4.1

PROBLEMAS INVERSOS EXPLICITOS, ANTECEDENTES HISTORICOS.

Inverse and Ill-Posed Problems

Edited by Heinz W. Engl C. W. Groetsch

ACADEMIC PRESS, INC.

Preface

Inverse problems of mathematical physics may be broadly described as problems of determining the internal structure or past state of a system from indirect measurements. Such problems would include for example the determination of diffusivities, conductivities, densities, sources, geometries of scatterers and absorbers, and prior temperature distributions to name just a few typical applications. Inverse problems appeared as early as 1765 with Daniel Bernoulli's study of the inhomogeneous vibrating string; however, only recently has a systematic treatment of such problems begun to emerge. Developments in this area have been spurred by the needs of modern technology for reliable means of indirect measurement. At the same time, the main tool necessary for the solution of inverse problems has been supplied by modern technology in the form of the digital computer.

Many inverse problems can be modeled abstractly as $Kx = g$, where K is a given operator between appropriate function spaces, g is given by measured "external" parameters, and the desired solution x represents "internal" parameters, which are inaccessible to direct measurement.

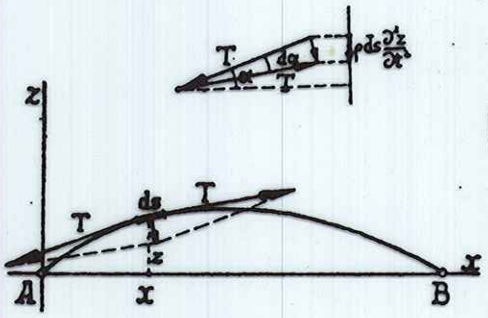
— NUEVO

LUEGO VEREMOS QUE SON LOS QUE
VAMOS A ESTUDIAR EN ESTE CURSO

2. La ecuación de la cuerda vibrante.— El primer ejemplo de problema con condiciones mixtas que vamos a desarrollar va a ser el estudio de las vibraciones de una cuerda tensa sujeta por los extremos A, B (cuerdas de los instrumentos musicales: piano, violín, guitarra, etc.).

Esta cuerda se aparta inicialmente ($t=0$) de su posición de equilibrio según una curva dada de ecuación $z=f(x)$ (referida a un eje x coincidente con la posición de equilibrio y a un eje z perpendicular) origen en un extremo A de la cuerda y se suelta sin velocidad inicial o, más generalmente, dando a cada punto una velocidad inicial en dirección del eje z que supondremos conocida $v(x)$.

Se trata de averiguar la ecuación $z=z(x, t)$ que da el movimiento de cada uno de sus puntos o, lo que es lo mismo, la configuración de la cuerda en función del tiempo. Como se ve, este problema es caso particular del de Cauchy, puesto que se conocen la curva inicial $z(x, o)=f(x)$ y la velocidad inicial $z_t(x, o)=v(x)$. Pero se añade a estas condiciones iniciales la sujeción en los extremos A, B, es decir, las condiciones de contorno $z(o, t)\equiv 0$, $z(l, t)\equiv 0$, siendo l la longitud de la cuerda.



Despreciando el peso de la cuerda frente a la tensión y a las fuerzas de inercia, en todo momento debe existir equilibrio entre las tensiones T aplicadas en los extremos de un elemento de cuerda ds y la fuerza de inercia cuya proyección sobre el eje z se formulará (llamando z a la distancia de dicho elemento a su posición de equilibrio, ρ a la masa por unidad de longitud) $\rho\, ds \dfrac{\partial^2 z}{\partial t^2}$, usando la notación de derivada parcial para indicar la derivación con respecto al tiempo, ya que z depende asimismo de la distancia x del elemento considerado al extremo A de la cuerda. Llamando, finalmente, α al ángulo que forma la tangente a la curva adoptada por la cuerda al deformarse, se tendrá, proyectando el triángulo de equilibrio (v. figura) sobre la vertical

$$T \operatorname{sen} \alpha - T \operatorname{sen} (\alpha - d\alpha) = \rho\, ds \frac{\partial^2 z}{\partial t^2}$$

Sustituyendo sen α por α, toda vez que se trata de ángulos muy pequeños, quedará

$$\frac{T}{\rho} \frac{d\alpha}{ds} = \frac{\partial^2 z}{\partial t^2}$$

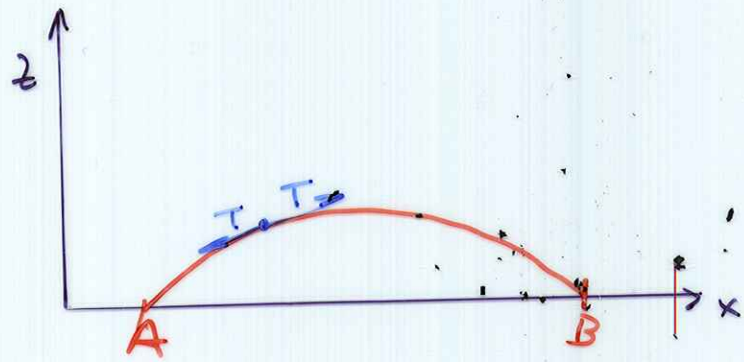
pero $\dfrac{d\alpha}{ds}$ = curvatura $= z''_{xx}$ en la hipótesis de pequeñez del ángulo α (véase tomo I, lecc. 31); por consiguiente, quedará la ecuación

$$a^2 \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2} \qquad [1]$$

donde la constante $a^2 = T : \rho$ es esencialmente positiva (*). Las soluciones de esta ecuación darán las configuraciones $z(x, t)$ de la cuerda al variar t.

Ahora bien, para la integración de esta ecuación podemos seguir dos caminos

CUERDA VIBRANTE



CADA PUNTO ESTÁ SOMETIDO A UNA TENSIÓN T (la misma para todos) POR AMBOS LADOS.

Ecuación

$$a^2 \frac{\partial^2 z(x,t)}{\partial x^2} = \frac{\partial^2 z(x,t)}{\partial t^2}$$

$a^2 = T/\rho$ POSITIVA SIEMPRE

CONDICIONES DE CONTORNO (LIMITES)

A PARA x_A $z = 0$ SIEMPRE

B PARA x_B $z = 0$ SIEMPRE

$$a^2 \frac{\partial^2 z(x,t)}{\partial x^2} = \frac{\partial^2 z(x,t)}{\partial t^2}$$

$F(x,\omega)$ es la T. Fourier $(t \to \omega)$ de $z(x,t)$

$$F(x,\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} z(x,t)\, dt$$

como

$$\int_{-\infty}^{+\infty} e^{-i\omega t} \frac{\partial}{\partial t} z(x,t)\, dt = i\omega \int_{-\infty}^{+\infty} e^{-i\omega t} z(x,t)\, dt$$

$$= i\omega F(x,\omega)$$

tomando la T.F. en la ecuación de la cuerda vibrante

$$a^2 \frac{\partial^2 F(x,\omega)}{\partial x^2} = -\omega^2 F(x,\omega)$$

RECUPERAMOS LA ECUACIÓN QUE TENÍAMOS PARA LAS ONDAS CUÁNTICAS PLANAS LIBRES: SIN SCATTERING POR UN POTENCIAL $\rho(x)$

EN REALIDAD POCO

PROBLEMA INVERSO DE ESTRUCTURA APARECE AQUÍ.

SE PUEDE CONCEBIR A PARTIR DE UNA SITUACIÓN ACTUAL, DETERMINAR PROPIEDADES DE LA CUERDA:

EXTERNAS T TENSIÓN APLICADA

INTERNAS ρ DENSIDAD

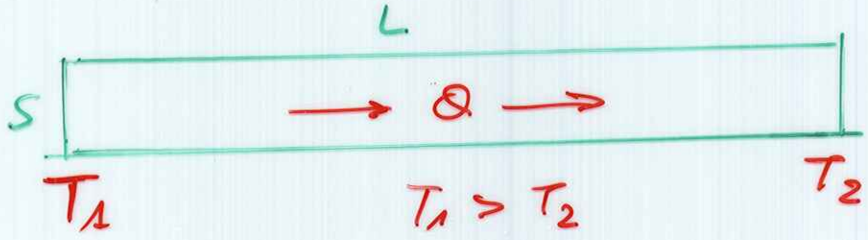
$$a^2 = T/\rho$$

PERO PUEDE PLANTEARSE EL PROBLEMA INVERSO DE DETERMINAR LAS CONDICIONES INICIALES A PARTIR DE LA MEDIDA / OBSERVACIÓN, DE UNA SITUACIÓN ACTUAL

EN REALIDAD, SI NO HAY PERTURBACIONES DE LAS ONDAS, EL PROBLEMA DEBIERA SER REVERSIBLE .... !!!

DIFUSIÓN DEL CALOR

CILINDRO AISLADO DE SECCIÓN S
Y LONGITUD L



LA CANTIDAD DE CALOR QUE SE PROPAGA POR UNIDAD DE TIEMPO

$$Q = \lambda \frac{S}{L} (T_1 - T_2)$$

$$\boxed{dQ = -\lambda \frac{dT}{dx} \, ds \, dt}$$

FOURIER

$\lambda = 1$ caloria /cm K s   cobre

$\lambda = 0.15$ calorias/cm K s   Hierro

$\lambda = 4 \cdot 10^{-3}$ calorias /cm K s   Ladrillo.

COMO VEREMOS SE TRATA DE UN PROBLEMA DE PLANTEO DIFERENCIAL Y TAL QUE LA SOLUCIÓN BUSCADA ES UNA TRANSFORMADA INTEGRAL

CALOR QUE EN UN dt SE QUEDA ENTRE x y $x+dx$

$\xrightarrow{dQ_x}$ $\qquad\qquad$ $\xrightarrow{dQ_{x+dx}}$

x $\qquad\qquad$ $x+dx$

$$dQ_x = -\lambda \left(\frac{\partial T}{\partial x}\right)_x dy\, dz\, dt$$

$$dQ_{x+dx} = -\lambda \left(\frac{\partial T}{\partial x}\right)_{x+dx} dy\, dz\, dt$$

$$\Delta Q = \lambda \frac{\partial^2 T}{\partial x^2} dx\, dy\, dz$$

$$= \lambda\, \Delta T\, dx\, dy\, dz$$

SE INVERTIRÁ EN AUMENTAR LA TEMPERATURA

$$\Delta Q = C_e M\, \Delta T$$

$$= C_e\, \rho\, dx\, dy\, dz\, \frac{\partial T}{\partial t} dt$$

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2}$$

$$D = \frac{\lambda}{C_e \rho}$$

$$\frac{\partial T}{\partial t} = D\, \Delta T$$

SOLUCIÓN DE

$$\frac{\partial T}{\partial t} = D\,\frac{\partial^2 T}{\partial x^2}$$

FACTORIZACIÓN

$$T(x,t) = \Theta(t)\,.\,R(x)$$

$$R(x)\,\frac{d\Theta(t)}{dt} = \Theta(t)\,\frac{d^2R(x)}{dx^2}\,D$$

$$\frac{1}{\Theta(t)}\,\frac{d\Theta(t)}{dt} = D\,\frac{1}{R(x)}\,\frac{d^2R(x)}{dx^2} = \pm D K^2$$

DK^2 CONSTANTE INDEPENDIENTE DE t, x

$$\Theta(t) = A\,e^{-DK^2 t}$$

SIGNO $-$ YA QUE NO HABIENDO FUENTES DE CALOR LA TEMPERATURA NO PUEDE CRECER INDEFINIDAMENTE

$$\frac{d^2R(x)}{dx^2} + K^2R(x) = 0 \qquad \text{LAPLACE}$$

$$R(x) = C_K^+\,e^{ikx} + C_K^-\,e^{-ikx} \qquad K \geq 0$$

$$R(x) = C_K\,e^{ikx} \qquad K>0 \text{ ó } K<0$$

ENTONCES

$$T(x,t) = \int_{-\infty}^{+\infty} dk \; C(k) \, e^{ikx} \, e^{-Dk^2 t}$$

CONDICIONES INICIALES

$t = 0 \qquad T(x,0) \equiv T_0(x) \qquad$ DATO

$$T_0(x) = \int_{-\infty}^{+\infty} dk \; C(k) \, e^{ikx}$$

FOURIER

$$\rightarrow \qquad C(k) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dx' \; T_0(x') \, e^{-ikx'}$$

LUEGO

$$T(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dx' \; T_0(x') \int_{-\infty}^{+\infty} dk \; e^{ik(x-x')} \, e^{-Dk^2 t}$$

LA ULTIMA INTEGRAL

$$= \sqrt{\frac{\pi}{Dt}} \; e^{-\frac{(x-x')^2}{4Dt}}$$

$$T(x,t) = \frac{1}{2\sqrt{\pi D t}} \int_{-\infty}^{+\infty} T_0(x') \, e^{-\frac{(x-x')^2}{4Dt}} \, dx'$$

PERO, POR SU IMPORTANCIA HISTÓRICA

1.1. Newton's Problem. One of the first inverse problems of dynamics which was solved in the past is Newton's problem on the determination of forces, under the action of which planetary motion takes place with the following properties (Kepler's laws) [3]:

A. The Sun is situated at one of the foci of the ellipses, which are the orbits of planetary motion.

B. The planets have a constant sector velocity.

C. The square of the time taken by a planet to complete a revolution around the Sun is proportional to the cube of the semi-major axis of its orbit.

It is well known that the solution of Newton's problem is obtained in the following way.

From the first two properties of motion, it immediately follows that the force is central, with the centre of field at the focus of the ellipse where the Sun is situated. The magnitude and the direction of the force causing the planet to move with given properties are determined by using the equation of motion of a particle in a central force field (in Binet's form). This force turns out to be attractive, directly proportional to the mass of the planet, and inversely proportional to the square of the distance between the centres of the Sun and the planet. Further, the third property of the motion of the planet is used to express the constant of proportionality in terms of universal constants (the mass of the Sun and the gravitational constant), and we get the familiar expression for the unknown force.

4.2 P.I. EXPLICITOS.

SCATTERING DE ONDAS.

ORIGINAL PUBLICATION INDEX

(The titles are in the language of original publication.)

1. "Über eine Frage der Eigenwerttheorie," *Z. f. Phys.*, vol. 53, pp. 690–695, 1929.

A PROBLEM IN THE THEORY OF EIGENVALUES

In a certain special case (oscillating string, the natural boundary conditions) the spectrum of eigenvalues determines uniquely the differential equation to which it corresponds (in Schrödinger's theory, the "equation of amplitudes").

In those fields of theoretical physics (wave mechanics, theory of oscillations) where eigenvalue problems arise, the question of the uniqueness of determination of the mechanical system (i.e., of the Hamiltonian) by the set of the eigenvalues of the corresponding linear equation can be important. If a spectrum really completely determines the differential equation, then in principle it would become possible to determine the structure of an atomic system from the frequencies it is radiating or absorbing. This would mean solving a problem which is inverse to the Schrödinger problem. However, an approach to the general problem leads to many difficulties. Therefore, below we consider only a special case.

We prove that among all equations

$$\mu \frac{d^2\varphi}{dx^2} - q(x)\,\varphi + \alpha\varphi = 0,$$

where α is a "parameter" of eigenvalues, μ is a constant, $q(x)$ is a continuous function, for "natural boundary conditions"

$$\varphi'(0) = \varphi'(\pi) = 0$$

only the equation of the oscillating string

$$\mu \frac{d^2\varphi}{dx^2} + \alpha\varphi = 0$$

has the eigenvalues

$$\alpha_n = \kappa n^2.$$

Methods of Inverse Problems in Physics

APARECE HACIA 1930
EL PROBLEMA DE
"SCATTERING DE ONDAS"

Author

Dilip N. Ghosh Roy
Technical Research Associates
Salt Lake City, Utah

CRC Press
Boca Raton Ann Arbor Boston

1991

Chapter 1

INTRODUCTION

It was perhaps John Tyndall[1] who observed more than a century ago the first scientific scattering of light by particles smaller in size than the wavelength of the incident radiation. Since Tyndall's time the science of scattering of waves and particles has undergone a long development. Rayleigh's[2] analysis of the bluish hue in the scattered light observed by Tyndall, Mie's[3] theory of scattering of electromagnetic waves by spherical particles, Einstein's[4] and Smolouchowski's[5] theory of light scattering by dielectric fluctuations caused by thermal agitation, Compton's[6] scattering of X-rays by atoms, Thomson's[7] scattering of plane electromagnetic waves by free electrons, well known phenomenon of neutron scattering,[8-10] scattering of waves and particles by quantum mechanical potentials,[11-15] Mandelshtam-Brillouin[16-18] scattering of light by acoustic phonons, and finally, Raman[19] scattering of photons by molecular vibrations that results in the frequency shift related to the internal dynamics of the molecules are some of the landmarks in this development. Sophisticated theories have been developed by Landau-Placzek,[20] Rytov,[21] and Komarov and Fisher[22] to explain the structures of the Mandelshtam-Brillouin doublet and of the nonpropagating entropy fluctuations in the Rayleigh lines. With the introduction of laser sources, the classical, spontaneous scattering of light has undergone dramatic advances both theoretically and experimentally, resulting in a number of powerful spectroscopic and other diagnostic techniques. The literature in this area is too extensive to mention in any detail here. However, an exhaustive account of these developments can be found in References 23 to 27. With the development of the Q-switched, pulsed, high-power laser pumps, stimulated scattering of light came into being. Almost every spontaneous scattering phenomenon was found to have its stimulated counterpart such as stimulated Brillouin scattering,[28-30] stimulated Raman scattering,[31-33] stimulated Rayleigh scattering,[34-35] and stimulated polariton scattering.[36] For details of these nonlinear optical phenomena, the reader is referred to Bloembergen,[37-38] Shen,[39] and a review article by Kaiser and Maier.[40] Scattering of acoustic waves has also undergone a lengthy development. The literature on acoustical scattering is extensive and reference is made to Morse and Ingard[41] and Beyer.[42]

Scattering is a very important tool in studying the structure and dynamics of matter and radiation, and there is hardly any important discipline of science and engineering that does not use scattering as an investigative probe. Whether in a highly sophisticated quantum mechanical problem of potential scattering or in the mundane task of locating a crack in an automobile crankshaft, method of scattering finds universal application. Fields of research as diverse as astrophysics and turbulence, plasma physics and flaw evaluation, quantum physics and medical imaging, lasers and seismology, nuclear physics and aerosol theory — the list can be extended easily — all use scattering phenomenon in one form or another in order to extract information about the system concerned. "Direct" or "forward" scattering involves determining the scattered fields or particles when the scattering potential or the distribution of the scatterers is known. The "inverse" or "backward" problem, on the other hand, consists of finding the distribution of the scatterers or the nature of the potential from the information on the scattered fields or particles. It is this backward scattering problem that has received by far the most attention in recent years and has been applied in almost all major branches of science and engineering.

Consider the equation

I. STRUCTURE OF AN INVERSE PROBLEM

Consider the equation

$$y''(x, k) + \left[k^2 - q(x)\right] y(x, k) = 0 \tag{1.1}$$

x is the space variable that may belong to either R^+, the positive half-line, or to R, the entire real axis. k may be real or complex. Appropriate boundary conditions must, of course, be prescribed. Equation 1.1 describes the scattering of plane waves by a "potential" q(x), the dependent variable y(x,k) constituting the "scattered" field. The direct problem associated with Equation 1.1 can be stated as follows. Given the function q(x), find the solution y(x,k) satisfying certain prescribed boundary conditions. The inverse problem, on the other hand, is the reverse: given a certain "data" associated with the solution y(x,k) satisfying certain boundary conditions, determine the function q(x). The direct problem consists of finding the solution of Equation 1.1 under prescribed conditions. On the other hand, the inverse problem is to reconstruct the differential equation *cum* boundary conditions from the data.

The formulation of a direct problem involves setting up a certain number of functions. One part of these functions determines the differential equation, for example, through the determination of the coefficients in the differential operator, while the other part involves determining the initial and boundary conditions of the problem. The end result of a direct problem is a new function, the solution, determined by this given set of functions. In an inverse problem, some of these functions may be unknown and are to be determined. This requires additional information on the solution of the direct problem in place of these unknown functions. The additional information may consist of the solution itself, defined on a certain set of independent variables, or some functionals of the solution.

The structure of an inverse problem may be described as follows. We follow Newton[43] here. A linear differential equation or a system of such equations is given. The system contains a function or functions such as q(x) in Equation 1.1 which are usually real valued. We will call these functions "potentials". Newton calls them "forces". The equation or a system thereof may be the Schrödinger equation with an atomic potential or may represent an electromagnetic problem, a transmission line, a problem of sound propagation in a medium, or a problem in elasticity. The corresponding "potential" may then be an actual atomic potential or an index of refraction, the transmission line impedance, the acoustic speed, the impedance of the elastic medium, or the elastic modulus. The solutions of the differential equation or the system of equations subject to certain prescribed boundary conditions lead to the determination of a set of quantities which can be observed either directly or indirectly such as the reflection or the transmission amplitudes of scattering. Such "observables" will be called the "scattering data" or the "excitation response". The data are usually complex valued and are not functions in the space of the differential equation. Instead, the data may depend on parameters such as frequency, energy or angles, the parameters that occur either in the differential equation itself or in the boundary conditions. This point will be discussed in Chapter 9.

UN PRIMER EJEMPLO MUY CONOCIDO

$$\frac{\partial p}{\partial x} = -[\,A(x)]^{-1}\,\frac{\partial u}{\partial t} \quad ; \quad \frac{\partial u}{\partial x} = -\,A(x)\,\frac{\partial p}{\partial t} \quad ; \qquad (1)$$

$$\frac{\partial^2 p}{\partial t^2} = [\,A(x)]^{-1}\,\frac{\partial}{\partial x}\,A(x)\,\frac{\partial p}{\partial x} \quad . \qquad (2)$$

The equation (2) is readily obtained from eqs. (1) for any solution (p,u) with continuous second order derivatives. A similar one, with A instead of A^{-1}, is obtained for u.

The system of equations (1) or eq. (2) is the telegraph equation, i.e. the Maxwell's equations in a medium, where units have been chosen in such a way that the product inductance - capacitance LC equals 1, and $A = C^{-1}$, p is the voltage, u the current.

It is also the acoustical equation in a tube where p is the pressure, u the volume velocity, and the units are chosen in such a way the velocity of sound $c = 1$, the density of air $\rho = 1$.

OTRO EJEMPLO

$$\frac{\partial \tilde{p}}{\partial x} = -[A(x)]^{-1} \omega^2 \tilde{w} \quad ; \quad \frac{\partial \tilde{w}}{\partial x} = A(x)\, \tilde{p} \qquad (3)$$

The system of equations (3) is obtained from eqs. (1) by setting $u = -\int i\omega e^{i\omega t}\, \tilde{w}\, dt$, and $p = \int e^{i\omega t}\, \tilde{p}\, dt$.

INDEPENDIENTEMENTE DE QUE LAS FUNCIONES DE UN SISTEMA SEAN LAS T.F. DE LAS FUNCIONES DEL OTRO

It governs the propagation of a stress wave in an elastic medium, where w is the displacement and p the isotropic stress.

If we assume that $A(x)$ is strictly positive with $p = \varphi / A(x)^{-1/2}$, eq (2) leads

$$\frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial^2 \varphi}{\partial x^2} - \{[A(x)]^{-1/2} \frac{d^2}{dx^2} [A(x)]^{1/2}\}\varphi \quad , (4)$$

which is a regular hyperbolic equation, in which the signal velocity is 1.

OTRO EJEMPLO

The equation

$$\frac{\partial^2\tilde{\varphi}}{\partial x^2} + [k^2 - q(x)]\tilde{\varphi} = 0 \qquad (5)$$

can be obtained from eq. (4) by setting

$\varphi = \int \tilde{\varphi}\, e^{i\omega t}\, dt$ and setting $q(x)$ by the formula

$$[\frac{d^2}{dx^2} - q(x)]\,[A(x)]^{1/2} = 0 \qquad (6)$$

The equation (5) governs the time-harmonic amplitude $\tilde{\varphi}(k,x)$ of the horizontally polarised electromagnetic field in an horizontal stratified ionised region. Of course, (5) is also the Schrödinger equation.

In the quantum scattering theory, the time domain formulation is scarcely used. Measurements give scattering cross sections, which are supposed to give the scattering matrix $S(k)$ (or the reflection coefficient $R(k)$ in the case of scattering on the line).

LES VIBRATIONS PROPRES DE LA TERRE

Comme tout corps élastique fini (corde vibrante, peau de tambour, cloche, etc.) la Terre est susceptible de mouvements stationnaires, appelés vibrations propres. Si pour un déplacement $u(M)\,e^{i\omega t}$, la force de cohésion, due à l'élasticité du milieu, est donnée par : $L(u)\,e^{i\omega t}$, une vibration propre de période $2\pi/\omega$ correspond à un alignement de la force et du déplacement;

$$L(u) = -\rho\omega^2 u. \tag{6}$$

Les conditions aux limites correspondantes sont l'annulation des contraintes à la surface libre, la continuité du déplacement et du vecteur contrainte sur toute surface de discontinuité en milieu solide, l'annulation du cisaillement à un interface solide-liquide. Pour un modèle de Terre à symétrie sphérique, dépourvu de rotation, on peut montrer que deux types d'oscillations sont possibles : les *modes sphéroïdaux*, correspondant à des mouvements sans rotation des particules autour de la verticale correspondant à chacune d'elles; les *modes toroïdaux*, sans déplacement vertical ni changement de volume.

FORMA DE TRABAJAR

5. El problema de la adaptación a las condiciones iniciales y de contorno. Valores propios y funciones propias.—Una vez obtenidas las soluciones matemáticas de tipo particular que satisfacen la ecuación diferencial,

- y que dependen de ciertos parámetros* -

VALORES PROPIOS

el físico matemático tiene que resolver aún una cuestión con frecuencia de mucha mayor dificultad: la de *seleccionar* y *combinar* adecuadamente estas soluciones hasta conseguir la que se adapte a las condiciones iniciales y de contorno de cada caso concreto.

1.° Averiguar para qué valores de los parámetros introducidos en el cálculo (α, β, o bien δ, ω, μ, ν) son posibles las condiciones fijadas en los límites, o en el contorno.

Estas condiciones establecen una *selección de valores posibles* de dichos parámetros (*valores propios* o *autovalores* del problema) y de las correspondientes soluciones (*funciones propias* o *autofunciones*).

2.° Combinar linealmente estas soluciones hasta conseguir la que se adapte a la distribución inicial

I. Si el conjunto de *valores propios* de $\alpha\beta$ es una sucesión numerable $\alpha_1\beta_1$, $\alpha_2\beta_2$, $\alpha_3\beta_3$, ... se procura conseguir la adaptación mediante una suma o una serie uniformemente convergente de soluciones propias.

En el caso de ser α ó β imaginarias se obtienen series de términos trigonométricos, muy frecuentes en las soluciones de muchas cuestiones de la Física matemática, de modo que el mencionado problema de adaptación a las condiciones iniciales es un problema de análisis armónico.

II. Si el conjunto de valores propios es continuo se pueden ensayar soluciones en forma de integral (paso al límite continuo) de las soluciones propias.

Más precisiones en la definición, cuando se precisen más los problemas.

QUANTUM SCATTERING

The problem of inverse scattering has received by far the most attention in quantum mechanics. The fundamental problem here is to reconstruct the potential from the scattering data which reduces to reconstructing the Schrödinger equation from the asymptotic information on its solutions. For scattering on the entire real axis, the scattering data consist of the "spectral transform" of the potential which contains information of the continuous spectrum (via the reflection coefficient), of the bound states (through the point spectrum) and the bound state normalization constants. For scattering on the half-axis (radial problem), it is the information of the partial-wave phase shifts and the bound states that constitute the scattering data. The physical picture here is that of a rectilinearly propagating energy E directed parallel to a vector k and incident upon a central field of force V(r). In quantum mechanics, such a propagation is described in terms of the momentum eigenfunction exp(ik·r), which is a plane wave with k as its wave vector. The scattering is, therefore, that of a plane wave from the spherically symmetric potential V(r). The process is described by a stationary scattering wavefunction which is expanded in terms of the spherical harmonics. The components of this expansion depend upon the angular momentum quantum number l (in addition to the momentum k) and are called "partial waves" of angular momentum l. Expanding the momentum eigenfunction exp(ik·r) in terms of the spherical harmonics and comparing with the partial wave expansion of the scattering wavefunction shows that the effect of the central potential is to introduce phase shifts (relative to the plane wave) in the outgoing scattered waves. The analysis of the scattering data in terms of these partial wave-phase shifts dates back to Faxen and Holtsmark.[63] The questions of whether the phase shifts alone can determine the potential uniquely, whether phase shifts at all angular momentum at one energy or at one angular momentum at all energies are required, or whether the scattering matrix itself is sufficient to determine the potential uniquely when the bound states are present were investigated by Levinson,[64,65] Bargmann,[66,67] Marcenko,[68] and Borg.[69] A brief and insightful account of these pioneering attempts, including that of Ambartsumian[70] in a slightly different context, has been given by Levitan and Gasymov.[49] Although of central importance to the inverse solution of the scattering potential, these uniqueness proofs do not lead to the actual determination of the potential.

* PORQUE LAS ONDAS PLANAS SON LAS SOLUCIONES PROPIAS DE LA ECUACIÓN CUANDO NO HAY POTENCIAL PERTURBADOR (EL QUE PRODUCE EL SCATTERING)

The research reached a new height when Gel'fand and Levitan[48] published their now celebrated, classic paper on the determination of the Schrödinger equation on the half-line from the spectral function. In this famous formalism, the wave function is represented as a linear Volterra integral equation of the second kind in terms of the "free" wave function exp(ikx), i.e., the solutions of Equation 1.1 with the potential q(x) set to zero. Specifically,

$$y(x,k) = \exp(ikx) + \int_x^{\infty} K(x,x')\, y(x',k)\, dx'$$

The kernel K(x,x′) is triangular, independent of k, and known as the "Povzner-Levitan transform"

The Gel'fand-Levitan theory was applied immediately by Jost and Kohn[80] and by Levinson[81] to the inverse scattering problem.

The Jost-Kohn or the Gel'fand-Levitan formalism has been extended to various scattering contexts such as coupled equations,[91-93] spin 1/2 particles,[94] the Klein-Gordon equation,[95] the Dirac equation,[96,97] and to separable potentials.[98]

INCREASING SOLUTIONS OF THE SCHROEDINGER EQUATION

L. D. Faddeev

1. Formulation of the Problem.

Many interesting solutions of the Schroedinger equation $Hu \equiv -\Delta u + v(x)u = \lambda u$ [$x = (x_1, \ldots, x_n)$ is a point in Euclidean space E_n, λ is a complex number] with a decreasing potential $v(x)$ are obtained as the solutions of integral equations of the form

$$u(x) = u_0(x) - \int \Gamma(x-y, \lambda)\, v(y)\, u(y)\, dy, \qquad (1)$$

where $u_0(x)$ is the solution of the equation $\Delta u_0 + \lambda u_0 = 0$, $\Gamma(x, \lambda)$ is the Green's function of the Helmholtz equation

$$\Delta\Gamma(x, \lambda) + \lambda\Gamma(x, \lambda) = -\delta(x).$$

Here $\delta(x)$ is the δ-function at the coordinate origin. The integration in (1) and throughout this paper extends over all E_n unless otherwise stated. u_0 is usually taken in the form of a plane wave $\exp\{i(k, x)\}$, $k = (k_1, \ldots, k_n) \in E_n$, $(k, x) = k_1x_1 + \ldots + k_nx_n$, and $\lambda = k^2 = (k, k)$. Various choices of the Green's function $\Gamma(x, \lambda)$ correspond to various systems of solutions of the equation $Hu = k^2u$.

SOVIET PHYSICS–DOKLADY

VOL. 10, NO. 11 MAY, 1966

Translated from Doklady Akademii Nauk SSSR, Vol. 165, No. 3
pp. 514-517, November, 1965
Original article submitted March 31, 1965

Gel'fand, I. M., and Levitan, B. M. On the determination of a differential equation from its spectral function. [Гельфанд, И. М. и Левитан, Б. М. Об определении дифференциального уравнения по его спектральной функции.] Izv. Akad. Nauk SSSR. Ser. Mat. 15, 309–306 (1951).

American Mathematical Society Translations
Series 2 Vol 1 1955. p. 253

ON THE DETERMINATION OF A DIFFERENTIAL EQUATION FROM ITS SPECTRAL FUNCTION

I. M. GEL'FAND AND B. M. LEVITAN

RUSSIAN MATHEMATICAL SURVEYS

VOLUME XIX

1964

A translation of the survey articles and of selected biographical articles in USPEKHI MATEMATICHESKIKH NAUK

DETERMINATION OF A DIFFERENTIAL EQUATION BY TWO OF ITS SPECTRA

B.M. LEVITAN and M.G. GASYMOV

On the n-dimensional Ambarzumyan's theorem

Hua-Huai Chern and Chao-Liang Shen

Institute of Mathematics, National Tsing Hua University, Hsinchu, Taiwan 30043, Republic of China

Inverse Problems 13 (1997) 15–18. Printed in the UK

RESUMEN PARCIAL { GEL'FAND, LEVITAN 1955
LEVITAN, GASYMOV 1964 }

SI $f_k(\bar{r})$ SATISFACE LA ECUACIÓN DE SCHRÖDINGER PARA EL AUTOVALOR K

$$\Delta f_K(\bar{r}) + [K^2 - V(\bar{r})] f_K(\bar{r}) = 0$$

SATISFACE LA ECUACIÓN INTEGRAL

$$f_k(\bar{r}) = g_k(\bar{r}) + \lambda_k \int_x^\infty K(\bar{r}, \bar{r}') f_k(\bar{r}') d\bar{r}'$$

DONDE $g_K(\bar{r})$ ES UNA SOLUCIÓN DE LA ECUACIÓN HOMOGENEA:

$$\Delta g_K(\bar{r}) + k^2 g_k(\bar{r}) = 0$$

ES DECIR, SIN SCATTERING (SIN POTENCIAL DIFUSOR), O SEA LA ECUACIÓN DE ONDAS LIBRES.

EL NUCLEO $K(\bar{r}', \bar{r}')$

DEPENDE DEL POTENCIAL DIFUSOR
$V(\bar{r})$ Y DE LAS CONDICIONES
DE CONTORNO

SE TRATA DE

PARTIENDO DE LOS VALORES Y
FUNCIONES PROPIAS
QUE SON OBSERVABLES
Y OBSERVADOS

ENCONTRAR (SINTETIZAR)

EL NUCLEO $K(\bar{r}, \bar{r}')$

Y LUEGO EL POTENCIAL
DIFUSOR $V(\bar{r})$

"l'équation de Schrödinger

sur la ligne"

$$\left[- \frac{d^2}{dx^2} + V(x)\right] \varphi(x) = k^2 \qquad (1)$$

Pour k réel, on peut définir l'unique solution de (1) asymptotique à $\exp[ikx]$ pour $x \to \infty$, soit $f_1(k,x)$, et celle asymptotique à $\exp[-ikx]$ pour $x \to -\infty$, soit $f_2(k,x)$. Ce sont les solutions de Jost. Leurs transformées de Fourier, solutions de l'équation de Schrödinger dépendant du temps, sont des ondes progressives qui se propagent respectivement vers $x = +\infty$ et $x = -\infty$. L'étude de (1) permet donc d'écrire

$$T(k)\, f_2(k,x) = f_1(-k,x) + R(k)\, f_1(k,x) \qquad (2)$$

$T(k)$ est le coefficient de transmission de droite à gauche, $R(k)$ le coefficient de réflexion.

La donnée de ce dernier est le "résultat" pris en considération dans le problème inverse.

En premier lieu,

on peut démontrer que la fonction

$$K(x,y) = (2\pi)^{-1}\int_{-\infty}^{+\infty} dk\, \exp[-iky]\{f_1(k,x)-\exp[ikx]\} \quad (3)$$

est nulle pour $x > y$

La formule de transformation inverse donne alors

$$f_1(k,x) = \exp[ikx] + \int_x^{\infty} dy\, \exp[iky]\, K(x,y) \quad (4)$$

qui fait apparaître $K(x,y)$ comme le noyau d'un "opérateur de transformation" faisant passer des solutions de Jost du problème pour $V = 0$

$$\exp[ikx]$$

a celles pour V :

$$f_1(k.x).$$

Pareillement pour $f_2(k.x)$

Maintenant on montre que f_1 (resp. f_2) est solution d'une équation de Volterra, soit en posant

$$F_1(k,x) = \exp[-ikx]\, f_1(k,x)_\infty$$

on peut trouver l'équation

$$F_1(k,x) = 1 - \int_x dy\, L(x,y)\, F_1(k,y) \tag{5}$$

où $L(x,y) = -k^{-1} \sin[k(y-x)] \exp[ik(y-x)]\, V(y)$ (6)

et pareillement pour f_2.

En substituant (5) dans (3) et en réexprimant f_1 à l'aide de (4), on obtient l'équation intégrale ($y > x$)

$$K(x,y) = \frac{1}{2} \int_{\frac{1}{2}(x+y)}^{\infty} ds \left\{ V(s) - \int_0^{\frac{1}{2}(y-x)} dt\, V(s-t) K(s-t, s+t) \right\}. \tag{7}$$

pour la fonction potentiel $V(s)$.

Ainsi le problème se réduit-il à la construction de $K(x,y)$ à partir de $R(k)$, es decir de las medidas de las soluciones que se propagan según los dos sentidos de x, eq. (2)

GENERALIZACION A 5 DIMENSIONES

Geophys. J. R. astr. Soc. (1975) 42, 375–383

Well-posed Inverse Eigenvalue Problems

Victor Barcilon

(Received 1974 September 23)

Without pretending to be exhaustive, let me divide eigenvalue problems into two classes depending upon whether they are associated with ordinary or partial differential equations, i.e. depending upon whether they are related to one-dimensional or multi-dimensional problems.

The following Sturm–Liouville problem:

$$u''+\{\lambda-q(x)\}u = 0$$
$$u(0) = u(1) = 0 \tag{1.1}$$

would fall into the first class. Within this class of one-dimensional problems, some would be of higher order than others. Indeed, we have already mentioned that the eigenvalue problem associated with the spheroidal modes of vibration is of sixth order.

The eigenvalue problem associated with a vibrating membrane, namely

$$\nabla^2 u+\lambda u = 0 \text{ in } D,$$
$$u = 0 \text{ on } \partial D, \tag{1.2}$$

provides the classical example of an eigenvalue problem in the second class.

3. The construction of a solution

Having discussed the uniqueness and existence of the solution of the inverse Sturm–Liouville problem, let me now turn to the actual construction of this solution. There exists at least two such constructions: one due to Krein (1951) and the other due to Gelfand & Levitan (1955). I do not want to go into the construction procedures here. I shall simply say that they do not lend themselves readily to computations, and at least in so far as I could see, they are not amenable to generalization for higher order equations or systems. I was therefore led to look for another procedure for constructing the solution of the inverse Sturm–Liouville problem (Barcilon 1974a). Let me present it by means of a specific example.

Given two admissible spectra $\{\lambda_i\}$ and $\{\mu_i\}$ associated with the following two eigenvalue problems

$$\left.\begin{aligned} u_i'' + \{\lambda_i - q(x)\}u_i &= 0 \\ u_i(0) = u_i(1) &= 0 \end{aligned}\right\} \tag{3.1}$$

and

$$\left.\begin{aligned} v_i'' + \{\mu_i - q(x)\}v_i &= 0 \\ v_i'(0) = v_i(1) &= 0 \end{aligned}\right\} \tag{3.2}$$

construct $q(x)$, i.e. find the functional

$$q = q[\{\lambda_i\}, \{\mu_i\}]. \tag{3.3}$$

The last part of the above sentence was written down facetiously: q is a very complicated functional of the two spectra and it is highly unlikely that an analytic expression for (3.3) can be found.

Geophys. J. R. astr. Soc. (1975) 42, 375–383

Well-posed Inverse Eigenvalue Problems

Victor Barcilon

(Received 1974 September 23)

Perhaps the inverse eigenvalue problem which is best known to geophysicists is that related to the normal modes of vibration of the Earth. It can be stated thus: assuming that the Earth is spherically symmetric and given the natural frequencies ${}_nS_l$ and ${}_nT_l$ of the spheroidal and torsional modes of vibration, can we deduce the density $\rho(r)$ and the speed of propagation $V_p(r)$ and $V_s(r)$ of the P and S waves? Although geophysicists have been able to extract an enormous amount of information from an analysis of normal modes of vibration (see e.g. Jordan & Anderson 1974), from a mathematical point of view the answer to the above question is still not known. The reason lies in the fact that the eigenvalue problem for the spheroidal modes is a sixth-order system of differential equations (see e.g. Alterman, Jarosch & Pekeris 1959; Takeuchi & Saito 1972) and very little is known about inverse eigenvalue problems of order higher than two.

4. Higher order inverse eigenvalue problems

Can some of the results obtained for the inverse Sturm–Liouville problem be generalized to higher order inverse eigenvalue problems? If so, what can be said about the inverse eigenvalue problem for the spheroidal modes?

One of the difficulties associated with higher order eigenvalue problems stems from the fact that the eigenvalues need not be simple.* For the case of simple spectra, I was recently able to generalize Borg's uniqueness theorem and to show that for inverse eigenvalue problems associated with the $2n$-th order operator

$$\mathscr{L} = \frac{d^{2n}}{dx^{2n}} - \frac{d^{n-1}}{dx^{n-1}}\left(p_1 \frac{d^{n-1}}{dx^{n-1}}\right) + \ldots + p_n \qquad (4.1)$$

$n+1$ distinct spectra are required (Barcilon 1974b) for the determination of the $p_1, p_2, \ldots, p_n$. The $n+1$ required spectra $\{\lambda_i^{(k)}\}_{i=1}^{\infty}$, $k = 1, 2, \ldots, n+1$ are associated with the $n+1$ eigenvalue problems

$$\left.\begin{aligned} \mathscr{L}u &= \lambda u \\ \mathscr{A}^{(k)}u(0) &= 0 \\ \mathscr{B}u(1) &= 0 \end{aligned}\right\} k = 1, 2, \ldots, n+1, \qquad (4.2)$$

CONCLUSIÓN IMPORTANTE

EN LA SOLUCION FORMAL (TEORICA) DE LOS PROBLEMAS INVERSOS TIPO SCATTERING DE ONDAS APARECEN ECUACIONES INTEGRALES

LUEGO VEREMOS QUE ESTAS ECUACIONES INTEGRALES SON EL NUCLEO DE OTRA FAMILIA MUY IMPORTANTE DE PROBLEMAS INVERSOS: LOS LLAMADOS POR GEL'FAND DE GEOMETRÍA INTEGRAL

4.3

PROBLEMAS INVERSOS EXPLICITOS:

1. CLASIFICACION DE GEL'FAND

CLASIFICACIÓN DE GEL'FAND

II. TWO DIFFERENT SETTINGS OF INVERSE PROBLEMS

Inverse problems seem to have arisen in two entirely different settings. The first, and the one to command the most attention, is in the context of scattering of plane waves by a potential, as represented by Equation 1.1. These are the inverse problems in scattering. Their importance derives primarily from the Schrödinger problem in quantum mechanical potential scattering.[11-15] In the context of scattering in a dielectric or an elastic medium. Helmholtz's equation[44] is used, namely,

$$\Delta f(x) + n(x) f(x) = 0 \tag{1.2}$$

where Δ is the Laplacian and $n(x)$ represents the inhomogeneity of the medium (refractive index for an electromagnetic and impedance for an elastic medium) to be reconstructed. The solutions of these inverse scattering problems are usually expressed as integral equations of the second kind. It may be recalled that an integral equation of the second kind is of the form

$$f(x) = g(x) + \lambda \int_a^b K(x, x') f(x') \, dx' \tag{1.3}$$

where $f(x)$ is the unknown function to be determined, $g(x)$ is an inhomogeneous term which is the solution of the scattering free problem, i.e., with no potential, and $K(x,x')$ is the "kernel". If the integration limits are fixed, then the equation is of the Fredholm type. If the upper limit is the parameter x itself such that $K(x,x') = 0$, $x' > x$, then the kernel is said to be "triangular" and the equation is of the Volterra type. λ is called the "characteristic value" of the equation.[45] The eigenfunctions of the Schrödinger equation (1.1) admit of such representations. This will be discussed in Chapters 4, 5, and 10. All well-known inverse solutions of scattering problems such as the Lippmann-Schwinger equation,[46,47] the Gel'fand-Levitan equation on the semi-axis,[48,49] and the Marcenko equation[50,51] on the entire real axis are all Fredholm's integral equations of the second kind. The kernel of the Lippmann-Schwinger equation is the Green's propagator in free space. The Gel'fand-Levitan equation has the Povzner-Levitan[52,53] transform as the kernel, whereas Marcenko's integral equation contains the Levin transforms[54,55] of the Jost functions or the Jost solutions[12,13,55] in the integral operator. The integral equations of this type are usually characteristics of the boundary value problems. In quantum scattering, for example, it is the asymptotic data, i.e., data at infinity which are often used in the reconstruction of the potential.

The second type of inverse problems arises in a nonboundary value context and is concerned with the reconstruction of geometrical or configurational distributions of some parameter or parameters from a set of "integrated" data, i.e., data which contain simultaneous contributions from many different elements of the distribution. The word "distribution" simply means a physical distribution and not in the sense of distribution in the theory of generalized functions. Following a terminology introduced by Gel'fand,[56] these will be referred to as inverse problems in "integral geometry". An integral-geometric problem is, therefore, a problem in which a function is to be determined from its integrals over a family of hyperplanes or hypersurfaces, depending on the dimension of the space involved. The parameters to be reconstructed in these problems can be refractive index, density, temperature, emission/attenuation coefficients, velocity, so forth and so on. Problems of this nature include inversions of astronomical data, of line-integrated spectral intensities in spectroscopy, determination of plasma parameters, imaging of human anatomy by X-ray transform, and a host of others. Figure 1 illustrates a physical situation typical of the problems in this category. The information emanating from the source S reaches the detector D and in the transit is integrated by the intervening hyperplanes of the object. The hyperplanes reduce to lines and planes for two-dimensional and three-dimensional problems, respectively.

The well-known solution of this problem is credited to Johannes Radon.[57] Extensive discussions of the transform bearing his name are given in References 58 to 60. The well-known Abel's inversion[43] is a special case of Radon's inversion. The integral equations associated with the inverse problems of the second type are generally of the first kind, i.e., of the form

$$F(x) = \lambda \int_a^b K(x, x')\, f(x')\, dx'$$

F is the integrated data, that is, the integral of the function f(x) over a family of hyperplanes or surfaces. From these data, the distribution f(x) is to be reconstructed. The limits are almost always fixed and the equations are, therefore, of the Fredholm type.

NO SIEMPRE, EL MEJOR CONTRAEJEMPLO

ABEL

GEL'FAND J. M.

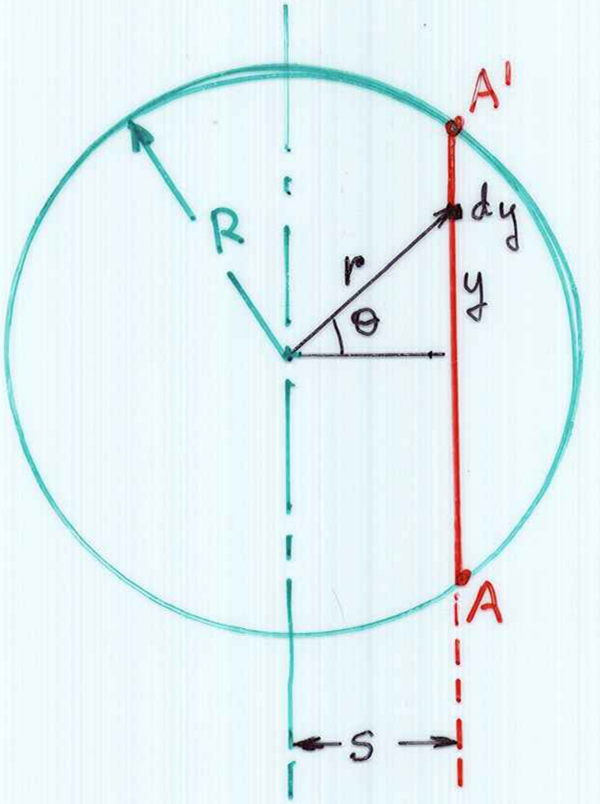
Integral geometry and its connection with representation theory

Ups. Mat. Nauk. 15, 1960

CLASIFICA LOS PROBLEMAS DE ESTE TIPO, COMO:

PROBLEMAS DE GEOMETRÍA INTEGRAL

PROBLEMAS EN LOS CUALES HAY QUE DETERMINAR UNA FUNCION $f(x)$ $x \in \mathbb{R}^n$ A PARTIR DE SUS INTEGRALES SOBRE SUPERFICIES.



$$y = \sqrt{r^2 - s^2}$$

$$r \geq s$$

$\rho(r)$ DENSIDAD REAL DE OBJETOS

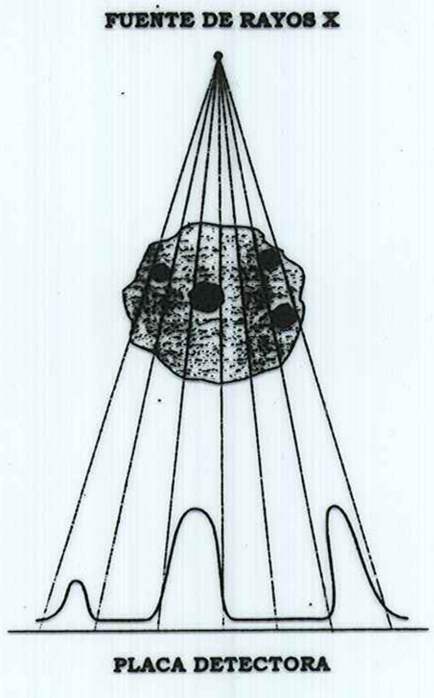
$\sigma(s)$ DENSIDAD SUPERFICIAL OBSERVADA

$$\sigma(s) = \int_A^{A'} \rho(r)\, dy$$

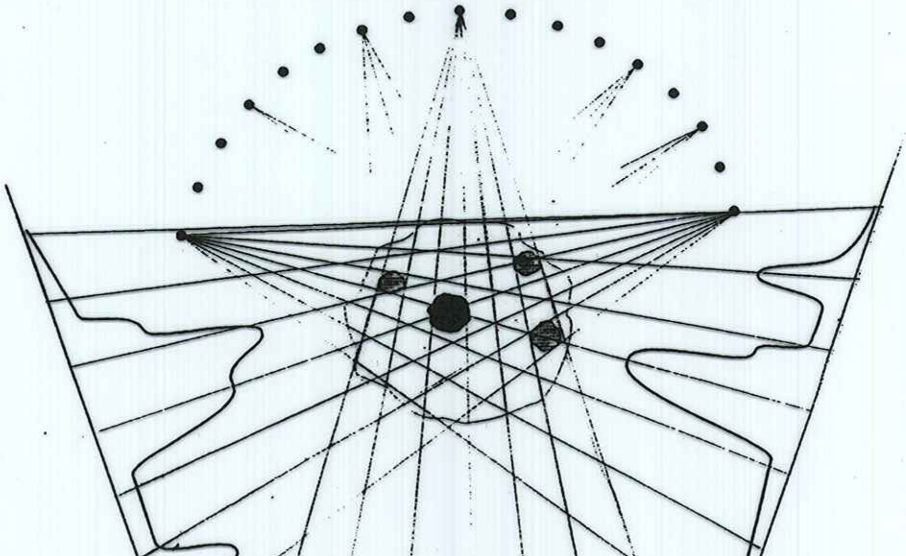
$$\sigma(s) = 2\int_s^R \rho(r) \frac{r\, dr}{\sqrt{r^2 - s^2}}$$

TRANSFORMADA DE ABEL

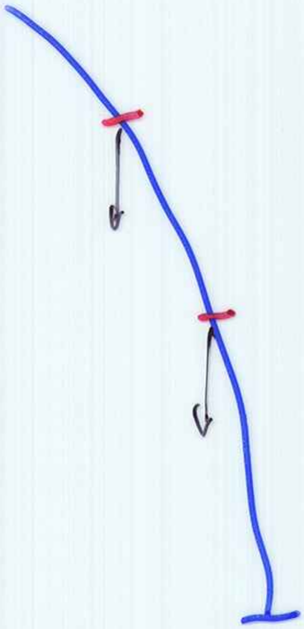
RADIOGRAFÍA



ESCÁNER



ABEL 1825



TAUTOCRONA

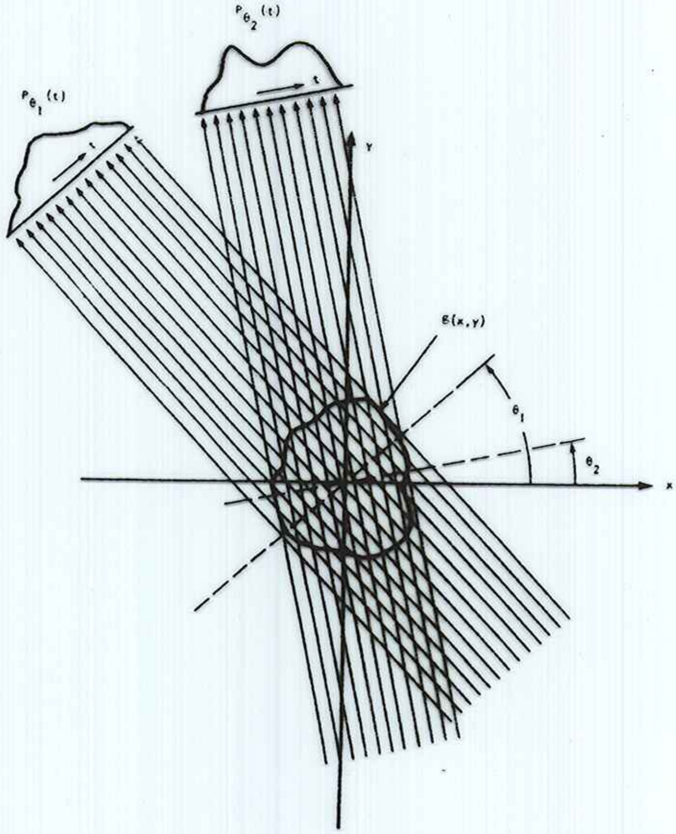
CURVA SOBRE LA CUAL

SI UN CUERPO PESADO DESLIZA, SIN ROZAMIENTO, BAJO LA ACCIÓN GRAVITATORIA

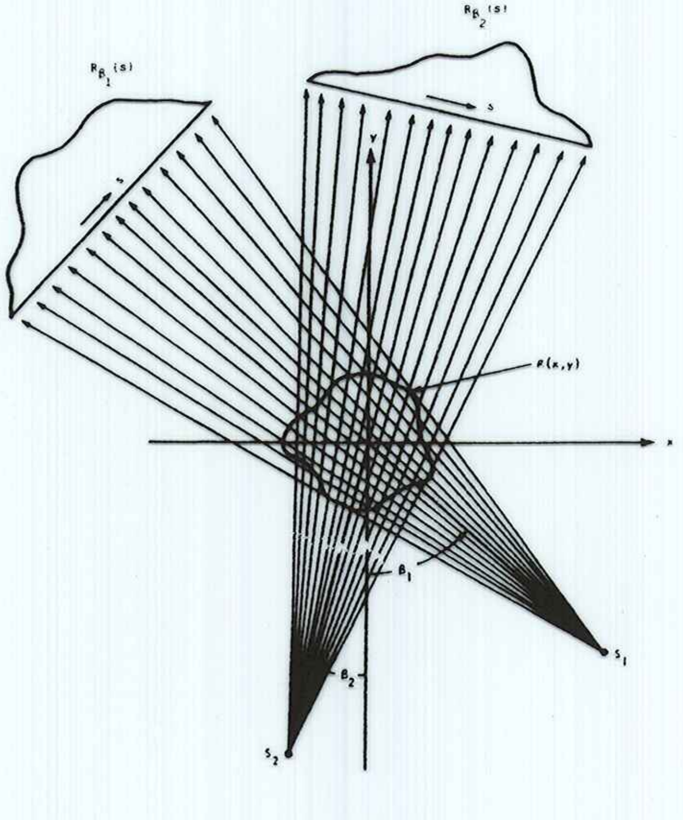
TARDA EN LLEGAR AL PUNTO INFERIOR EL MISMO TIEMPO, PARTA DE DONDE PARTA

$$g(y) = \int_y^{1/\infty} \frac{f(x)}{\sqrt{x-y}}\,dx$$

$$\Rightarrow \quad f(x) = -\frac{1}{\pi}\frac{d}{dx}\int_x^{1/\infty} \frac{g(y)}{\sqrt{y-x}}\,dy$$



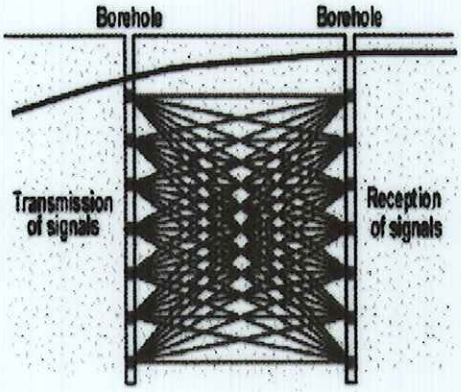
$P_{\theta_2}(t)$
$P_{\theta_1}(t)$
t
y
$f(x,y)$
θ_1
θ_2
x

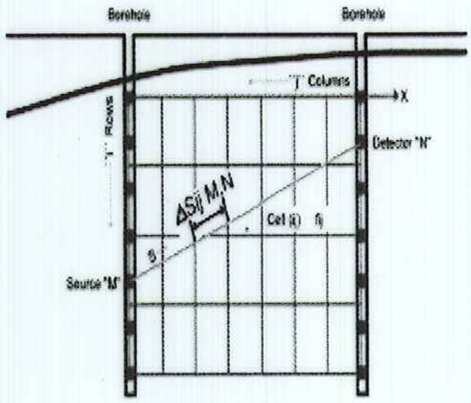


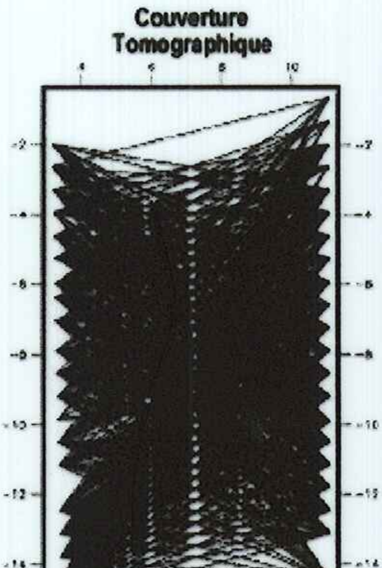
$R_{\beta_2}(s)$
$R_{\beta_1}(s)$
s
y
$f(x,y)$
x
β_1
β_2
S_1
S_2

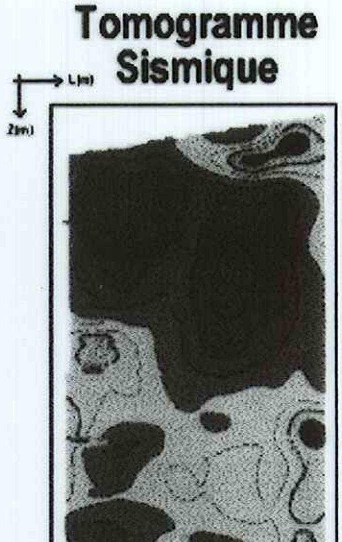
3.2.2.3. TOMOGRAFÍA SÍSMICA

La tomografía sísmica permite obtener una imagen de la distribución espacial de la velocidad de propagación de las ondas sísmicas en la sección del terreno que queda entre dos sondeos. Consiste en la generación de impulsos sísmicos mediante medios mecánicos desde el interior de sondeos y en la superficie del terreno, recibiendo las señales en geófonos instalados en múltiples puntos del interior de sondeos y/o de la superficie. Se estudia así la respuesta del terreno ante múltiples impulsos sísmicos desde multitud de puntos, midiendo los tiempos de llegada de las ondas. La sección de terreno afectada por el ensayo se divide en "celdillas" a las que se asigna un valor de velocidad de propagación sísmica para cada lugar.



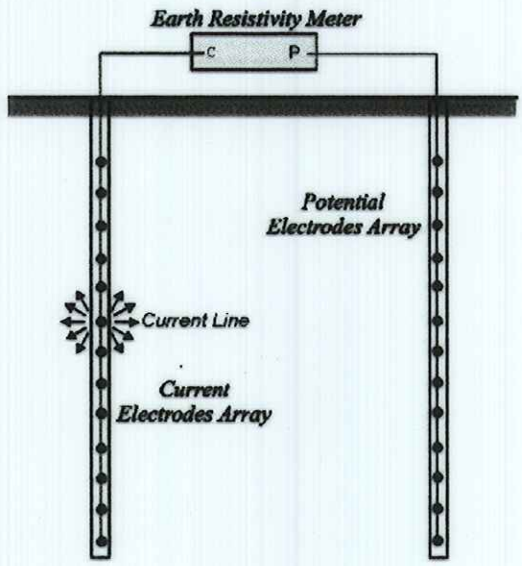


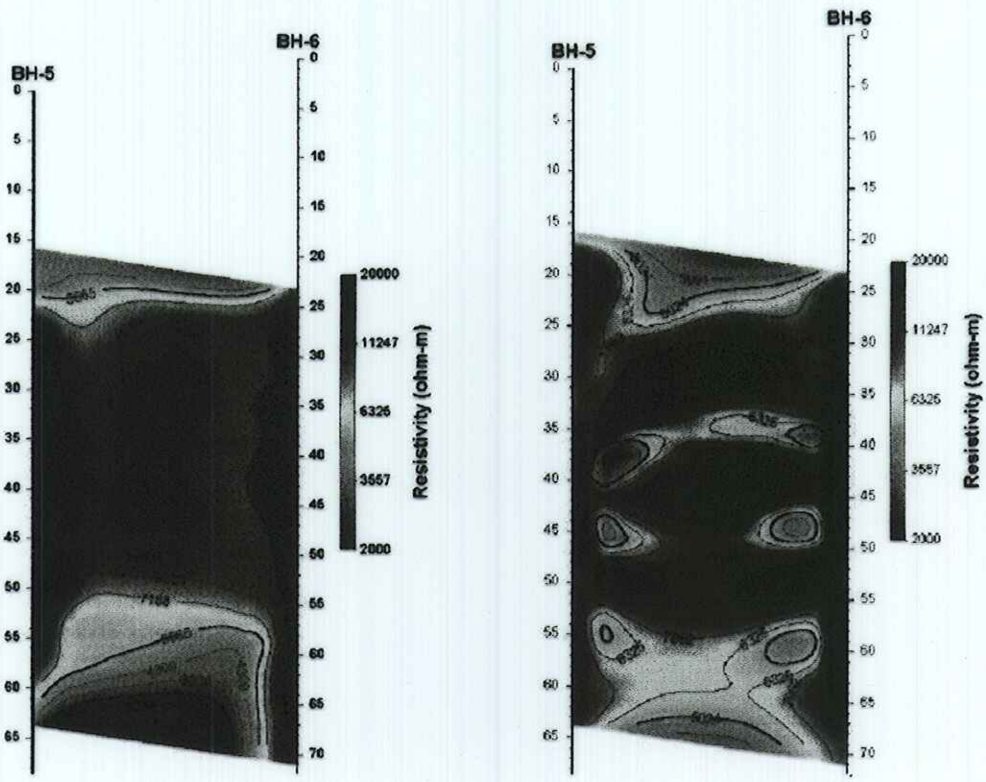




3.2.3. TOMOGRAFÍA ELÉCTRICA EN SONDEOS

Del mismo modo que con los métodos sísmicos, la tomografía eléctrica permite obtener imágenes de la distribución de la resistividad eléctrica del terreno existente entre dos sondeos. Requiere un cableado especial con para ser introducido en sondeos y que no se vea afectado por la presencia de agua. La transmisión de la electricidad al terreno se hace por el contacto entre los electrodos y el agua que, necesariamente, ha de rellenar el interior de la perforación.





MUY PRONTO DECADA de los 30

APARECEN, PRECISAMENTE EN ASTRONOMIA PROBLEMAS DEL TIPO:

ENCONTRAR $f(x,y,z)$ / $f(x,y)$

SI SE CONOCEN TODAS SUS INTEGRALES SOBRE LAS SUPERFICIES

$$ax + by + cz = 1 \quad \forall, a, b, c$$

OSEA, SI SE CONOCEN TODAS

$g(a,b,c)$

$$g(a,b,c) = \iiint dx\, dy\, dz\, \delta(ax+by+cz=1)\, f(x,y,z)$$

$$\rightarrow \iint d\cancel{x}\, dy\, f(x,y,z)\, \delta\left\{z = \frac{ax+by-1}{c}\right\}$$

SE TRATA DE UN PROBLEMA INVERSO DE GEOMETRÍA INTEGRAL (GEL'FAND)

VAMOS A DEDICAR A ELLOS PRACTICAMENTE TODO EL CURSO

4

P. I TEORICOS

P. I. APROXIMADOS

ACABAMOS DE VER QUE EN EL TRATAMIENTO MATEMATICO DE LOS P.I. RELACIONADOS CON EL SCATTERING DE ONDAS (CUANTICAS O NO)

APARECEN ECUACIONES INTEGRALES

EN PARTICULAR LA TRANSFORMADA DE RADON

PARA ESTAS ECUACIONES INTEGRALES SE DESARROLLA UNA METODOLOGIA NUMERICA QUE ALGUIEN PROPUSO APLICAR AL PROBLEMA DE SCATTERING DE ONDAS

SIN EMBARGO MUY POCAS VECES SE TRATAN NUMERICAMENTE ESTE TIPO DE PROBLEMAS

PERO, LAS TRANSFORMADAS INTEGRALES ACABARÁN SIENDO EL EJEMPLO PARADIGMATICO DE P.I.

EN PRINCIPIO TAL TIPO DE P.I. SE PLANTEÓ COMO UN PROBLEMA TEÓRICO: DESARROLLO DE METODOS DE INVERSION EXACTOS

The word exact refers to the fact that the inversion methods to be used are, in principle, not approximations. Assuming that waves are described by standard wave and constitutive equations, these exact inversion methods should give new equations(*) that would, in principle, allow the exact inference of a parameter (e.g. impedance) from a knowledge of the reflection amplitude.

For the sake of simplicity, we shall study only the one-dimensional inverse scattering problem, which can always be reduced to one of the equations or systems of equations.

(*) GENERALMENTE ECUACIONES INTEGRALES

T

EN LA SOLUCIÓN FORMAL (TEÓRICA) DE ESTE PROBLEMA, APARECEN ECUACIONES INTEGRALES DEL TIPO DE LAS QUE VEREMOS CUANDO ESTUDIEMOS LA OTRA GRAN FAMILIA DE P.I.

AQUELLOS DE GEOMETRÍA INTEGRAL
"RECONSTITUCIÓN DE ESTRUCTURAS A PARTIR DE INTEGRALES SOBRE LINEAS O PLANOS":
TOMOGRAFÍA COMPUTARIZADA

It has been however suggested recently that the well-known techniques used in computerized tomography for medical uses would be useful here for inversing the Radon transform. This is not our personal belief. These methods (iterative methods, Fourier or convolution methods) are convenient to treat very large sets of data with strong smoothness a priori assumptions. Here the physics is different and we make a point of this difference.

COMO YA HEMOS DICHO, PRACTICAMENTE TODOS LOS TRABAJOS SOBRE ESTE TIPO DE P.I. SON PURAMENTE TEÓRICOS.

al final de la exposición

P.I. TEORICOS

DATOS MATEMATICAMENTE EXACTOS
SOLUCIÓN MATEMATICAMENTE EXACTA

P.I. APROXIMADOS

DATOS MATEMATICAMENTE APROXIMADOS
SOLUCIÓN APROXIMADA (generalizada bajo el punto de vista matemático)

PERO: EL ESTUDIAR LA SOLUCIÓN TEORICA DE UN P.I. A PARTIR DE DATOS TEORICAMENTE EXACTOS, NOS PUEDE PERMITIR ENCONTRAR <u>METODOS NUMÉRICOS OPTIMOS PARA RESOLVER APROXIMADAMENTE UN PROBLEMA EN EL QUE LOS DATOS SEAN APROXIMADOS</u>

CUIDADO

APLICAR NUMERICAMENTE
EL TRATAMIENTO MATEMÁTICO
DE UNA SOLUCIÓN EXACTA

NOS LLEVA GENERALMENTE
A RESULTADOS CATASTROFICOS

SIEMPRE APARECEN DERIVADAS
DE LOS DATOS (VER ANTERIOR-
MENTE)

Y DERIVAR NUMERICAMENTE
DATOS EXPERIMENTALES O
OBSERVACIONALES, ES UN
PROCESO MUY DELICADO.

LA IMPRECISIÓN EN LAS
DERIVADAS NUMÉRICAS, SE
AMPLIFICAN EXTRAORDINARIAMENTE
EN EL RESTO DEL PROCESO
DE INVERSIÓN.

HAY QUE TRATAR DE EVITAR
MUCHAS DE LAS OPERACIONES QUE
APARECEN EN LAS SOLUCIONES
TEÓRICAS.