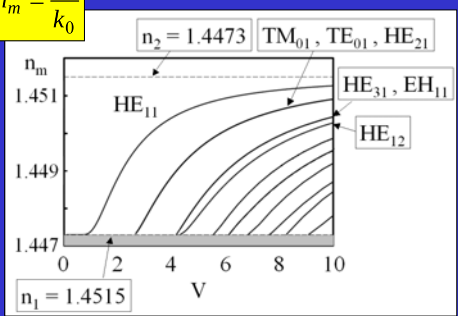
- LP aproximation: $\delta n << n_{núcleo}$ and $\mu_1 = \mu_2$

$$\left.\begin{array}{l} \dfrac{K_n(x)}{xK_{n+1}(x)} = \dfrac{J_n(y)}{yJ_{n+1}(y)} \\ x^2 + y^2 = V^2 \end{array}\right\}$$

$$V = k_0\, a\sqrt{n_{nucleo}^2 - n_{cubierta}^2}$$

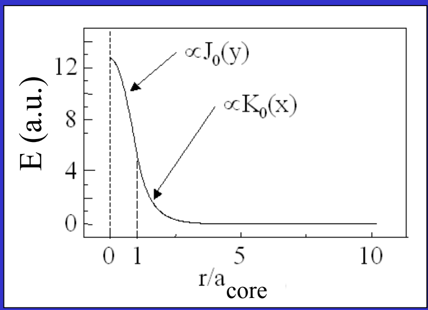
$$\Rightarrow \beta = \sqrt{k_0^2 n_{cubierta}^2 + (x/a)^2}$$

$$n_m = \frac{\beta}{k_0}$$



- Nomenclature:
  - $HE_{11} \rightarrow LP_{01}$
  - TM_{01}, TE_{01}, $HE_{21} \rightarrow LP_{11}$
  - HE_{31}, $EH_{11} \rightarrow LP_{21}$
  - $HE_{12} \rightarrow LP_{02}$

• The fundamental optical core mode
Gaussian approximation



$$\vec{\mathbf{E}}_0 = \sqrt{\frac{4Z_0 P}{\pi n w^2}} \exp\left[-\frac{x^2+y^2}{w^2}\right] \vec{\mathbf{u}}_x$$

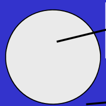
$$\frac{w}{a} = 0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6}$$

$$V = k_0\, a \sqrt{n_{core}^2 - n_{cladding}^2}$$

$$\vec{\mathbf{E}}(x,y,z,t) = \vec{\mathbf{E}}_0\, e^{-j\beta z}\, e^{j\omega t} \quad ; k_{core} > \beta > k_{clad} \quad (\beta \cong k_{core})$$

$$n_{mode} = \frac{\beta}{k_0} \cong n_{core}$$

• Step index fiber: cladding modes

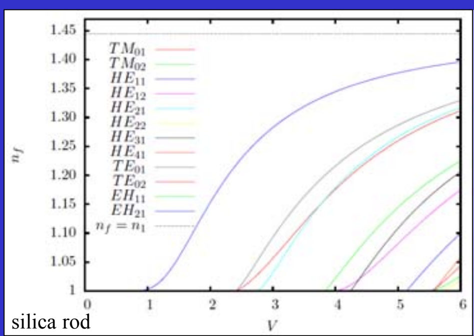


Medium 1: cladding of radius b, ε_1 and μ_1

External medium 2: ε_2 and μ_2

$$[\varepsilon_{r1} G_n + \varepsilon_{r2} F_n][\mu_{r1} G_n + \mu_{r2} F_n] - \left(\frac{n\beta}{k_0}\right)^2 \left(\frac{1}{x^2} + \frac{1}{y^2}\right)^2 = 0$$

$$x^2 + y^2 = V^2$$



silica rod

$$G_n = \frac{J_n'(y)}{y\, J_n(y)} \quad F_n = \frac{K_n'(x)}{x\, K_n(x)} \quad x = hb \quad y = kb$$

$$V = \omega\, b \sqrt{\varepsilon_1 \mu_1 - \varepsilon_2 \mu_2} \quad \beta = \sqrt{h^2 + \omega^2 \varepsilon_2 \mu_2}$$

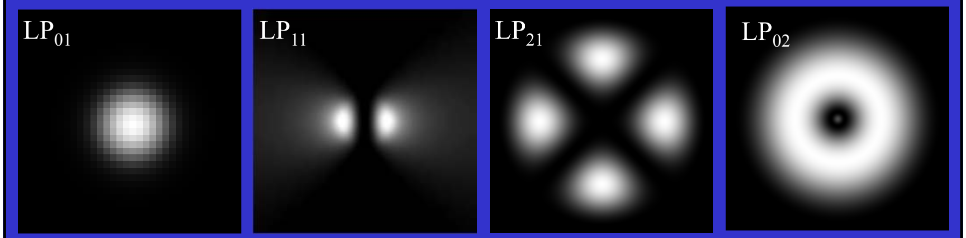
- Silica rod in air ($\mu_1 = \mu_2$)

$$V = k_0\, b \sqrt{n_{cladding}^2 - n_{air}^2}$$

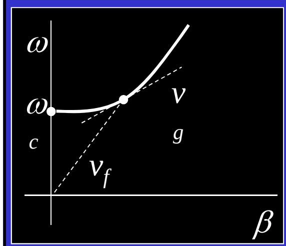
$$k_{clad} > \beta > k_{air} \quad (\beta \cong k_{clad})$$

$$n_{mode} = \frac{\beta}{k_0} \cong n_{clad}$$

- Representation of the field itensity



- Some basic properties



– The modes define a basis of the vector space
– Modes are orthogonal: no transfer of power between modes is possible
– The eigenvalues, i.e., the propagation constants, determine the phase velocity, de mode index, the group velocity and the group index of the modes

$$v_f = \frac{\omega}{\beta} \quad , \quad n_f = \frac{c}{v_f} \quad , \quad v_g = \frac{\partial \omega}{\partial \beta} \quad , \quad n_g = \frac{c}{v_g}$$

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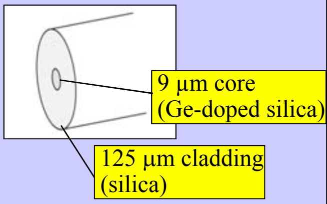
In-fiber acousto-optics

• Effects of strain on the propagation of the optical modes

Conventional optical fibre



$$\vec{\mathbf{H}}(x,y,z,t) = \vec{\mathbf{h}}(x,y)\,\mathrm{e}^{-j\beta z}\,\mathrm{e}^{j\omega t}$$

$$\Phi = \beta z$$

$$\beta = k_0 n_{\mathrm{mode}} \Rightarrow \Phi = k_0 n_{\mathrm{mode}} z$$

$$\varepsilon = \frac{\Delta L}{L} = \mathrm{E}\frac{F}{S} \;\Rightarrow\; d\Phi = k_0\left(z dn_{\mathrm{mode}} + n_{\mathrm{mode}} dz\right)$$

$$dn_{\mathrm{mode}} \cong dn = -n\ p_e\ \varepsilon \quad \text{and} \quad dz = z\varepsilon \;\Rightarrow\; d\Phi = \Phi\left[1 - p_e\right]\varepsilon$$

Effective refractive index n_{eff} :

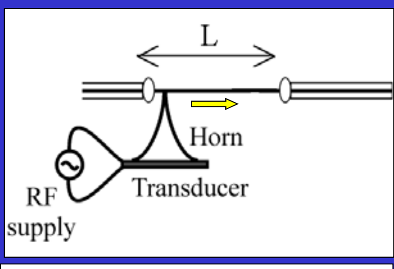
$$d\Phi = k_0 z\, dn_{eff} \rightarrow \frac{dn_{eff}}{n_{eff}} = \left[1 - p_e\right]\varepsilon$$

Effective length z_{eff} :

$$d\Phi = k_0 n_{\mathrm{mode}}\, dz_{eff} \rightarrow \frac{dz_{eff}}{z_{eff}} = \left[1 - p_e\right]\varepsilon$$

• Effects of strain on the propagation of the optical modes

Flexural waves

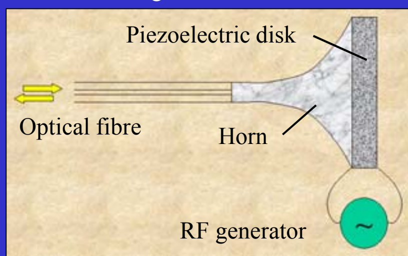


$$u(t,z) = u_0 \cos\left(\Omega_s t - k_s z\right)$$

$$\delta n_{ef} = \delta n_0 \cos\left(\Omega_s t - k_s z\right)$$

$$\delta n_0 = n_{ef}\,(1 - p_e) k_s^2\, u_0\, y$$

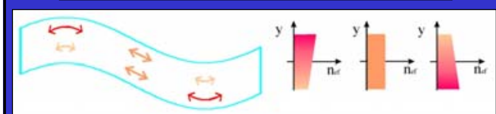
Longitudinal waves

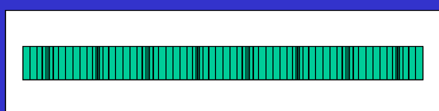


$$\varepsilon(t,z) = \varepsilon_0 \cos\left(\Omega_s t - k_s z\right)$$

$$\delta n_{ef} = \delta n_0 \cos\left(\Omega_s t - k_s z\right)$$

$$\delta n_0 = n_{ef}\,(1 - p_e)\,\varepsilon_0$$





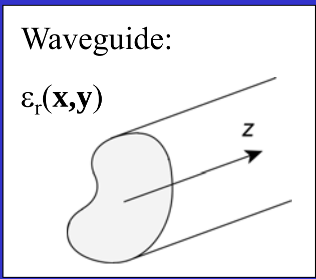
VNIVERSITAT DE VALÈNCIA

I. Fundamentals and applications

I.3. Acousto-optic interaction: coupled modes theory

Laboratorio de Fibras Ópticas

• Basis properties of the modes of a waveguide



$$\mathbf{H}(x,y,z,t) = \mathbf{h}(x,y)\,e^{-j\beta z}\,e^{j\omega t} \quad ,$$
$$\mathbf{h} = \mathbf{h}_t + \mathbf{h}_z \quad y \quad \mathbf{h}_t = (h_x, h_y)$$

$$\left(\frac{1}{k_0^2}\nabla_t^2 + \varepsilon_r\right)\mathbf{h}_t = n_m^2\,\mathbf{h}_t\,, \quad n_m = \frac{\beta}{k_0} = \frac{c}{v_f}$$

• There is no transfer of power between modes propagating along a waveguide

$$\frac{\partial \mathbf{H}}{\partial z} = -j\beta\,\mathbf{H} \qquad \int_S \mathbf{e}_{ti} \times \mathbf{h}_{tj}^*\, d\mathbf{S} = P_i\delta_{ij}$$

$$\mathbf{H} = \sum_{i=1}^{2}\mathbf{H}_i \Rightarrow P = \frac{1}{2}\int \mathbf{e}_t \times \mathbf{h}_t^*\, d\mathbf{S} = P_1 + P_2$$

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• Coupling between two modes by introducing a perturbation

Original waveguide $\tilde{\varepsilon}_r(x,y)$

$$\frac{\partial \tilde{\mathbf{H}}_1}{\partial z} = -j\tilde{\beta}_1\tilde{\mathbf{H}}_1 \qquad \frac{\partial \tilde{\mathbf{H}}_2}{\partial z} = -j\tilde{\beta}_2\tilde{\mathbf{H}}_2$$

Introducing a perturbation $\varepsilon_r(x,y) \neq \tilde{\varepsilon}_r(x,y)$

$$\frac{\partial \tilde{\mathbf{H}}_1}{\partial z} = -j\tilde{\beta}_1\tilde{\mathbf{H}}_1 + jk_{12}\tilde{\mathbf{H}}_2 \qquad \frac{\partial \tilde{\mathbf{H}}_2}{\partial z} = -j\tilde{\beta}_2\tilde{\mathbf{H}}_2 + jk_{21}\tilde{\mathbf{H}}_1$$

$$\left.\begin{aligned}\tilde{\mathbf{H}}_1(x,y,z,t) &= \tilde{\mathbf{h}}_1(x,y)\,e^{-j\tilde{\beta}_1 z}\,e^{j\omega t}\\ \tilde{\mathbf{H}}_2(x,y,z,t) &= \tilde{\mathbf{h}}_2(x,y)\,e^{-j\tilde{\beta}_2 z}\,e^{j\omega t}\end{aligned}\right\} \Rightarrow \begin{aligned}\frac{\partial \tilde{\mathbf{h}}_1(z)}{\partial z} &= jk_{12}\tilde{\mathbf{h}}_2(z)\,e^{j(\tilde{\beta}_1-\tilde{\beta}_2)z}\\ \frac{\partial \tilde{\mathbf{h}}_2(z)}{\partial z} &= jk_{21}\tilde{\mathbf{h}}_1(z)\,e^{j(\tilde{\beta}_2-\tilde{\beta}_1)z}\end{aligned}$$

$\Rightarrow$ **Transfer of power between the original modes of the waveguide**

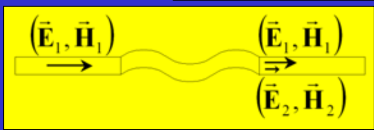
• Qualitative derivation of the coupling coefficient k_{12}

$$\vec{\nabla}\times\vec{\mathbf{E}} = -\frac{\partial\vec{\mathbf{B}}}{\partial t} \qquad \vec{\nabla}\times\vec{\mathbf{H}} = \frac{\partial\vec{\mathbf{D}}}{\partial t}$$

Introducing a perturbation $\varepsilon \to \varepsilon + \delta\varepsilon$

$$\vec{\mathbf{J}}_D = \frac{\partial\vec{\mathbf{D}}}{\partial t} = j\omega\varepsilon\,\vec{\mathbf{E}} \to \delta\vec{\mathbf{J}}_D = j\omega\,\delta\varepsilon\,\vec{\mathbf{E}}$$

Poynting Theorem: $P = \frac{1}{4}\int_V \vec{\mathbf{J}}\,\vec{\mathbf{E}}^*\,dV$



$$P_{12} \propto \int_V \delta\vec{\mathbf{J}}_1\,\vec{\mathbf{E}}_2^*\,dV \propto \int_V \delta\varepsilon\,\vec{\mathbf{E}}_1\vec{\mathbf{E}}_2^*\,dV$$

$\delta\varepsilon \propto \delta n$

$$k_{12} \propto \int_V \delta n\,\vec{\mathbf{E}}_1\vec{\mathbf{E}}_2^*\,dV$$

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• Coupling coefficient

$$\delta n_{eff} = \delta n_0 \cos(\Omega_s t - k_s z)$$

$$k_{12} \propto \int_V \delta n\,\vec{\mathbf{E}}_1\vec{\mathbf{E}}_2^*\,dV \cong \int_S dxdy \int_z^{z+\Delta z} dz\,\delta n_0 \cos(\Omega_s t - k_s z)\vec{\mathbf{E}}_1\vec{\mathbf{E}}_2^*$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{e}}_1(x,y)\,\mathrm{e}^{-j\beta_1 z}\,\mathrm{e}^{j\omega_1 t}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{e}}_2(x,y)\,\mathrm{e}^{-j\beta_2 z}\,\mathrm{e}^{j\omega_2 t}$$

$$\cos(\Omega_s t - k_s z) = \frac{1}{2}\left(\mathrm{e}^{-jk_s z}\,\mathrm{e}^{j\Omega_s t} + \mathrm{e}^{jk_s z}\,\mathrm{e}^{-j\Omega_s t}\right)$$

- **Coupling coefficient**

$$k_{12} \propto \int_{z}^{z+\Delta z} dz \left[\mathrm{e}^{j(\beta_2-\beta_1-k_s)z}\ \mathrm{e}^{j(\omega_1-\omega_2+\Omega)t} + \mathrm{e}^{j(\beta_2-\beta_1+k_s)z}\ \mathrm{e}^{j(\omega_1-\omega_2-\Omega)t} \right] \cdot$$

$$\cdot \int_{-\infty}^{+\infty} \delta n_0(x,y)\, \vec{\mathbf{e}}_1\, \vec{\mathbf{e}}_2^{*}\, dxdy$$

Bragg condition (phase matching): $\beta_1 - \beta_2 \approx \pm k_s$

Conservation of energy: $\omega_1 - \omega_2 \approx \pm\Omega$

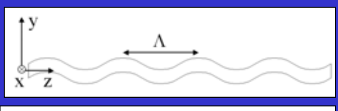
$$\bar{k}_{12} \propto \int_{-\infty}^{+\infty} \delta n_0(x,y)\, \vec{\mathbf{e}}_1\, \vec{\mathbf{e}}_2^{*}\, dxdy$$

Contents

In-fiber acousto-optics

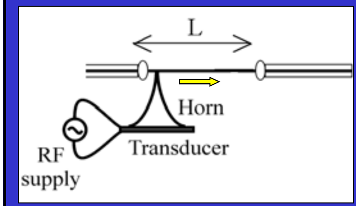
- The fundamental flexural acoustic mode

For an optical fiber, $2a \sim 125$ µm, and at f < 10 MHz, the fundamental flexural mode will have a velocity: $c_{flex} = 205.9\,(fa)^{1/2}$
and the displacement will be approximately transversal: $A\vec{\mathbf{u}}_y \exp[i(\Omega t - \kappa z)]$



Bragg condition (phase matching): $\beta_1 - \beta_2 \approx \pm k_s$

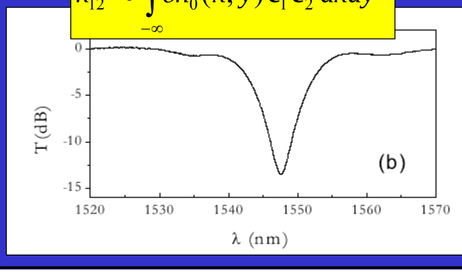
Conservation of energy: $\omega_1 - \omega_2 \approx \pm\Omega$



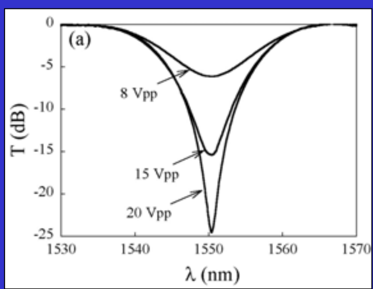
$$u(t,z) = u_0 \cos(\Omega_s t - k_s z)$$
$$\delta n_{ef} = \delta n_0 \cos(\Omega_s t - k_s z)$$
$$\delta n_0 = n_{ef}(1 - p_e) k_s^2\, u_0\, y$$

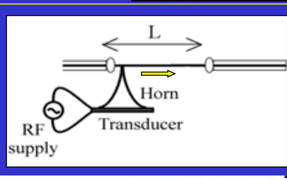
$LP_{01,\,core}$ $LP_{11,\,cladding}$

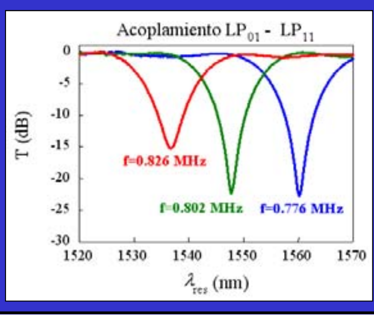
$$\bar{k}_{12} \propto \int_{-\infty}^{+\infty} \delta n_0(x,y)\, \vec{\mathbf{e}}_1 \vec{\mathbf{e}}_2^{\,*}\, dxdy$$

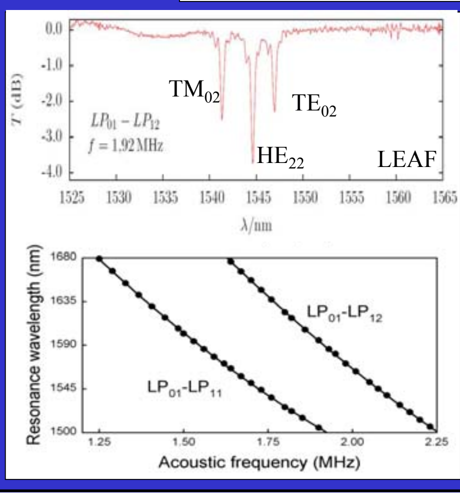


- The fundamental flexural acoustic mode

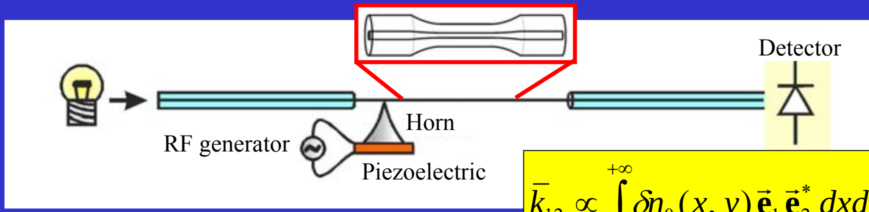




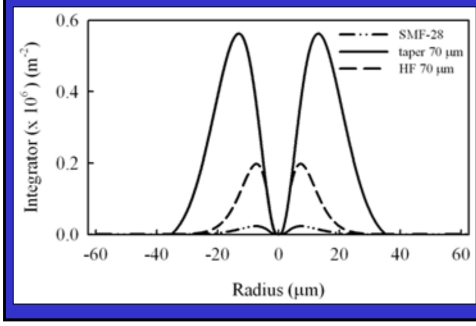


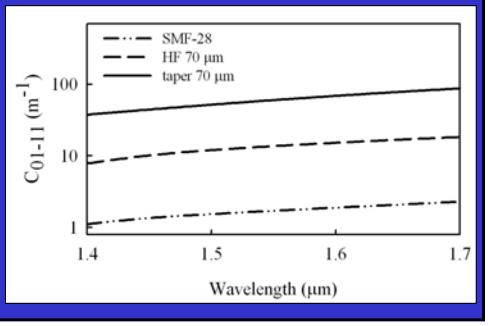


- The fundamental flexural acoustic mode: Optimization



$$\bar{k}_{12} \propto \int_{-\infty}^{+\infty} \delta n_0(x,y)\,\vec{\mathbf{e}}_1\,\vec{\mathbf{e}}_2^{*}\,dxdy$$





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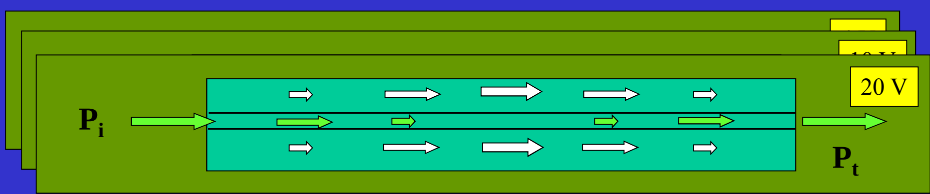
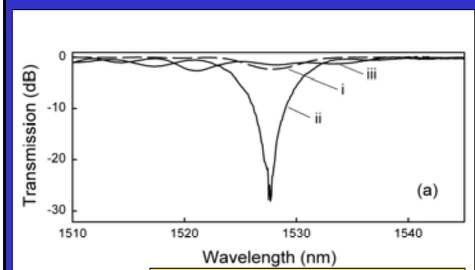
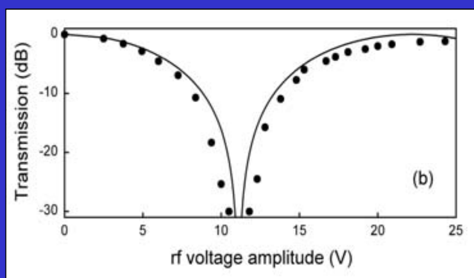
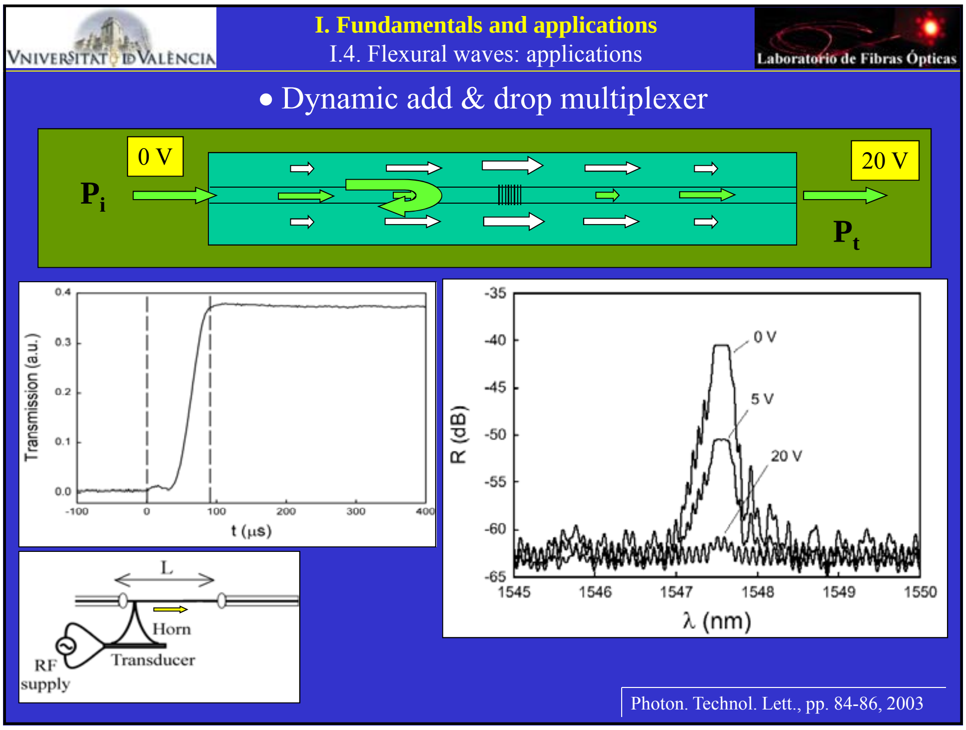
- Periodic transfer of power with flexural waves





(i) 4 V, (ii) 10 V and (iii) 20 V

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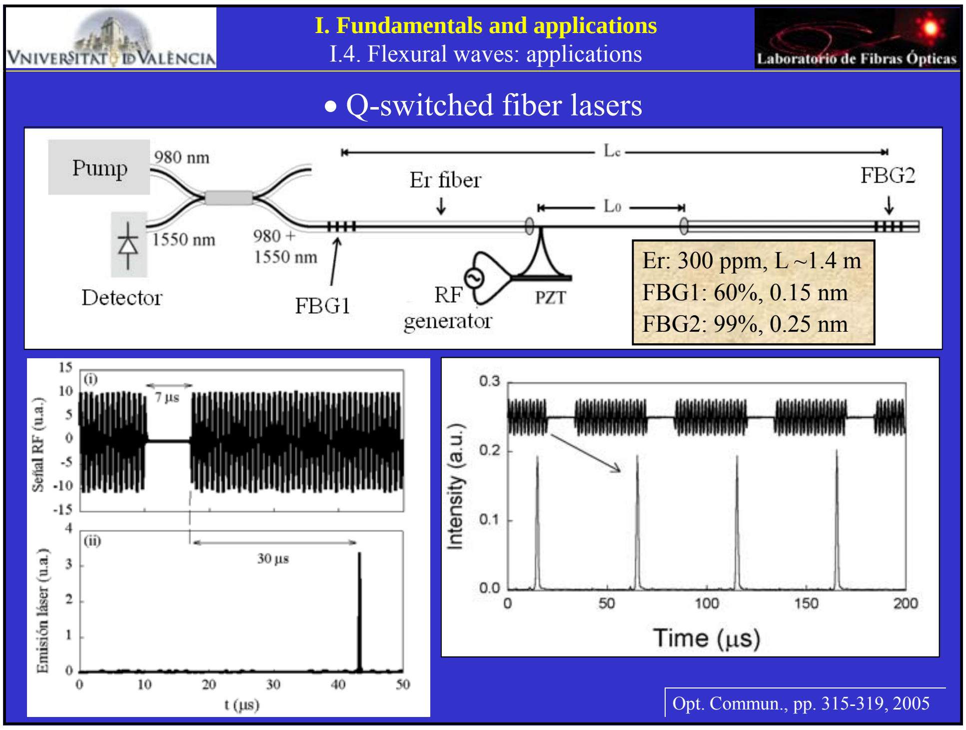
• Dynamic add & drop multiplexer
0 V
20 V
Pi
Pt
Transmission (a.u.)
t (μs)
R (dB)
λ (nm)
0 V
5 V
20 V
L
Horn
Transducer
RF
supply
Photon. Technol. Lett., pp. 84-86, 2003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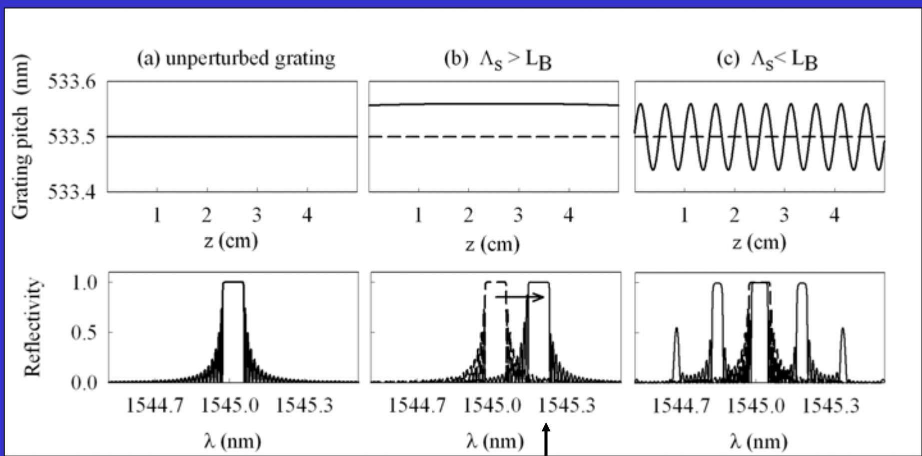
• Q-switched fiber lasers
Pump
980 nm
1550 nm
Detector
980 +
1550 nm
FBG1
Er fiber
RF
generator
PZT
FBG2
Er: 300 ppm, L ~1.4 m
FBG1: 60%, 0.15 nm
FBG2: 99%, 0.25 nm
Señal RF (u.a.)
7 μs
30 μs
Emisión láser (u.a.)
t (μs)
Intensity (a.u.)
Time (μs)
Opt. Commun., pp. 315-319, 2005

Contents

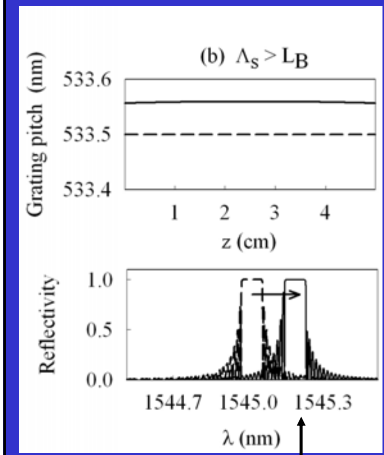
In-fiber acousto-optics

- **The fundamental longitudinal acoustic mode + fiber Bragg grating**

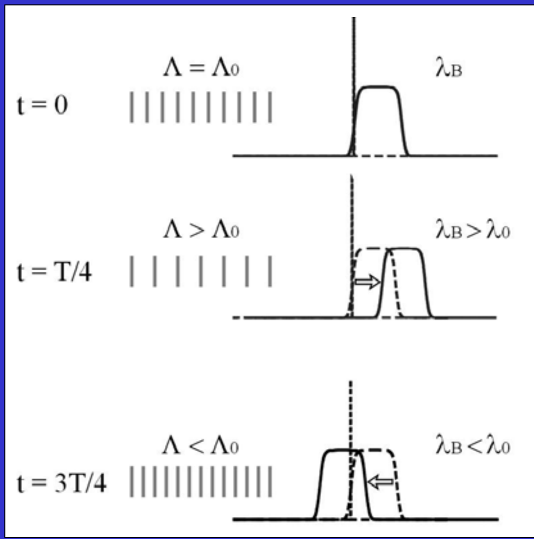


$$\varepsilon_0 = \left[2P_s / EAv_{gs}\right]^{1/2}$$

• Longitudinal waves + fiber Bragg gratings: $\Lambda_s > L_{FBG}$



$$\varepsilon_0 = \left[2P_s / EAv_{gs}\right]^{1/2}$$



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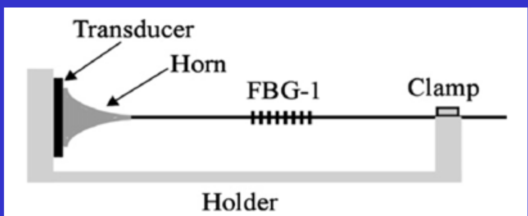
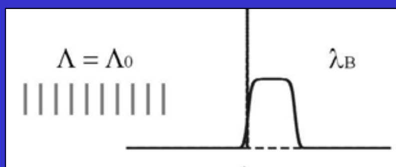
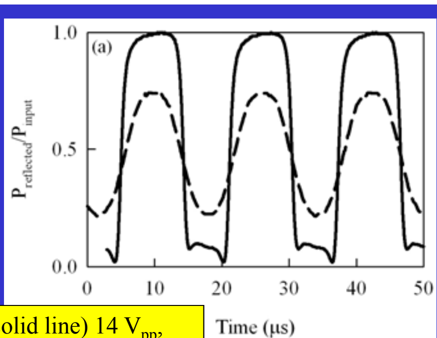
• Longitudinal waves + fiber Bragg gratings: $\Lambda_s > L_{FBG}$

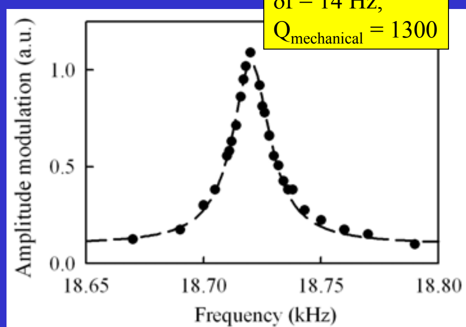




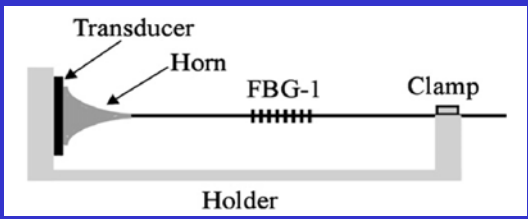
δf = 14 Hz,
$Q_{mechanical}$ = 1300

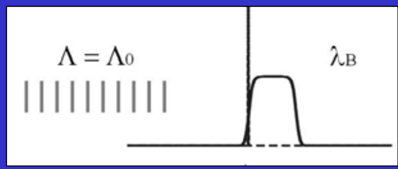


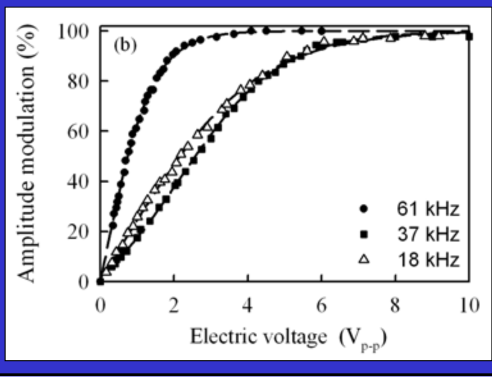
(solid line) 14 V_{pp},
(dashed lines) 4.2 V_{pp}



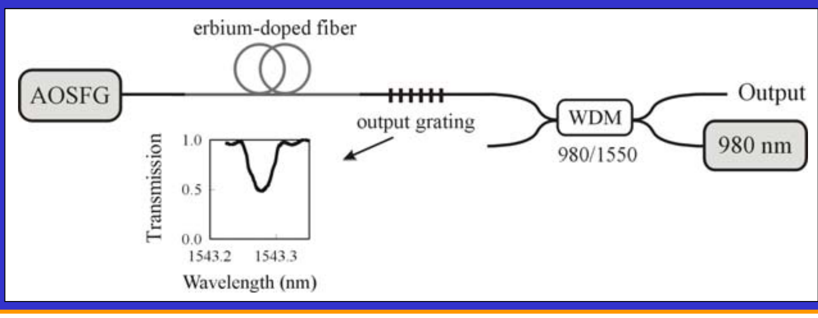
• Longitudinal waves + fiber Bragg gratings: $\Lambda_s > L_{FBG}$

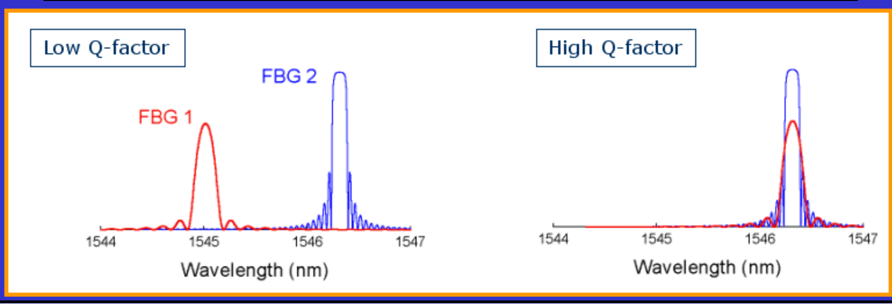






• Longitudinal waves + fiber Bragg gratings: $\Lambda_s > L_{FBG}$





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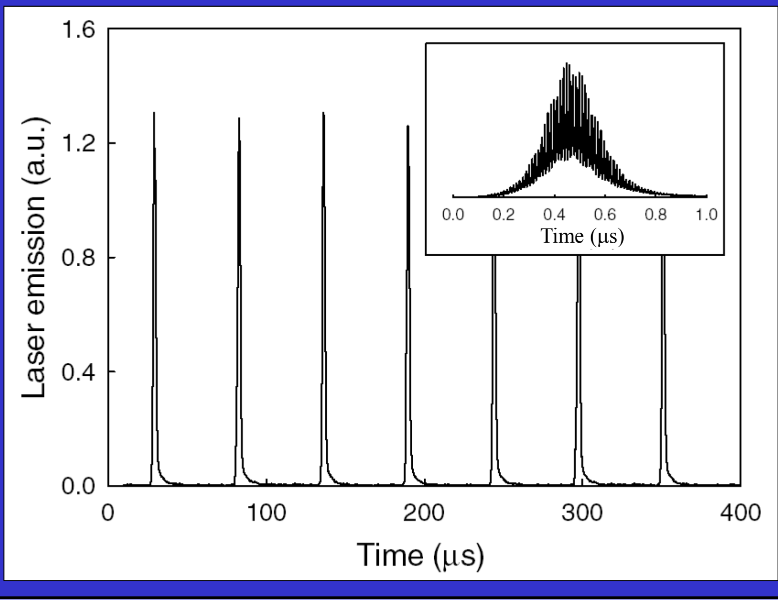
• Longitudinal waves + fiber Bragg gratings: $\Lambda_s > L_{FBG}$



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I.5. Longitudinal waves: applications

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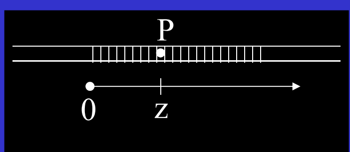
• Longitudinal waves + fiber Bragg gratings: $\Lambda_s << L_{FBG}$

For an optical fiber, 2a ~ 125 µm, and at f < 10 MHz, the fundamental longitudinal mode will have constant velocity c_{long} = 5760 m/s, and the strain will be approximately axial ($u_z \neq 0$ and $u_r \cong 0$)

Bragg grating: $\delta n = \delta n_0 \cos\{Kz + \phi\}$

Bragg grating perturbated by a longitudinal wave: $\delta n = \delta n_0 \cos\{K(z + \delta z) + \phi\}$

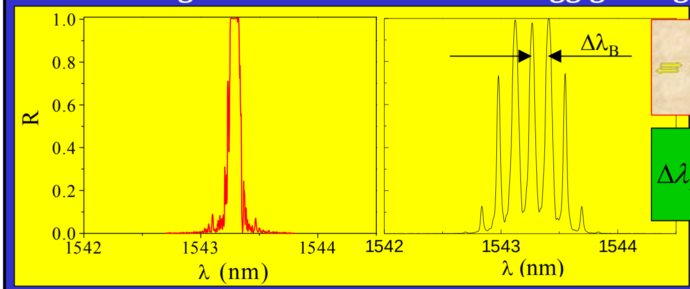


$$\delta z = \int \varepsilon_0 \cos\{\Omega_s t - k_s z\} dz = -\frac{\varepsilon_0}{k_s} sen\{\Omega_s t - k_s z\}$$

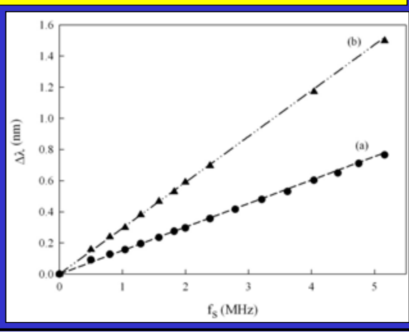
$$\delta n = \delta n_0 \cos\left\{Kz - \frac{K\varepsilon_0}{k_s} sen\,(\Omega_s t - k_s z)\right\} \Rightarrow \lambda_{Bm} = \lambda_{B0}\left(1 \pm m\frac{\Lambda}{\lambda_s}\right) \;\; y \;\; \omega_m = \omega \mp m\Omega_s$$

$$k_{12} \propto \int_V \delta n \, \vec{\mathbf{E}}_1 \vec{\mathbf{E}}_2^* \, dV$$

• Longitudinal waves + fiber Bragg gratings: $\Lambda_s << L_{FBG}$



$$\Delta\lambda_B = \frac{\lambda_{B0}^2}{2\,n}\frac{f_s}{v_s}$$



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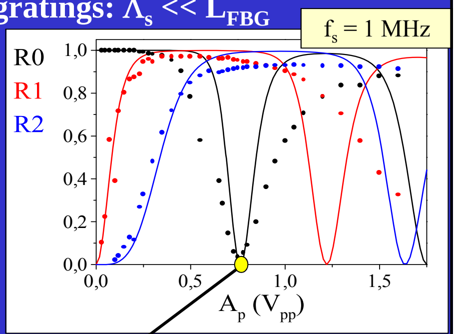
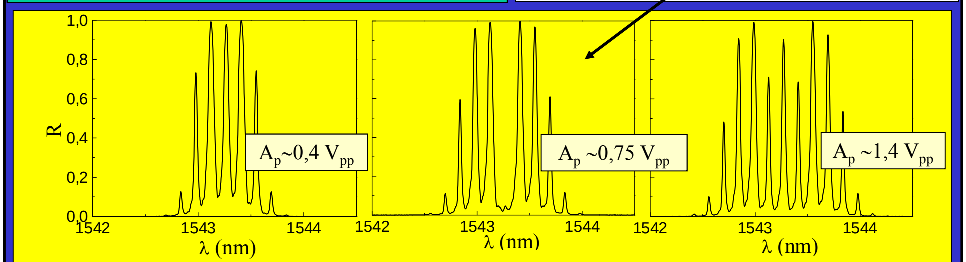
• Longitudinal waves + fiber Bragg gratings: $\Lambda_s << L_{FBG}$

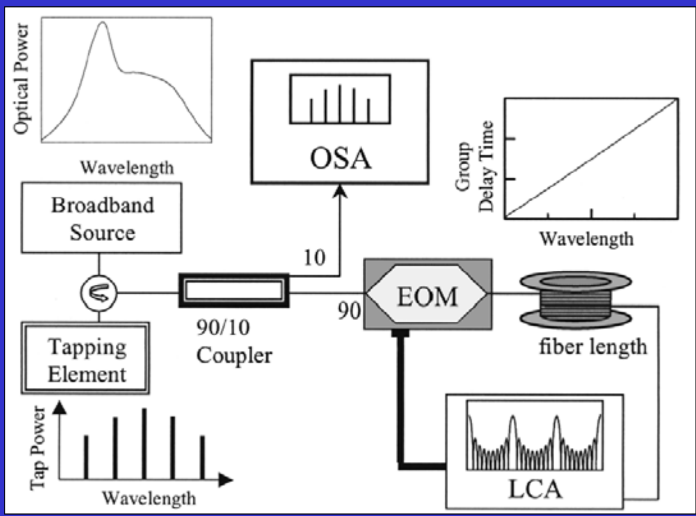
$$R_m = \tanh^2\left[\kappa\, L\, J_m\left(\frac{\lambda_{B0}}{\Delta\lambda_B}\,\varepsilon_0\right)\right]$$

$$\varepsilon_0 = \left[2P_s / EAv_{gs}\right]^{1/2}$$





- **Tunable microwave optical filter based on a broadband source sliced by an acousticaly modulated FBG**



Opt. Lett., pp. 8-10, 2005

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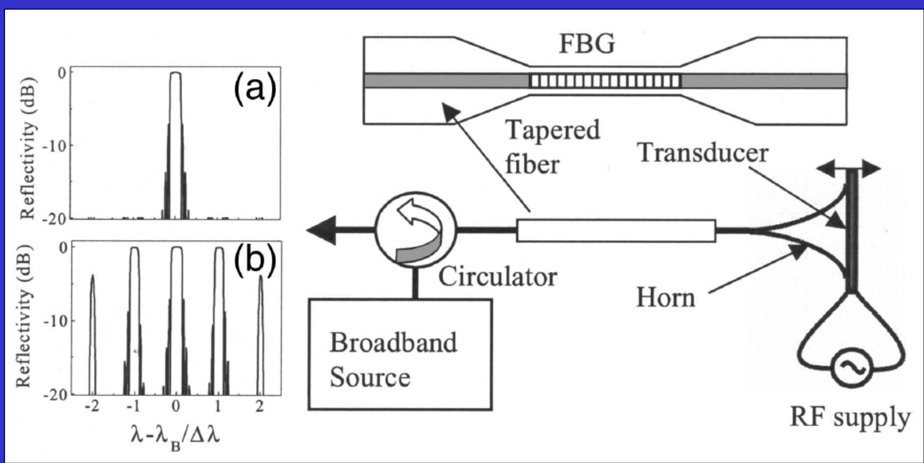
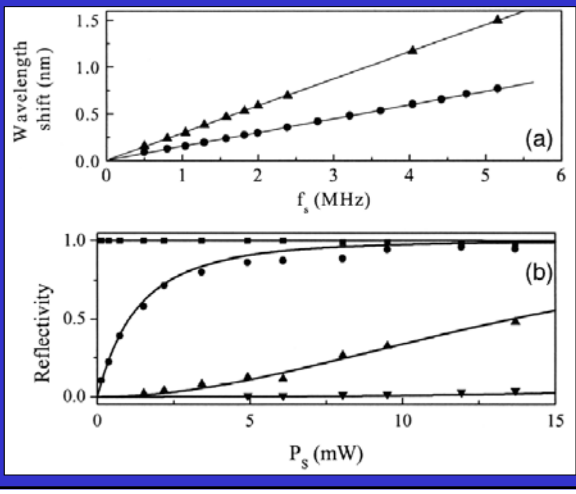
- The reflection from the acousticaly modulated FBG provides the taps required to form the transversal filter



- Tunable: the wavelength separation between taps is adjustable with the frequency of the acoustic wave
- Reconfigurable: the amplitude of the taps can be controlled with the amplitude of the acoustic wave



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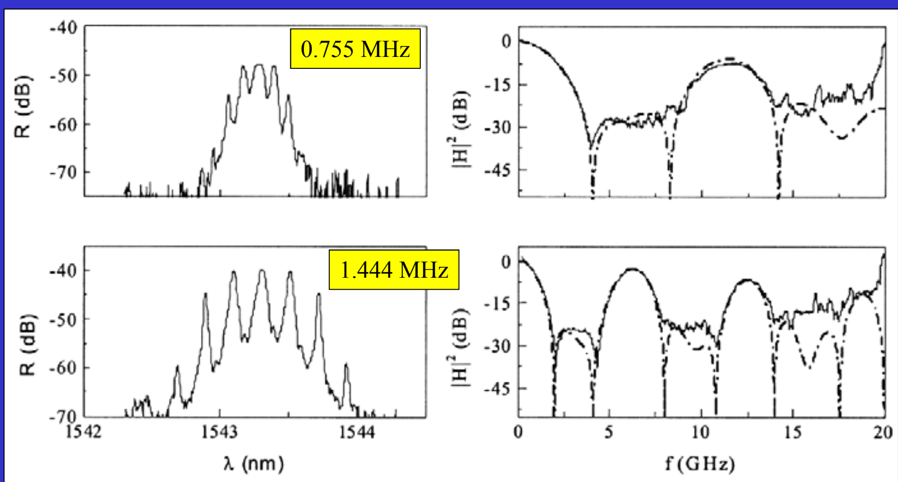
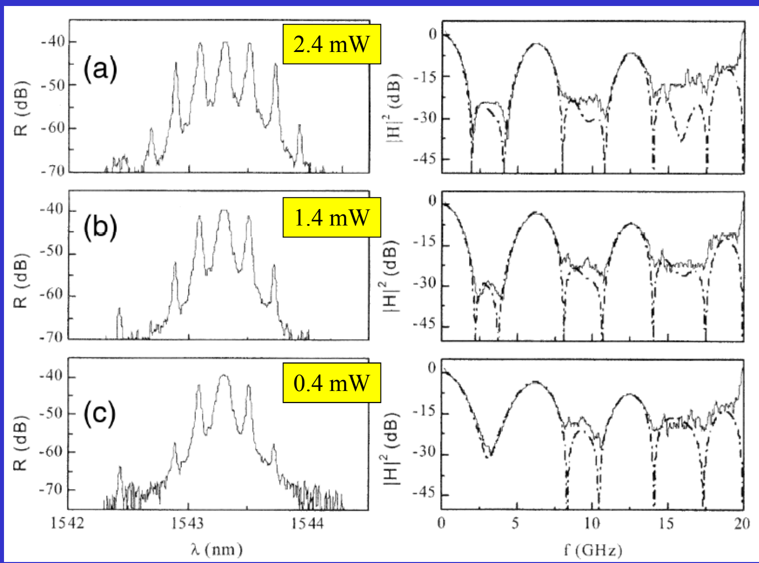
- The transmission bands of the filter are tunable



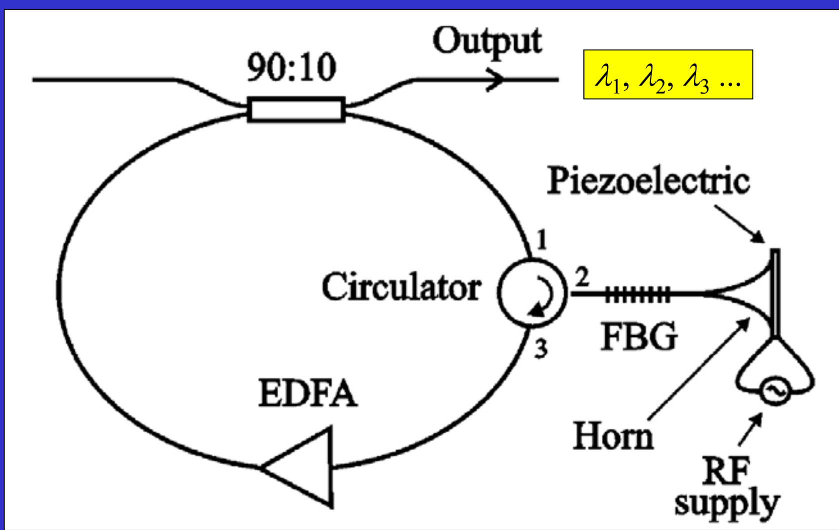
Constant amplitude: acoustic power 2.4 mW

- The relative weight of the taps can be changed



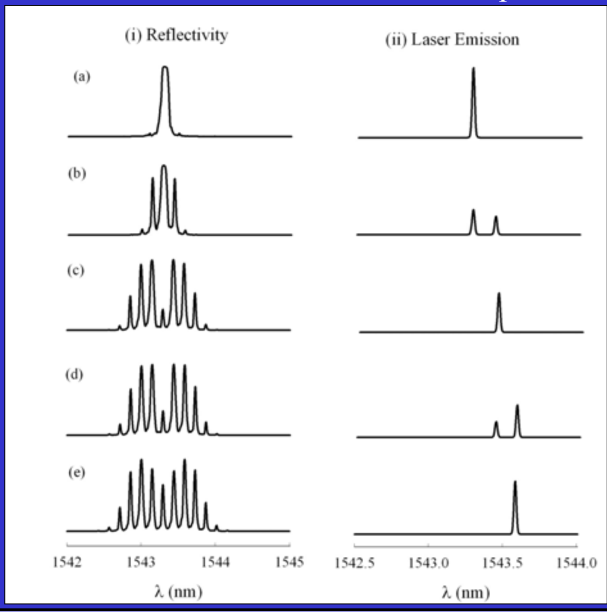
Constant frequency: 1.444 MHz

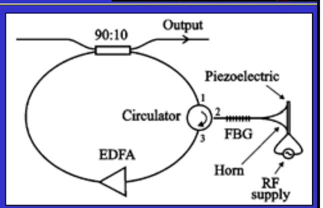
- **Wavelength switchable fiber laser**



Photon. Technol. Lett., pp. 552-554, 2005.

- Emission as a function of acoustic amplitude





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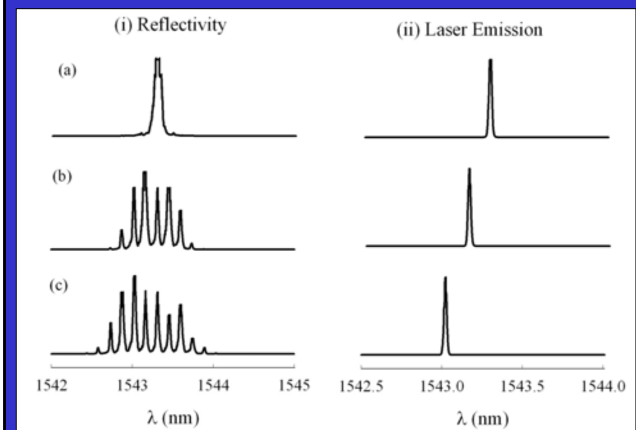
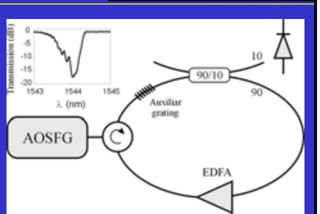
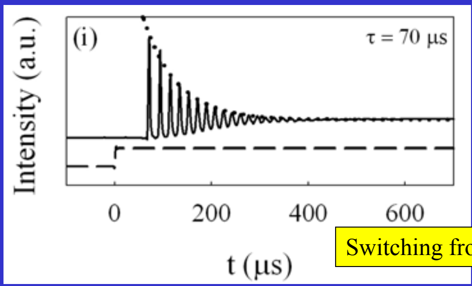
- Emission as a function of acoustic amplitude

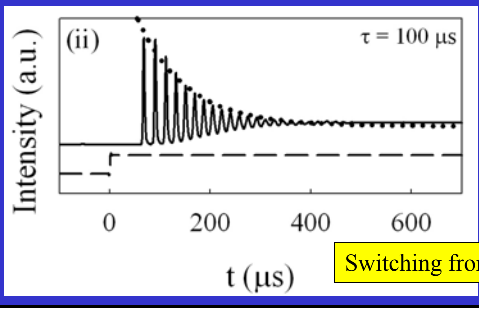




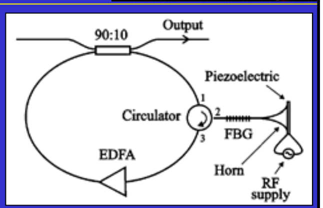
• Emission as a function of acoustic amplitude



Switching from order 0 to order 1



Switching from order 0 to order 2



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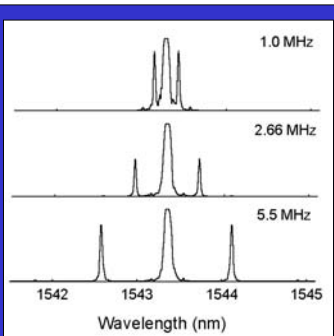
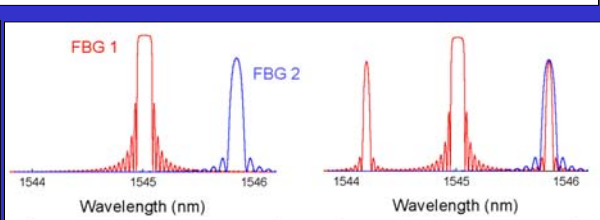
• **Q-switched fiber laser**

Er fibre : 300 ppm, L ~1 m, FBG2: 60%, 0.05 nm, FBG1: 99%, 0.25 nm





Wave OFF ➔ Low Q-factor

Wave ON ➔ High Q-factor

Opt. Express, pp. 1106-1112, 2006.

• Spectrum, peak power and pulse width