

Convolution Integral Equations

with special function kernels

VOLTERRA

H. M. SRIVASTAVA

Professor of Mathematics

University of Victoria, British Columbia, Canada

R. G. BUSCHMAN

Professor of Mathematics

University of Guelph, Ontario, Canada

28434



WILEY EASTERN LIMITED

New Delhi • Bangalore

1977

ISBN 0 85226 834 3

CHAPTER 3

METHODS AND EXAMPLES

3.1 Rodrigues' formula	33
3.2 Resolvent kernel	35
3.3 Laplace transformation	36
3.4 Mellin and other transformations	40
3.5 Fractional integrals	42
3.6 Mikusiński's operators	45
3.7 Other methods	46

CHAPTER 4

MISCELLANEOUS RESULTS

4.1 Some immediate results from tables of integral transforms	50
4.2 Simplifications of generalized hypergeometric function kernels	53
4.3 Confluent hypergeometric functions of several variables	63
4.4 Some open questions	67

APPENDIX List of symbols	69
--------------------------	----

INVERSION TABLES	73
------------------	----

BIBLIOGRAPHY	143
--------------	-----

AUTHOR INDEX	158
--------------	-----

SUBJECT INDEX	161
---------------	-----

41.7
PRI
92

Theory and Applications of Convolution Integral Equations

by

H. M. Srivastava

*Department of Mathematics and Statistics,
University of Victoria,
Victoria, British Columbia, Canada*

and

R. G. Buschman

*Department of Mathematics,
College of Arts and Sciences,
University of Wyoming,
Laramie, Wyoming, U.S.A.*

By the late Dr. H. M. Srivastava, it is well known that convolution integral equations have been extensively studied and applied in the physical and engineering sciences. This book is a comprehensive treatment of the theory and applications of convolution integral equations. It is written in a clear and concise style, and is suitable for use as a textbook or as a reference work.

The book is divided into two parts. The first part deals with the theory of convolution integral equations, and the second part deals with their applications. The theory part covers the basic properties of convolution, the existence and uniqueness of solutions, and the methods of solution. The applications part covers the use of convolution integral equations in the theory of probability, the theory of statistics, and the theory of queueing systems.

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

1992

ISBN 0 7923 1891 9

TABLE OF CONTENTS

PREFACE	xi
ACKNOWLEDGEMENTS	xiv
INTRODUCTION	xv
CHAPTER 1. PRELIMINARIES ON SPECIAL FUNCTION KERNELS	
1.1. Algebraic kernels	1
1.2. Exponential, logarithmic, and trigonometric kernels	2
1.3. Chebyshev polynomials	3
1.4. Legendre polynomials and functions	4
1.5. Gegenbauer and Jacobi polynomials	5
1.6. Laguerre and Hermite polynomials	6
1.7. Bessel functions	7
1.8. Kummer's and Whittaker's functions	8
1.9. Gauss' hypergeometric function	9
1.10. Generalized hypergeometric and other functions	10
1.11. Further integral equations with multivariable kernels	13
CHAPTER 2. BASIC PROPERTIES AND THEOREMS	
2.1. Convolutions and relations among equations	15
2.2. The Theorem of Titchmarsh and the uniqueness of solutions	19
2.3. Associated integral transformations	20
2.4. Associated fractional integrals	21
2.5. Simple kernel variations	24
CHAPTER 3. METHODS AND ILLUSTRATIVE EXAMPLES	
3.1. Rodrigues' formula	26
3.2. Resolvent kernel	27
3.3. Laplace transformation	29
3.4. Mellin and other transformations	42
3.5. Fractional integrals	45
3.6. Mikusiński operators	48
3.7. Other methods	53

CHAPTER 4. MISCELLANEOUS RESULTS AND OPEN QUESTIONS

4.1. Some immediate consequences from tables of integral transforms.....	56
4.2. Simplifications of generalized hypergeometric kernels.....	59
4.3. Confluent hypergeometric functions of several variables.....	64
4.4. Some open questions.....	68

CHAPTER 5. EQUATIONS OF THE SECOND AND OTHER KINDS

5.1. General properties of equations of the second kind.....	70
5.2. Algebraic kernels.....	72
5.3. Exponential, trigonometric, and hyperbolic kernels.....	73
5.4. Higher transcendental functions.....	75
5.5. Integro-differential equations.....	76
5.6. Equations of the third kind.....	77
5.7. Non-linear equations.....	79

CHAPTER 6. CONVOLUTIONS OVER OTHER INTERVALS

6.1. Convolution over (x, ∞)	81
6.2. Equations on the interval (x, ∞)	84
6.3. Equations on the interval $(0, \infty)$	86
6.4. Equations on the interval $(-\infty, +\infty)$	100
6.5. Equations on the interval $[a, b]$	101

APPENDIX. LIST OF SYMBOLS..... 104

INVERSION TABLES. INDEX..... 114

TABLE 1. $\int_0^x k(x-t)f(t)dt = g(x)$	116
TABLE 2. $g(x) + \int_0^x k(x-t)f(t)dt = f(x)$	158
TABLE 3. $\int_x^\infty k(t-x)f(t)dt = g(x)$	167
TABLE 4. $\int_0^x k(x/t)f(t)dt = g(x)$	172
TABLE 5. The forms $\int_0^\infty k(xt)f(t)dt$ and $\int_0^\infty t^{-1}k(x/t)f(t)dt$	180
TABLE 6. The form $\int_0^\infty k(x-t)f(t)dt$	187
TABLE 7. The form $\int_{-\infty}^{+\infty} k(x-t)f(t)dt$	189
TABLE 8. The form $\int_a^b k(x-t)f(t)dt$	192

Table of Contents

ix

BIBLIOGRAPHY 195

AUTHOR INDEX 230

SUBJECT INDEX 234